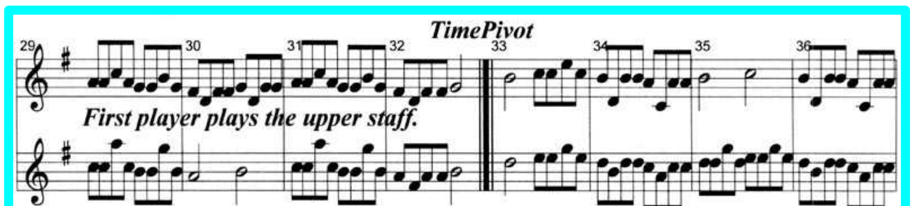
Introduction:

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}$$
$$= \bar{\psi}_i(i\gamma^\mu\partial_\mu - m)\psi_i - gG^a_\mu\bar{\psi}_i\gamma^\mu T^a_{ij}\psi_j - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu},$$



Mozart: Inverted retrograde canon in G

Patterns, Symmetry (obvious & hidden), interpretation

Notes ⇒ themes ⇒ Melody/Harmony ⇒ interaction/counterpoint ⇒ structure

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}$$
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Mozart: Inverted retrograde canon in G

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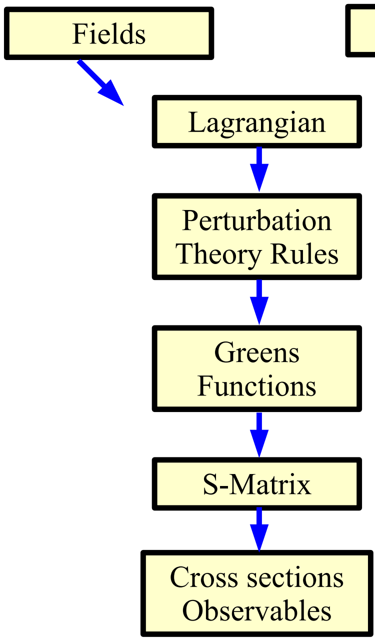
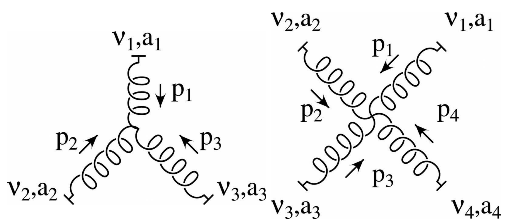
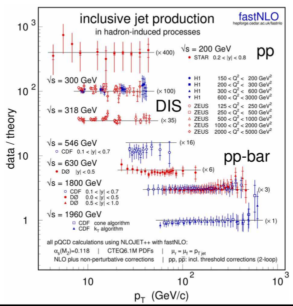
Notes ⇒ themes ⇒ Melody/Harmony ⇒ interaction/counterpoint ⇒ structure

The Game

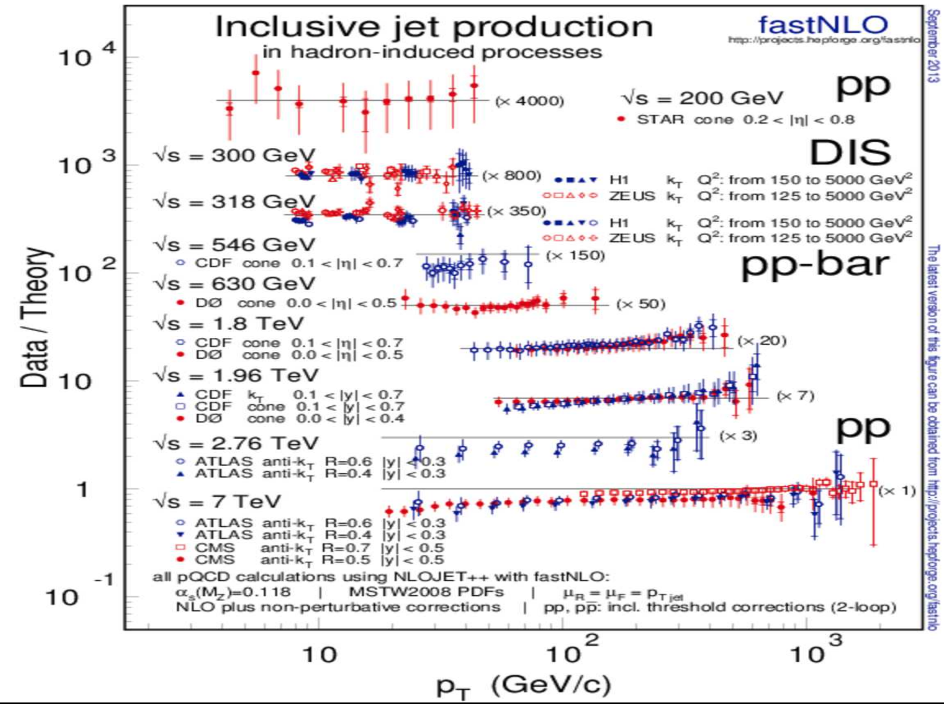


$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$





QCD does a remarkable job!!!

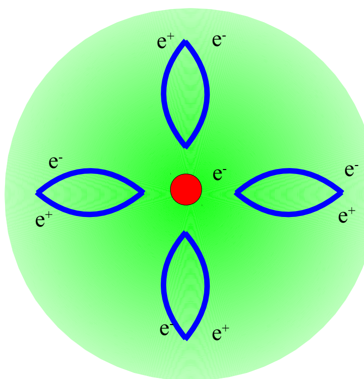


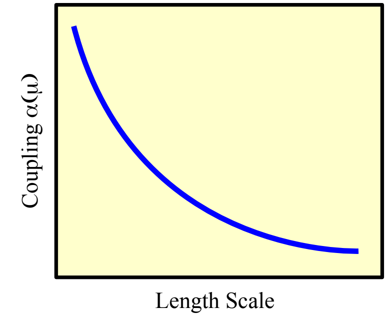
QCD is just like QED,

.... only different

QCD is just like QED, only different ...

