PHYS 5382
Fall 2020
TE Coan
Due: 25 Sep '20 6:00 pm

Homework 4

0. Box your **entire** final answer (not just its right hand side!) for each problem or lose points.

1. SKIP Show that the operator $\hat{C}$ defined through $[\hat{A}, \hat{B}] = i\hat{C}$ is hermitian if the operators $\hat{A}$ and $\hat{B}$ are. The relation $(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger \hat{A}^\dagger$ may be useful.

2. We have used many times a Taylor series expansion for expressions that contain an operator in the exponent. For example, we did this when investigating angular momentum when we treated the operator just like a variable from calculus. However, operators in exponents must be handled with care. For example, show that

$$e^{\hat{A}+\hat{B}} \neq e^{\hat{A}} e^{\hat{B}}$$

unless the operators $\hat{A}$ and $\hat{B}$ commute.

3. A spin-1 particle is in the state

$$|\Psi\rangle \underset{\mathrm{S_z basis}}{\longrightarrow} \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2i \\ i \end{pmatrix}.$$

(a.) What is $\langle S_y \rangle$? Note that the matrix operator for S_y and J_y we saw in lecture are the same in this case since they both refer to spin.

(b.) What is the probability that a measurement of S_y will yield the value of $-\hbar$ for this state?

4. A bit of drill. Use the matrix representation of the spin-$\frac{1}{2}$ angular momentum operators $\hat{S}_x, \hat{S}_y$ and $\hat{S}_z$ in the S_z basis to verify explicitly through matrix multiplication that

$$[\hat{S}_x, \hat{S}_y] = i\hbar \hat{S}_z.$$

By the way, commutation relations are independent of the basis they are expressed in, as long as all the operators are expressed in the same basis.

5. A spin-$\frac{3}{2}$ particle is in the state

$$|\Psi\rangle \underset{\mathrm{S_z\,basis}}{\longrightarrow} N \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4i \end{pmatrix}$$

(a.) Determine N so that $|\Psi\rangle$ is appropriately normalized.

(b.) What is $\langle S_x \rangle$ for this state? The matrix representation of $\hat{S}_x$ can be found in Townsend in Example 3.4.

(c.) What is the probability P that measuring S_x for this state will yield a value of $-\hbar/2$? You can use the following representations of the $\hat{S}_x$ eigenstates $|s, m\rangle_x$ written in the S_z basis to help you.

$$\left|\frac{3}{2}, \frac{3}{2}\right\rangle_x \underset{\mathrm{S_z\,basis}}{\longrightarrow} \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ \sqrt{3} \\ \sqrt{3} \\ 1 \end{pmatrix} \qquad \left|\frac{3}{2}, \frac{1}{2}\right\rangle_x \underset{\mathrm{S_z\,basis}}{\longrightarrow} \frac{1}{2\sqrt{2}} \begin{pmatrix} \sqrt{3} \\ 1 \\ -1 \\ -\sqrt{3} \end{pmatrix}$$

$$\left|\frac{3}{2}, -\frac{1}{2}\right\rangle_x \underset{\mathrm{S_z\,basis}}{\longrightarrow} \frac{1}{2\sqrt{2}} \begin{pmatrix} \sqrt{3} \\ -1 \\ -1 \\ \sqrt{3} \end{pmatrix} \qquad \left|\frac{3}{2}, -\frac{3}{2}\right\rangle_x \underset{\mathrm{S_z\,basis}}{\longrightarrow} \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ -\sqrt{3} \\ \sqrt{3} \\ -1 \end{pmatrix}$$