PHYS 5382
Fall 2020
TE Coan
Due: 11 Sep '20 6:00 pm

Homework 2

1. Verify that $\Delta S_y = \sqrt{\langle S_y^2 \rangle - \langle S_y \rangle^2} = 0$ for the state $|+\mathbf{y}\rangle$.

2. Show explicitly that neither the probability of obtaining the result a_i nor the expectation value $\langle A \rangle$ is affected by $|\Psi\rangle \to e^{i\phi}\,|\Psi\rangle$, that is, by an overall phase change for the state $|\Psi\rangle$. This is an important general result for quantum mechanics.

3a. With some state $|\Psi\rangle$ of a spin-$\frac{1}{2}$ particle you measure S_z and find that 90% of the time you measure $S_z = \hbar/2$. Furthermore, it is known there is a 20% chance of obtaining $S_y = \hbar/2$ if you measure S_y for the same state. Determine the spin state of this particle as completely as possible from this information. Hint: You need to determine the relative *phases* between the kets $|+\mathbf{z}\rangle$ and $|-\mathbf{z}\rangle$ when you write $|\Psi\rangle$. See Townsend pp.18-19.

(b.) What is the probability of obtaining $S_x = \hbar/2$ if a measurement of S_x is made?

4. Use Dirac notation (the properties of kets, bras and inner products) directly without explicitly using matrix representations to show that the projection operator $\hat{P}_+$ is Hermitian. **Hint:** Place the operator $\hat{P}_+$ between a bra and a ket, and then use what you know about complex conjugation and the "daggering" of an operator to demonstrate the desired result. A few lines should be enough.

(b.) Use the fact that $\hat{P}_+^2 = \hat{P}_+$ to demonstrate that the eigenvalues of the projection operator are 1 and 0. **Hint:** Let $\hat{P}_+^2 = \hat{P}_+$ act on a ket and remember the general form of an eigenvalue equation.

5. Determine the column vectors representing the states $|+\mathbf{x}\rangle$ and $|-\mathbf{x}\rangle$ using the states $|+\mathbf{y}\rangle$ and $|-\mathbf{y}\rangle$ as a basis.