QED: Abelian U(1) Symmetry





Perturbation theory at large distance is convergent

$$\alpha(\infty) \sim \frac{1}{137}$$

$$\alpha(M_Z) \sim \frac{1}{128}$$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \,,$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - 0$$

Abelian

α is good expansion parameter

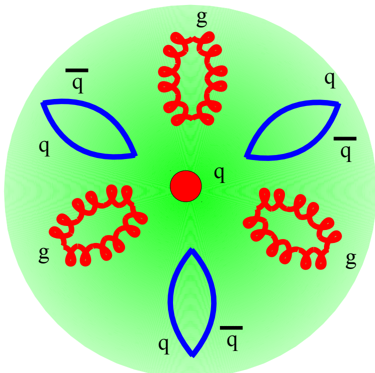
QCD is Non-Abelian SU(3) Symmetry; Quarks are Confined!!!

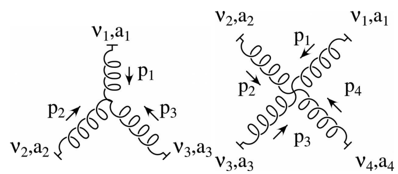
$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

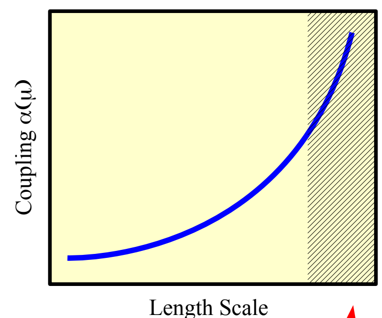
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

Non-Abelian

$$[T_a, T_b] = i\sum_{c=1}^{8} f_{abc} T_c$$



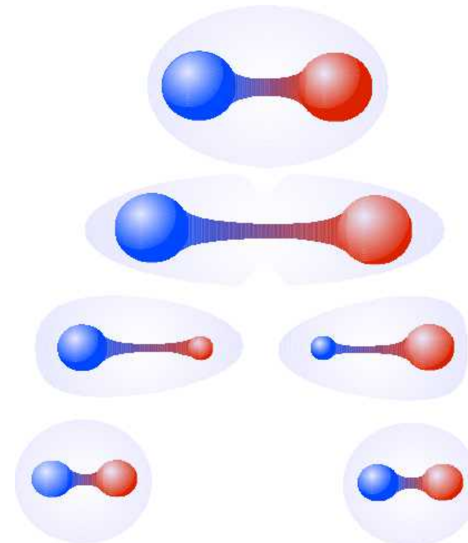




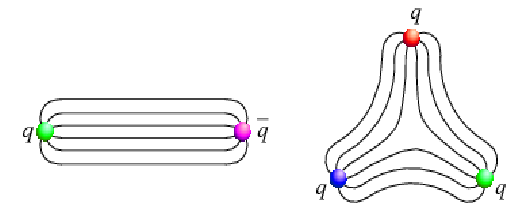
$\alpha_s(M_Z) \sim 0.118$

Cannot perform Perturbation theory at large distance

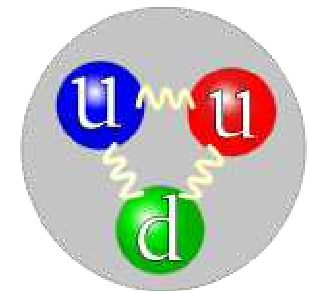
Quark Confinement & String Interpretation



Thomas Lippert, NIC-ZAM, Jülich, for the SESAM Collaboration)



http://www.scholarpedia.org/



http://en.wikipedia.org/

Quarks are confined

Statement of the problem

Theorist #1: The universe is completely described by the symmetry group SO(10)

Theorist #2: You're wrong; the correct answer is SuperSymmetric flipped SU(5)xU(1)

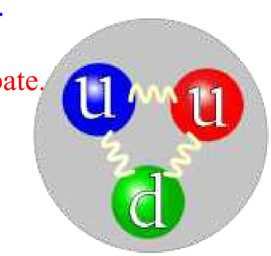
Theorist #3: You've flipped! The only rational choice is E8xE8 dictated by SuperString Theology.

Experimentalist: Enough of this speculative nonsense. I'm going to measure something to settle this question. What can you predict???

Theorist #1: We can predict the interactions between fundamental particles such as quarks and leptons.

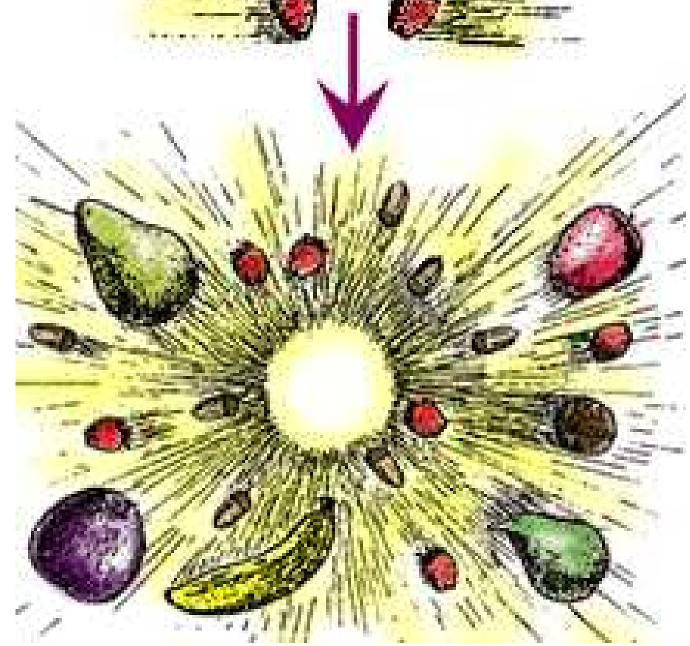
Experimentalist: Great! Give me a beam of quarks and leptons, and I can settle this debate.

