

# General Relativity and Cosmology

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The End of Absolute Space

Cosmological Principle

Black Holes

CBMR and Big Bang

# The End of Absolute Space (AS)

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- Special Relativity (SR) abolished AS only for the special case of inertial frames
- Inertial frames (IF) are reference frames that can be moving uniformly or can be stationary. Each IF has the ability to say that it is at rest and all the other frames are in motion.
- But what about frames which are being accelerated? Or rotated?

- Think of an elevator that is cut loose and is accelerated toward the earth. A ball dropped by a person in the elevator would, from their perspective, stay exactly where they let go of it. From our perspective the ball would be falling toward the earth. Who is right?
- Now think of that same elevator out in space being accelerated with a rope. From the outside we know that they are not in a gravitational field, but being accelerated by something else. Inside the elevator however, all the physics remain the same as when they were in the gravitational field. Who is right?

- Now allow a beam of light to enter the accelerated elevator from one side and pass to the other side. In free space the light appears to travel in a straight line to those inside the elevator, while the outside observer sees the light as having fallen. Is this effect the same in a gravitational field? Who is right?
- In General Relativity (GR) both the internal and external observers are correct in all of the above scenarios.

# Basic Principles of GR

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- Equivalence Principle: Inertial Mass and Gravitational Mass are the same thing.
- Light always follows geodesics.  
(Geodesics are the shortest path between any two points on any surface)
- Locally space-time is flat.
- Universally space-time is curved by massive objects.

# Cosmological Principle

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- GR severed the last tie with the idea that Earth was unique.
- Combining Hubble's Law (velocity is directly proportional to the object's distance from us) and the concept of isotropy Einstein came up with the Cosmological Principle (CP).
- CP : No one place is preferred over any other (I.e. nothing can be considered the center of the expansion.)

# Predictions

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- Precession of Mercury
- Bending of Light
- Gravitational Lensing
- Black Holes
- Curvature of Space is related to the total density of the universe
- Big Bang

# Precession of Mercury

- The precession of Mercury's orbit (and that of all the other planets) could not be totally accounted for by Newtonian mechanics
- SR and GR corrections to the equations govern the precession of planets account for the difference between Newtonian calculations and observed values.

Body	(seconds of arc per century)	
	Newton-observation	GR
Mercury	43.11+0.45	43.03
Venus	8.4+4.8	8.6
Earth	5.0+1.2	3.8
Icarus	9.8+0.8	10.3

# Bending of Light

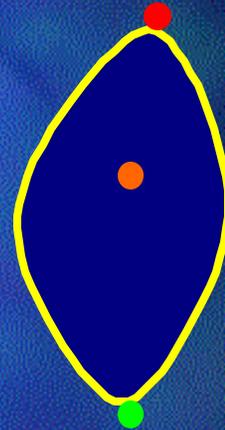
- Light has energy. Energy can be related to mass by  $E=mc^2$ . All mass is equivalent. Therefore light acts just like a massive particle in a gravitational field.
- First test of GR. Newtonian mechanics also predicted bending of light but was off by a factor of two when the test was performed. GR was right on the money. (1919)

# Gravitational Lensing

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- Distance in astronomical terms is only relative to some known distance and is usually an approximation. Gravitational Lensing provides an absolute way to measure distances.
- Ingredients: A massive object whose distance you want to measure and a star somewhere behind it.

- Case I: the source is exactly behind the lens => Einstein Ring
- Case II: the source is not co-linear with us and the lens => light will arrive at different times and from different angles.



$$D = \Delta t / |\mathbf{q}_1 - \mathbf{q}_2|$$

# Metrics

- Metrics are the measure of proper distance in a space.

- Euclidean Metric :  $dl^2 = dx^2 + dy^2 + dz^2$

- Flat space-time metric:

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

- Schwarzschild Metric:

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 (d\mathbf{q}^2 + \sin^2 \mathbf{q} d\mathbf{f}^2)$$

# Black Holes

- Schwarzschild radius is the radius at which the Schwarzschild metric becomes crazy.  $R = 2M$  ( $R = 2GM/c^2$ )
- For  $R > 2M$  everything works normally.
- For  $R < 2M$ : radius becomes a time like coordinate, mathematical white holes form, infinite in fall, etc...

# CBMR and Big Bang

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- Cosmic Background Microwave Radiation indicates that at some point in the history of the universe radiation dominated matter and the universe was in thermal equilibrium.
- In order for this and other data (like the expansion of the universe) to fit together with GR the Big Bang theory came about.

# Big Problems on the Horizon

- Recently observations have been made that the universe is flat. Since curvature is related to total density this implies that the universe has the critical density needed to make it flat.
- This density cannot be accounted for by present observations of matter and radiation densities. Even if the dark matter that is needed to make small-scale structure to work is included, the density is still not enough. Dark Energy is postulated to solve this problem.
- Some other recent data indicates that the universe is accelerating, not decelerating as it should after the Big Bang. Some solutions include a new inflationary period and dark energy.