PHYS 3344
Fall 2021
TE Coan
Due: 24 Sep '21 6:00 pm

Homework 3

1. Even though the total force on a system of N particles ($\sum_\alpha \mathbf{F}_\alpha^{\text{ext}} + \sum_\alpha \sum_{\beta \neq \alpha} \mathbf{F}_{\alpha\beta}$) is zero, the net torque may not be zero. Show that the net torque has the same value in any coordinate system. **Hint:** Calculate the net torque on a system of particles in any two fixed coordinate systems whose origins differ by a fixed vector amount **a**.

2. The speed v of some strangely shaped projectile of mass m varies with distance x as $v(x) = \alpha x^{-n}$. Assume that at $t = 0$, $v(x = 0) = 0$.

a) What force $F(x)$ is responsible for this state of affairs?

b) What is $x(t)$?

c) Finally, what is $F(t)$?

3. A DART train moves along the tracks at a constant speed u. A woman on the train throws a ball, for reasons mysterious to this day, of mass m straight ahead with a speed v with respect to *herself*.

a) What is the kinetic energy gain $\Delta\text{KE}_{\text{train}}$ of the ball as measured by a person on the train?

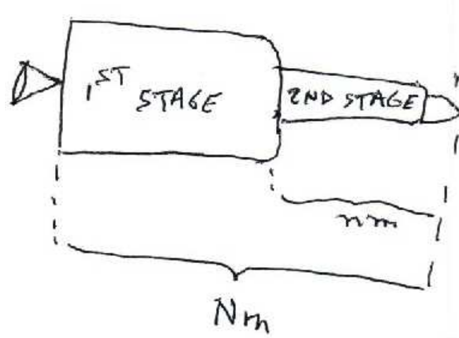
b) Compute the same quantity as in (a) but for a person standing by the *track*?

c) How much work W_{Woman} is done by the woman throwing the ball?

d) How much Work W_{Train} is done by the train?

4. A chain of lenght b and mass ρb is suspended from one end at a height b above a table so that the free end barely touches the tabletop. At time $t = 0$, the fixed end of the chain is released. Find the force that the tabletop exerts on the chain after the original fixed end has fallen a vertical distance x.

5. This is a somewhat involved problem, but fully within your ability to solve. It is designed to show you the advantages of a multi-stage rocket compared to a single-stage rocket. Since the commercialization of space by Mr. Amazon and Mr. Tesla seems approaching, it seems sensible to know something about it. Suppose that the payload (e.g., a capsule carrying some hyper-affluent people, or maybe something more interesting) has a mass m and is mounted on a two-stage rocket (see the fig.). The *total* mass –both rocket stages fully fueled, plus the payload – is Nm. The mass of the second-stage plus the payload, after first-stage burnout and separation, is nm. In each case, the ratio of the burnout mass (casing) to initial mass (casing plus fuel) is r, and the exhaust speed of the gas with respect to either stage is v_{ex}.



a) Show that the velocity v_1 gained after the first stage burn, starting from rest (and ignoring gravity), is given by

$$v_1 = v_{ex} \ln \left[\frac{N}{rN + n(1-r)} \right]$$

b) Obtain a corresponding expression for the additional velocity v_2 gained from the second-stage burn. Box your answer.

c) Adding v_1 and v_2, you have the payload velocity v in terms of N, n, and r. Taking N and r as constants, find the value of n that maximizes v. **HINT:** This will turn out to be a simple function of N. The algebra can seem messy so I recommend NOT combining the natural logs when you go to maximize v. You can also use Mathematica. Box your answer.

d) Show that the condition for v to be a maximum corresponds to having equal velocity gains for each of the two stages. Find the maximum value of v. Comment on why this expression makes sense. Box your answer.

6. A particle of mass m at the end of a light string wraps itself about a fixed vertical cylinder of radius a (see the figure below). All the motion is in the horizontal plane (ignore gravity). The angular velocity of the string is ω_0 when the distance from the particle to the point of contact of the string and cylinder is b. Find the angular velocity and tension T in the string after the the string has turned through an additional angle θ.