Accelerator Operator: Sorry, quarks only come in a 3-pack and we can't break a set!



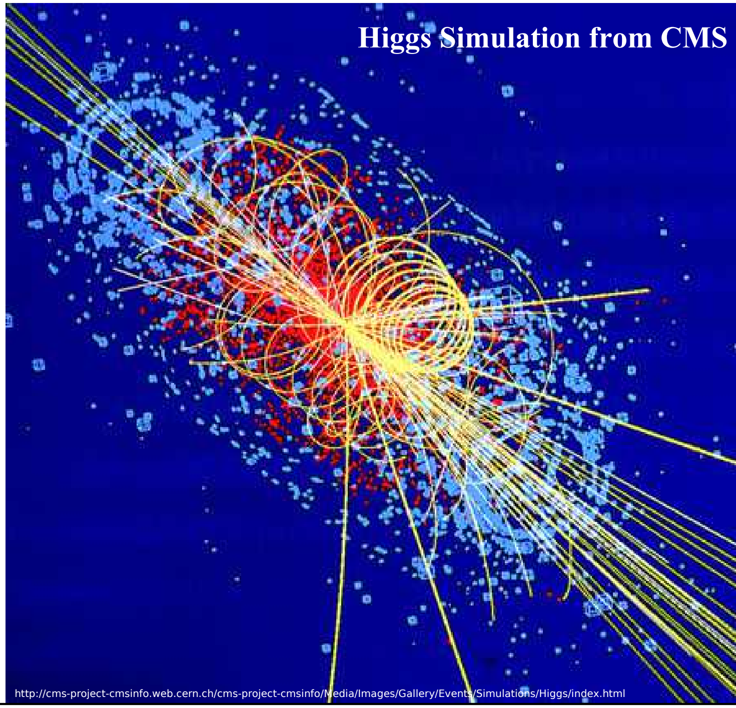
http://en.wikipedia.org/

One interpretation of a hadron-hadron collision

Did we find the Higgs?

A bit more realistic interpretation of a hadron-hadron collision



OUTLINE

QCD is a theory with a rich structure

Working in the limit of a spherical horse ...

We are going to look at the essence of what makes QCD so different from the other forces.

As a consequence, we will need to be creative in how we study the properties, now we define observables, and interpret the results.

QCD has a history of more than 40 years, and we are still trying to fully understand its structure.

The goal of these lectures

Provide pictorial/graphical/heuristic explanations for everything that confused me as a student

BEFORE

$\Rightarrow$

AFTER

Lecture 1:

Overview& essential features
Nature of strong coupling constant
& how it varies with scale
Issues beyond LO and SM
Renormalization Group Equation &
Resummation
Scaling and the proton Structure

Lecture 2:

The structure of the proton
Deeply Inelastic Scattering (DIS)
The Parton Model
PDF's & Evolution
Scaling and Scale Violation

cf., Simona Malace
cf., Pavel Nadolsky

Lecture 3:

Issues at NLO
Collinear and Soft Singularities
Mandelstam Variables
An example from Freshman Physics
Regularized Distributions
Extension to higher orders

Lecture 4:

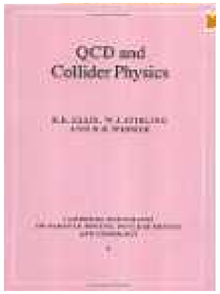
Drell-Yan and e^+e^- Processes
W/Z/Higgs Production & Kinematics
3-body Phase Space & Dalitz Plots
Sterman-Weinberg Jets
Infrared Safe Observables
Rapidity & Pseudo Rapidity
Jet Definitions

cf., Radja Boughezal
cf., Dave Soper & Andrew Larkoski

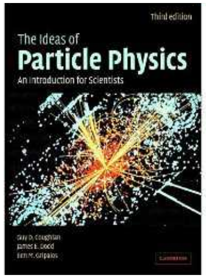
Homework:

Physics is not a spectator sport

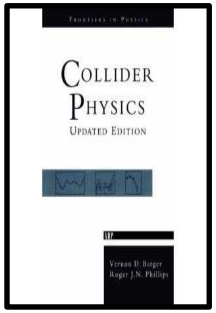
Useful References & Thanks:



Ellis, Stirling, Webber



Coughlan, Dodd, Gripaios



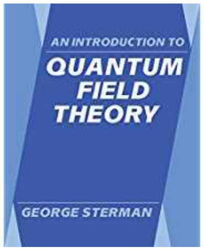
Barger & Phillips

CTEQ Handbook
Reviews of Modern Physics

An Introduction to QFT
Peskin & Schroeder

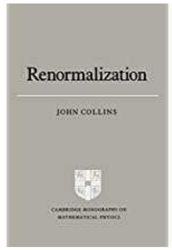
Particle Data Group
http://pdg.lbl.gov

More Useful References

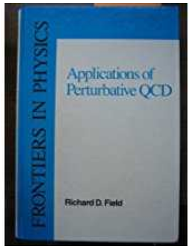


An Introduction to
Quantum Field Theory
George Sterman

Foundations of
Perturbative QCD
John C. Collins



Renormalization:
John C. Collins



Applications of
Perturbative QCD,
Richard D. Field

More Useful References

The CTEQ Pedagogical Page
Linked from **`cteq.org`**

Everything you wanted to know about Lambda-QCD but were afraid to ask
Randall J. Scalise and Fredrick I. Olness

Regularization, Renormalization, and Dimensional Analysis:
Dimensional Regularization meets Freshman E&M
Fredrick Olness & Randall Scalise
e-Print: arXiv:0812.3578

Calculational Techniques in Perturbative QCD: The Drell-Yan Process.
Björn Pötter has prepared a writeup of the lecture given by Jack Smith.
This is a wonderful reference for those learning to do real 1-loop calculations.

