Q1

a) $\boxed{R_a \simeq 1 \text{ Angstrom} = 10^{-10} m}$

b) $\boxed{R_N \simeq 1 \text{ fm} = 10^{-15} m}$

c) $M \simeq 90$ kg     Avogadro's # $N_0 = 6.023 \times 10^{23} \frac{\text{ATOMS}}{\text{MOLE}}$

ASSUME $^{12}C$ IS DOMINANT ATOM

$$\text{ATOMS} \simeq \frac{90 \times 10^3 \text{ gm}}{12 \text{ gm/mole}} * N_0$$

$\boxed{N_B \simeq 5 \times 10^{27}}$

d) $\boxed{\hbar = 1.05 \times 10^{-34} \text{ J-s}}$

e) $\boxed{e = 1.6 \times 10^{-19} \text{ C}}$

Q2

(a) MAKE $\hat{n} = \hat{x}$ : $\theta = \pi/2$ , $\varphi = 0$

$$\Rightarrow |+n\rangle = |+x\rangle = \cos\pi/4\,|+z\rangle + \sin\pi/4\,|-z\rangle$$

$$|+x\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$$ ✓

$|+y\rangle$: $\hat{n} = \hat{y}$ FOR ~~cos~~ $\theta = \pi/2$ , $\varphi = \pi/2$

$$|+n\rangle = |+y\rangle = \cos\pi/4\,|+z\rangle + e^{i\pi/2}\sin\pi/4\,|-z\rangle$$

$$|+y\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{i}{\sqrt{2}}|-z\rangle$$ ✓

(b) FIND $|C_{+z}|^2$ & $|C_{-z}|^2$

PROBABILITY OF MEASURING $+\hbar/2$

(i) $\boxed{P(+\hbar/2) = (\cos\theta/2)^2}$

ii $P(-\hbar/2) = |e^{i\varphi}\sin\theta/2|^2 = \sin^2\theta/2$

$\boxed{P(-\hbar/2) = \sin^2\theta/2}$

NOTE: $P(+\hbar/2) + P(-\hbar/2) = \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} = 1$

NO SURPRISE HERE.

(c) $(\Delta S_z)^2 = \langle S_z^2 \rangle - \langle S_z \rangle^2$

$$= \cos^2\frac{\theta}{4}\,\frac{\hbar^2}{4} + \sin^2\frac{\theta}{2}\left(-\hbar/2\right)^2$$

$$-\left[\cos^2\frac{\theta}{2}\,\hbar/2 + \sin^2\frac{\theta}{2}\left(-\hbar/2\right)\right]^2$$

$$= \hbar^2/4 - (\hbar/2)^2\left[\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right]^2$$

BUT $\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = \cos\theta$

so,

$$\Delta S_z^2 = \hbar^2/4 - \frac{\hbar^2}{4}\cos^2\theta$$

$$\Delta S_z = \frac{\hbar}{2}\left[1 - \cos^2\theta\right]$$

$$\boxed{\Delta S_z = \frac{\hbar}{2}\sin\theta}$$

Q3

FIND $\langle +y | +n \rangle$

(a)

$$|+y\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{i}{\sqrt{2}}|-z\rangle$$

$$\langle +y| = \frac{1}{\sqrt{2}}|+z\rangle - \frac{i}{\sqrt{2}}\langle -z|$$

So,

$$\langle +y | +n\rangle = \frac{1}{\sqrt{2}}\cos\frac{\theta}{2} \langle +z|-z\rangle$$

$$- \frac{i}{\sqrt{2}} e^{i\varphi} \sin\frac{\theta}{2} \langle -z|\cancel{+}z\rangle$$

$$= \frac{1}{\sqrt{2}}\cos\theta/2 + \frac{1}{\sqrt{2}} e^{i(\varphi - \pi/2)} \sin\theta/2$$

$$\text{w/ } i = e^{i\pi/2}$$

$$|\langle +y| +n\rangle|^2 = \left(\frac{1}{\sqrt{2}}\cos\frac{\theta}{2} + \frac{1}{\sqrt{2}} e^{i(\varphi - \frac{\pi}{2})}\sin\frac{\theta}{2}\right) * CC$$

$$= \frac{1}{2}\left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right) + \frac{1}{2}\cos\frac{\theta}{2}\sin\frac{\theta}{2}$$

$$* \left(e^{i(\varphi-\pi/2)} + e^{-i(\varphi-\pi/2)}\right)$$

$$= \frac{1}{2}\left(1 + \sin\theta\cos(\varphi - \pi/2)\right)$$

$$\boxed{|\langle +y|+n\rangle|^2 = \frac{1}{2}\left(1 + \sin\theta \sin\varphi\right)}$$

(b) $\langle +x | +y \rangle = \langle +y | x \rangle^*$

$$= \frac{1}{\sqrt{2}} \cos\frac{\theta}{2} + \frac{1}{\sqrt{2}} e^{-i(\varphi - \pi/2)} \sin\theta/2$$

PROBABILITY $= |\langle +x | +y \rangle|^2$

$$= \langle x | +y \rangle \langle +x | +y \rangle^*$$

$$= \langle +y | x \rangle^* \langle +y | +x \rangle$$

$$\boxed{\text{PROB} = |\langle +y | +x \rangle|^2 = \frac{1}{2}(1 + \sin\theta \sin\varphi)}$$

CONSISTENT w/ (a)

Q4.

