PHYS 5382
Fall 2020
TE Coan
Due: 4 Sep '20 6:00 pm

Homework 1

1. You should know some basic facts about our Universe. You live in it. **Box your answers.**

a) 1 pt. What is the radius R_a of a typical atom? Use SI units (i.e., your answer should be in meters!). Yes, I have asked this question in my other courses as well.

b) 1 pt. What is the radius R_N of a typical atomic *nucleus*? Use SI units (i.e., your answer should be in meters!).

c) 1 pt. Estimate the number of atoms N_B in your body. Notice the word "estimate." Box your answer appropriately.

d) 1 pt. What is the value of Planck's constant divided by 2π, $\hbar$? Use SI units. Your answer should in the form $\boxed{\hbar = \text{blah-blah-blah}}$. **Box the entire answer, not just the number.**

e) 1 pt. What is the value of the electronic charge e? Include the appropriate sign and SI unit. Box that answer. Really.

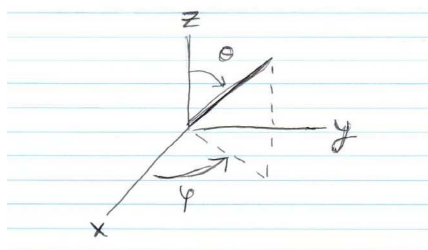
2. Later on, we shall see that the state of a spin-$\frac{1}{2}$ particle that is spin up along an axis whose direction is specified by the unit vector $\mathbf{n}$

$$\mathbf{n} = \sin\theta\cos\phi\,\mathbf{i} + \sin\theta\sin\phi\,\mathbf{j} + \cos\theta\,\mathbf{k}\,,$$

with θ and ϕ shown in the figure below, is given by

$$|+\mathbf{n}\rangle = \cos\frac{\theta}{2}\,|+\mathbf{z}\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|-\mathbf{z}\rangle\ .$$

(a) Verify that the state $|+\mathbf{n}\rangle$ reduces to the states $|+\mathbf{x}\rangle$ and $|+\mathbf{y}\rangle$ given in chapter 1 of Townsend for the appropriate choices of θ and ϕ.



(b) Suppose, suppose a measurement of S_z is carried out on a particle in the state $|+\mathbf{n}\rangle$. What is the probability that the measurement yields (i) $\hbar/2$, (ii) $-\hbar/2$?

(c) Determine the uncertainty ΔS_z in your measurement.

3a. This is a continuation of problem 2. What is the (probability) amplitude A to find a particle that is in the state $|+\mathbf{n}\rangle$ with $S_y = \hbar/2$? What is the probability P? Check your result by evaluating the probability for an appropriate choice of the angles θ and ϕ.

(b) Determine the amplitude A to find a particle in the state $|+\mathbf{y}\rangle$ with $S_n = \hbar/2$.

4. The state $|\Psi\rangle$ described by

$$|\Psi\rangle = \frac{1}{2}\,|+\mathbf{z}\rangle + \frac{i\sqrt{3}}{2}\,|-\mathbf{z}\rangle \ ,$$

is a state with $S_n = \hbar/2$ along a particular axis $\mathbf{n}$. Compare the state $|\Psi\rangle$ with the state $|+\mathbf{n}\rangle$ in problem 2 above to find $\mathbf{n}$. Determine $\langle S_x\rangle, \langle S_y\rangle$, and $\langle S_z\rangle$. *Note:* $\langle S_z\rangle$ and $\langle S_y\rangle$ for this state are given in Townsend Example 1.2 and 1.3, respectively.

5. Calculate $\langle S_x\rangle, \langle S_y\rangle$ and $\langle S_z\rangle$ for the state

$$|\Psi\rangle = -\frac{i}{2}\,|+\mathbf{z}\rangle + \frac{\sqrt{3}}{2}\,|-\mathbf{z}\rangle \ .$$

Compare your results with those from problem 4. What can you conclude about these two states?

6. The state $|\Psi\rangle$

$$|\Psi\rangle = \frac{1}{2}\,|+\mathbf{z}\rangle + \frac{\sqrt{3}}{2}\,|-\mathbf{z}\rangle \ .$$

is similar to the one in problem 4. It is “missing” the i. By comparing the state with the state $|+\mathbf{n}\rangle$ in problem 2, determine along which direction $\mathbf{n}$ the state is spin up. Calculate $\langle S_x\rangle$, $\langle S_y\rangle$ and $\langle S_z\rangle$ for the state $|\Psi\rangle$. Compare your results with those of Problem 4.