Thanks to ...

Thanks to:

Dave Soper, George Sterman,
John Collins, & Jeff Owens for ideas
borrowed from previous CTEQ
introductory lecturers

Thanks to Randy Scalise for the help on
the Dimensional Regularization.

Thanks to my friends at Grenoble who
helped with suggestions and corrections.

Thanks to Jeff Owens for help on
Drell-Yan and Resummation.

To the CTEQ and MCnet folks
for making all this possible.

LibreOffice
The Document Foundation

and the many web pages where I borrowed my figures ...

PowerPoint

The Strong Coupling, Scaling, and Stuff

QCD is Non-Abelian SU(3) Symmetry; Quarks are Confined

$$\mathcal{L}_{QCD} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

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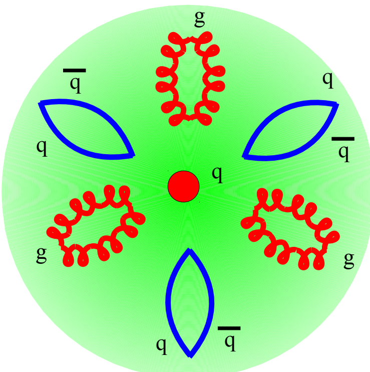
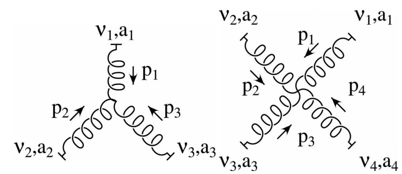
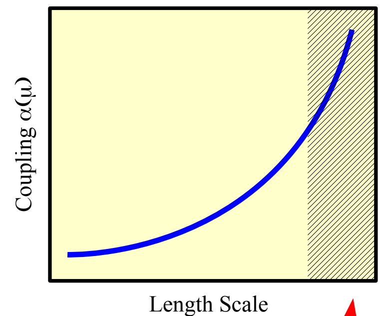






$\Lambda \sim 200\,\mathrm{MeV} \sim 1\ \mathrm{fm}$

Cannot perform Perturbation theory at large distance

25

Where did that β-function come from ???

Consider a physical observable: $R(Q^2/\mu^2, \alpha_S)$

Q is the characteristic energy scale of the problem

μ is an artificial scale we introduce to regulate the calculation (more later)

The Renormalization Group Equation (RGE) is: $\frac{dR}{d\mu^2} = 0$

$$\left\{ \mu^2 \frac{d}{d\mu^2} \right\} R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0$$

Using the chain rule:

$$\left\{ \mu^2 \frac{\partial}{\partial \mu^2} + \left[\mu^2 \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2} \right] \frac{\partial}{\partial \alpha_s(\mu^2)} \right\} R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0$$

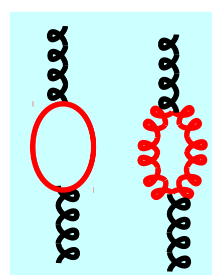
$$\beta(\alpha_s(\mu))$$

β function tell us how α_s changes with energy scale!!!

The β-function: $\beta(\alpha_S(\mu))$

β function tell us how α_S changes with energy scale!!!

$$\beta(\alpha_s(\mu)) = \mu^2 \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2} = \frac{\partial \alpha_s(\mu^2)}{\partial \ln \mu^2}$$

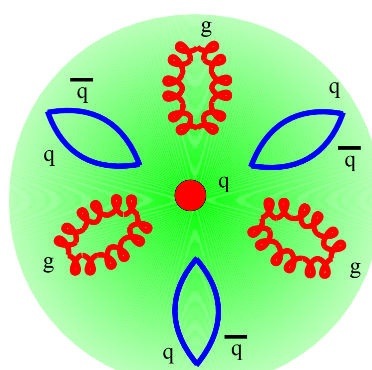


We can calculate this perturbatively

$$\beta(\alpha_s(\mu)) = -\left\{ b_0 \alpha_s^2 + b_1 \alpha_s^3 + b_2 \alpha_s^4 + \ldots \right\}$$

$$b_0 = \frac{33 - 2N_F}{12\pi}$$

$$\beta = -\alpha_S^2 \left[\frac{33 - 2N_F}{12\pi} \right] + \ldots$$



Note: b_0 and b_1 are scheme independent.

β is negative; let's find the implications

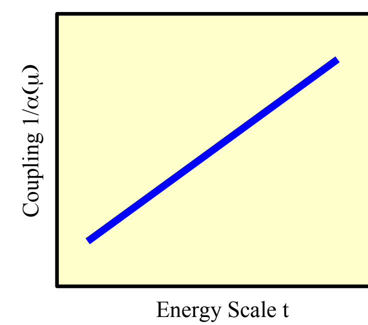
Solve for the running coupling

Let: *$t = \ln \mu^2$*

$$\beta = \frac{\partial \alpha_s}{\partial t} \simeq -b_0\, \alpha_s^2 + \ldots$$

$$\frac{\partial \alpha_s}{\alpha_s^2} = -b_0\, \partial t$$

$$\frac{1}{\alpha_s} = b_0\, t$$



$$b_0 = \frac{33 - 2N_F}{12\pi}$$

$$\beta = -\alpha_S^2 \left[\frac{33 - 2N_F}{12\pi} \right] + \ldots$$

Observe $\beta_{QCD} < 0$ for $N_F < 17$ or for 8 generations or less.