COMPARE $|\psi\rangle$ w/ $|+n\rangle$

$$\Rightarrow \theta = 2\pi/3$$
$$\varphi = \pi/2$$

STATE IS SPIN UP IN DIRECTION @ 120° TO Z-AXIS IN Y-Z PLANE

$$PA(S_x = +\hbar/2) = \langle +x|+\psi\rangle$$

$$= \left(\tfrac{1}{\sqrt{2}}\langle +z| + \tfrac{1}{\sqrt{2}}\langle -z|\right)\left(\tfrac{1}{2}|+z\rangle + i\tfrac{\sqrt{3}}{2}|-z\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{1}{2} + \frac{1}{\sqrt{2}}\cdot\frac{i\sqrt{3}}{2}$$

$$\boxed{\langle x|\psi\rangle = \frac{1}{2\sqrt{2}}(1 + i\sqrt{3})}$$

$$\text{PROBABILITY}(S_x = \hbar/2) = \left|\frac{1}{2\sqrt{2}}(1+i\sqrt{3})\right|^2$$

$$\boxed{\text{PROB} = \tfrac{1}{2}}$$

$$\boxed{\langle S_x\rangle = \tfrac{1}{2}(+\hbar/2) + \tfrac{1}{2}(-\hbar/2) = 0}$$

IN TEXT, EX 1.3 $\langle S_y\rangle = \frac{\sqrt{3}}{4}\hbar$

EX 1.2 $\langle S_z\rangle = -\hbar/4$

**Q5:**

**RAPID WAY**

$$|\psi\rangle = -\frac{i}{2}|+z\rangle + \frac{\sqrt{3}}{2}|-z\rangle$$

$$= -i\left(\frac{1}{2}|+z\rangle + \frac{i\sqrt{3}}{2}|-z\rangle\right)$$

$$= e^{-i\pi/2}\left(\frac{1}{2}|+z\rangle + \frac{i\sqrt{3}}{2}|-z\rangle\right)$$

∴ OVERALL PHASE DIFFERENCE FROM PROB 4. PROBABILITIES WILL NOT CHANGE, e.g., $|\langle +x|\psi\rangle|^2$ WON'T CHANGE.

$$\Rightarrow \boxed{\langle S_x\rangle, \langle S_y\rangle \;\&\; \langle S_z\rangle \text{ SAME AS PREVIOUS PROBLEM}}$$

Q6

COMPARE $|\psi\rangle$ w/ $|+n\rangle$

$\Rightarrow \theta = 2\pi/3$ & $\phi = 0$.

$|\psi\rangle$ IS SPIN UP @ $120°$ to $z$-axis IN $x-z$ PLANE

COMPARE w/ PROBLEM 4: $\langle S_x\rangle \leftrightarrow \langle S_y\rangle$

SO, PROB OF FINDING STATE $|\psi\rangle$ w/ $S_x = +\hbar/2$ IS:

$$\langle +x|\psi\rangle = \left(\tfrac{1}{\sqrt{2}}\langle +z| + \tfrac{1}{\sqrt{2}}\langle -z|\right)\left(\tfrac{1}{2}|+z\rangle + \tfrac{\sqrt{3}}{2}|-z\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{1}{2} + \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{2\sqrt{2}}\left(1+\sqrt{3}\right)$$

SO, $\boxed{|\langle +x|\psi\rangle|^2 = \frac{1}{2} + \sqrt{3}/4}$

& $\boxed{|\langle -x|\psi\rangle|^2 = 1 - |\langle +x|\psi\rangle|^2 = \frac{1}{2} - \frac{\sqrt{3}}{4}}$

$$\langle S_x\rangle = \left(\frac{1}{2} + \frac{\sqrt{3}}{4}\right)\frac{\hbar}{2} + \left(\frac{1}{2} - \frac{\sqrt{3}}{4}\right)\left(-\hbar/2\right)$$

$$\boxed{\langle S_x\rangle = \frac{\sqrt{3}}{4}\hbar}$$

SAME RESULT FOR $\langle S_y\rangle$ IN TEXT EXAMPLE 1.3