Thus, in general $\beta_{QCD} < 0$ in the QCD theory

Contrast with QED: $\beta_{QED} > 0 = +\alpha^2/3\pi + \ldots$

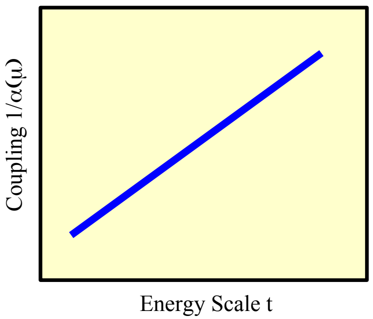
The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek "for the discovery of asymptotic freedom

Solve for the running coupling and Λ_{QCD}

Let: $t = \ln \mu^2$

$$\frac{1}{\alpha_S}\Big]_{\mu_0}^{\mu_1} = b_0\, t\Big]_{\mu_0}^{\mu_1}$$

$$\frac{1}{\alpha_S(\mu_1)} - \frac{1}{\alpha_S(\mu_0)} = b_0 \ln(\mu_1/\mu_0)$$



β functions gives us running, but we still need a reference

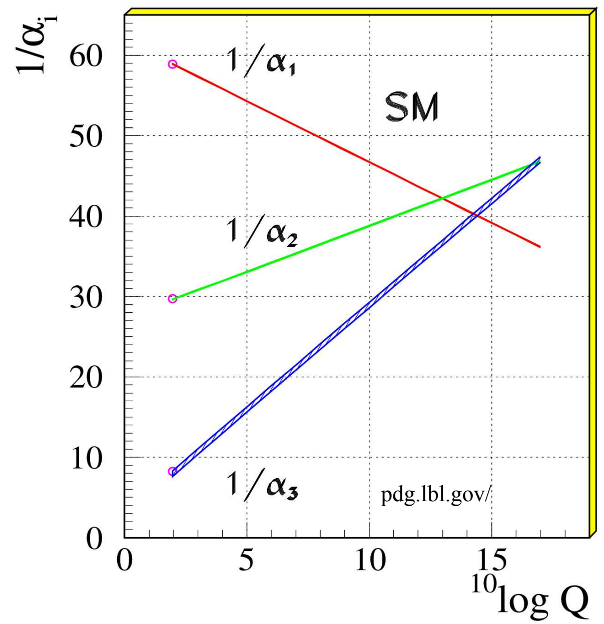
$$\alpha_s(\mu) = \frac{1}{b_0 \ln(\mu/\Lambda_{QCD})}$$

Landau Pole $\Lambda = \mu$

$$\Lambda = \mu\, e^{-1/(b_0\, \alpha_s(\mu))}$$

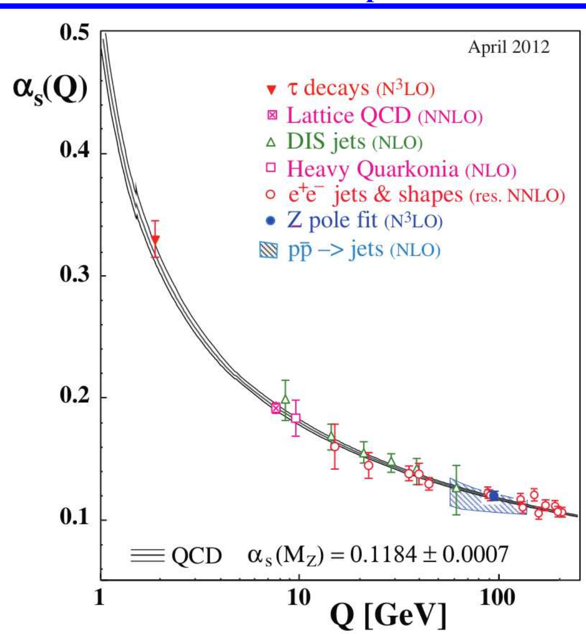
$$\Lambda_{QCD} \sim 200\,\mathrm{MeV} \sim 1\ \mathrm{fm}$$

The Standard Model (SM) Running Couplings: U(1), SU(2), SU(3)



$$b_1 = 0 + (2/3)N_F + \tfrac{1}{10}N_{Higgs}$$
$$b_2 = -22/3 + (2/3)N_F + \tfrac{1}{6}N_{Higgs}$$
$$b_3 = -11 + (2/3)N_F + 0$$

Comparison with data



Siegfried Bethke arXiv:1210.0325 [hep-ex]

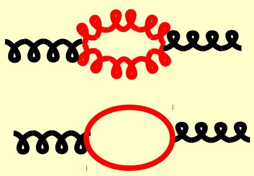
$$\alpha_s(M_Z) = 0.118$$

Low Q points have more discriminating power

Caution: α_s is NOT a physical observable

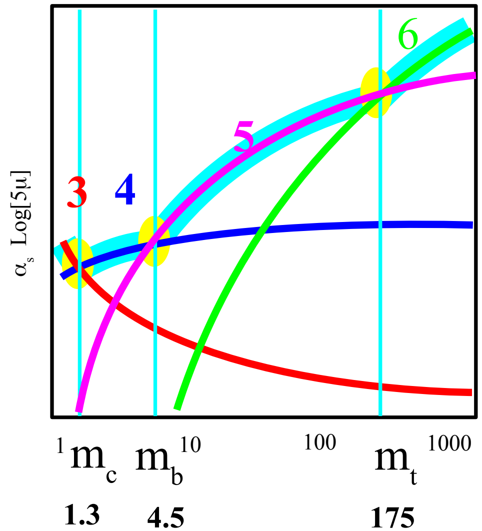
BEYOND NLO

N_F Matters



$$\beta = -\alpha_S^2 \left[\frac{33 - 2N_F}{12\pi} \right] + \dots$$

At 1-loop and 2-loops, continuous at thresholds

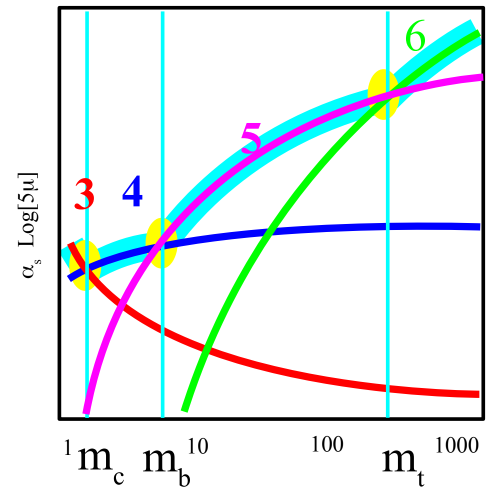


$\Lambda \sim 200\,\mathrm{MeV} \sim 1\ \mathrm{fm}$

$$\alpha_S^{N_F+1}(\mu^2) = \alpha_S^{N_F}(\mu^2) \left[\begin{array}{l} 1 \\ + \ \alpha_s^1 (c_{10} + c_{11} L^1) \\ + \ \alpha_s^2 (c_{20} + c_{21} L^1 + c_{22} L^2) \\ + \ \dots \end{array} \right]$$

This is zero

This is non-zero



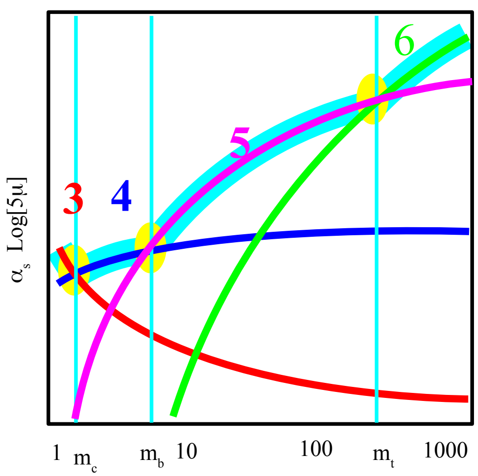
$L = \ln(\mu^2/m^2)$

$c_0 = 0$

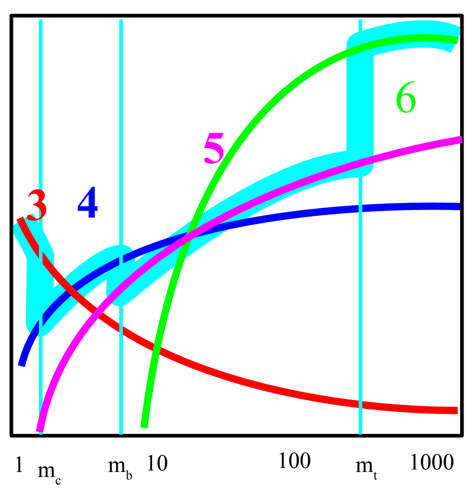
$c_1 = \dfrac{-11}{72\,\pi} \neq 0$

At $\mu = m$ $\quad \alpha_S^{N_F+1}(\mu^2) = \alpha_S^{N_F}(\mu^2)[1 + 0 + c_{20}\,\alpha_S^2]$

At 1-loop and 2-loops, continuous at thresholds

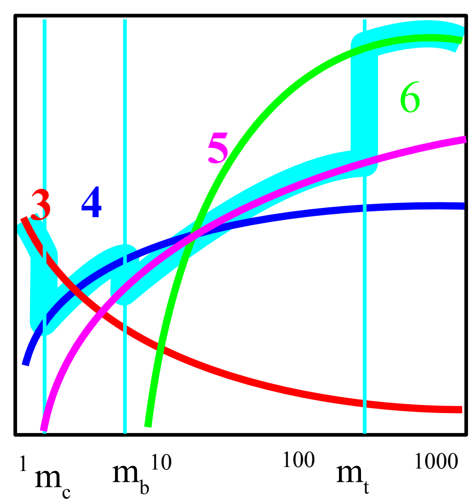


At $O(\alpha_s^3)$, not even continuous at thresholds



$$\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M) - \frac{11}{72\pi^2}\,\alpha^3_{(n_f-1)}(M) + \mathcal{O}(\alpha^4_{(n_f-1)})$$

At $O(\alpha_s^3)$, not even continuous at thresholds



Un-physical theoretical constructs:

(E.g., as, PDFs, ...)

Cannot be measured directly

Depends on Schemes

Renormalization Schemes:

MS, MS-Bar, DIS

Renormalization Scale m

Depends on Higher Orders

Physical Observables

Measure directly

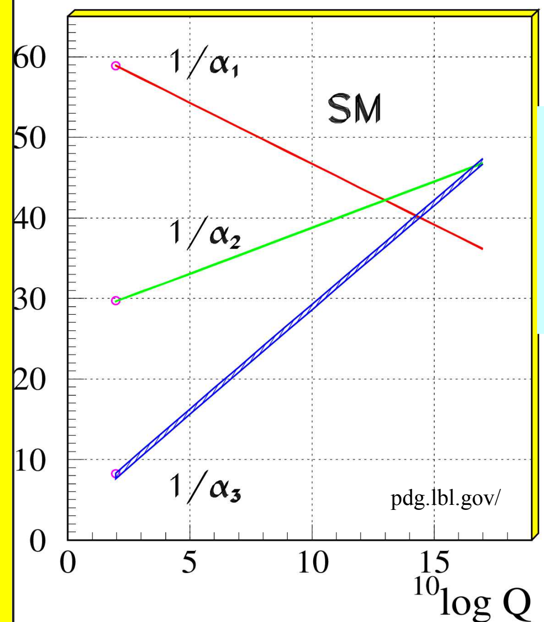
Independent of Schemes/Definitions

Independent of Higher Orders

$$\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M) - \frac{11}{72\pi^2}\,\alpha^3_{(n_f-1)}(M) + \mathcal{O}(\alpha^4_{(n_f-1)})$$

BEYOND SM

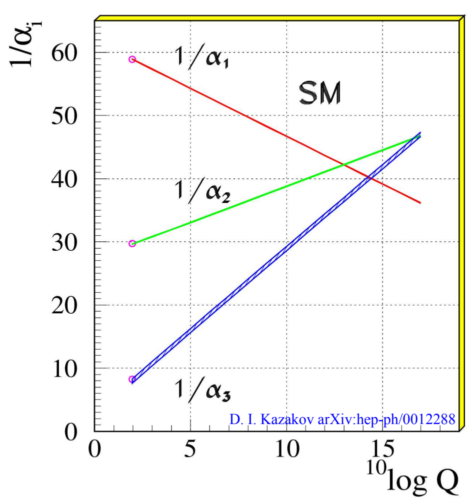
The Standard Model (SM) Running Couplings: U(1), SU(2), SU(3)

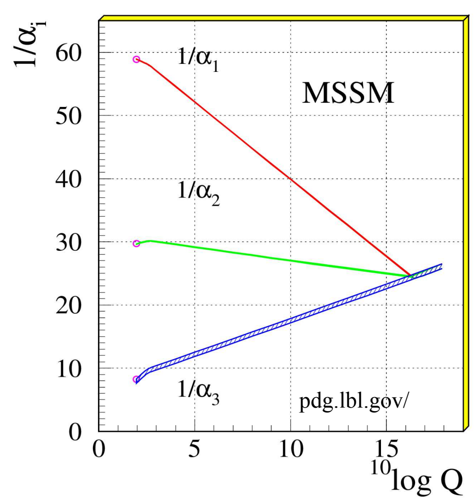


$$
\begin{aligned}
b_1 &= 0 + (2/3)N_F + \tfrac{1}{10}N_{Higgs} \\
b_2 &= -22/3 + (2/3)N_F + \tfrac{1}{6}N_{Higgs} \\
b_3 &= -11 + (2/3)N_F + 0
\end{aligned}
$$

Can we do better???

The Standard Model (SM) & SUSY Running Couplings





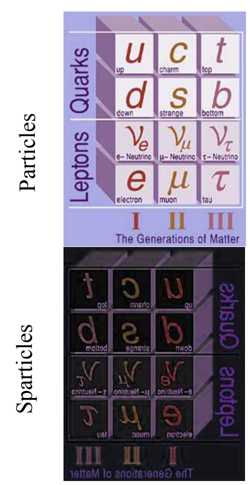
$$
\begin{aligned}
b_1 &= 0 + (2/3)N_F + \tfrac{1}{10}N_{Higgs} \\
b_2 &= -22/3 + (2/3)N_F + \tfrac{1}{6}N_{Higgs} \\
b_3 &= -11 + (2/3)N_F + 0
\end{aligned}
$$

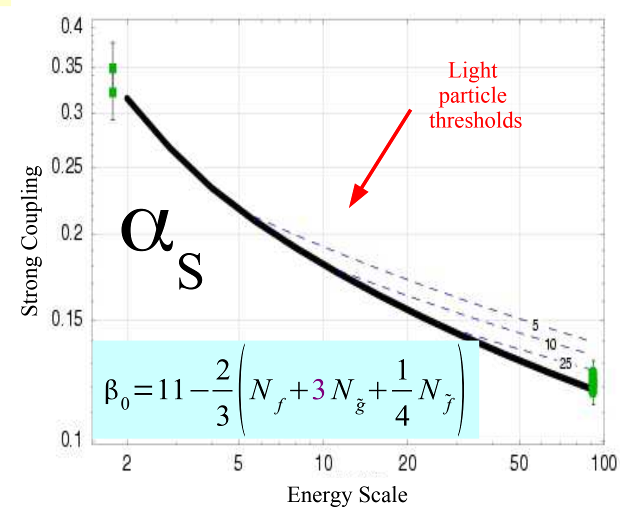
$$\beta_0 = 11 - \frac{2}{3}\left(N_f + 3N_{\tilde{g}} + \frac{1}{4}N_{\tilde{f}}\right)$$

The Standard Model (SM) & SUSY Running Couplings

We've only discovered half the particles

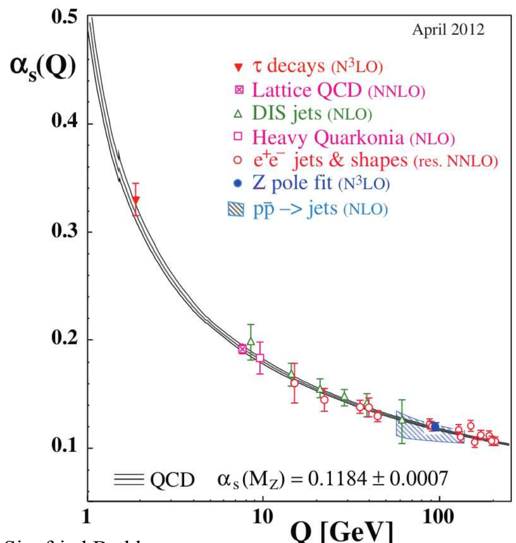
New particles effects evolution of $\alpha_s(\mu)$



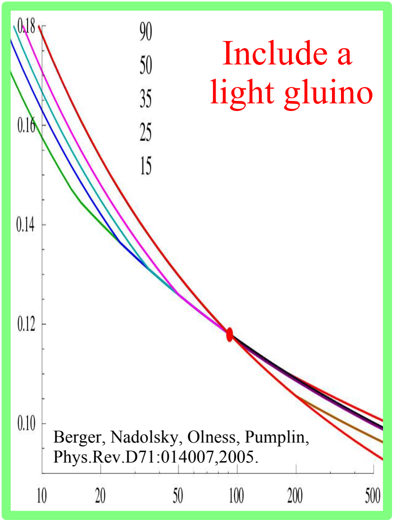


$$\beta_0 = 11 - \frac{2}{3}\left(N_f + 3N_{\tilde{g}} + \frac{1}{4}N_{\tilde{f}}\right)$$

The Standard Model (SM) & SUSY Running Couplings

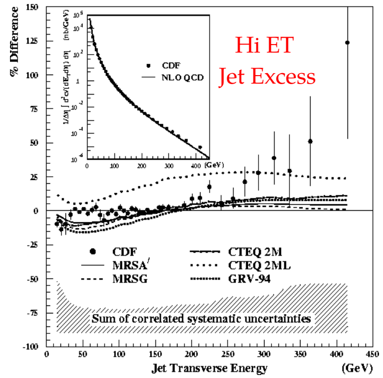


Siegfried Bethke
arXiv:1210.0325 [hep-ex]

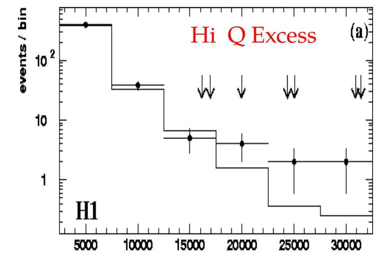


$$b_0 = \frac{1}{12\pi}\left\{33 - 2N_F - 6N_{\tilde{g}} - \frac{1}{2}N_{\tilde{F}} \ldots\right\}$$

GOT QCD ???



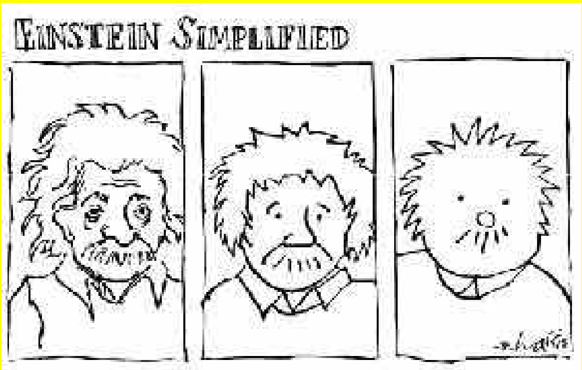
CDF Collaboration, PRL 77, 438 (1996)



H1 Collaboration, ZPC74, 191 (1997)
ZEUS Collaboration, ZPC74, 207 (1997)

Indispensable
for discovery of
"new physics"

RESUMMATION

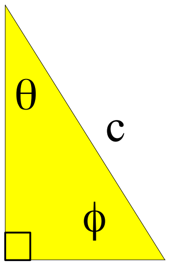


... over simplified

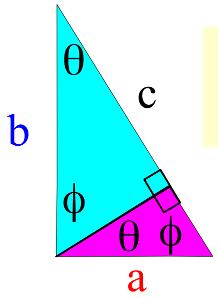
Warm up: Dimensional Analysis: Pythagorean Theorem

GOAL:
Pythagorean Theorem

METHOD:
Dimensional Analysis



$$A_c = c^2 f(\theta, \phi)$$



$$A_a + A_b = a^2 f(\theta, \phi) + b^2 f(\theta, \phi)$$

$$A_a + A_b = A_c$$

$$a^2 + b^2 = c^2$$

Two examples to come: 1) Resummation, and 2) Scaling

Resummation: Over-Simplified

$$\left\{\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s(\mu)) \frac{\partial}{\partial \alpha_s(\mu^2)}\right\} R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0$$

Logs can be large and spoil perturbation theory

If we expand R in powers of α_s, and we know β, we then know μ dependence of R.

$$R\left(\frac{\mu^2}{Q^2}, \alpha_s(\mu^2)\right) = R_0 + \alpha_s(\mu^2)\, R_1\left[\ln(Q^2/\mu^2) + c_1\right]$$
$$+ \alpha_s^2(\mu^2)\, R_2\left[\ln^2(Q^2/\mu^2) + \ln(Q^2/\mu^2) + c_2\right] + O(\alpha_s^3(\mu^2))$$

Since μ is arbitrary, choose μ=Q.

$$R\left(\frac{Q^2}{Q^2}, \alpha_s(Q^2)\right) = R_0 + \alpha_s(Q^2)\, R_1[0 + c_1] + \alpha_s^2(Q^2)\, R_2[0 + 0 + c_2] + \ldots$$

We just summed the logs

QCD is non-abelian SU(3) Symmetry ⇒ confinement

More Differential Quantities ⇒ More Mass Scales ⇒ More Logs!!!

$$\frac{d\sigma}{dQ^2} \sim \ln\left(\frac{Q^2}{\mu^2}\right) \qquad \frac{d\sigma}{dQ^2} \sim \ln\left(\frac{Q^2}{\mu^2}\right) \quad and \quad \ln\left(\frac{q_T^2}{\mu^2}\right)$$

How do we resum logs? Use the Renormalization Group Equation

For a physical observable R: $\mu \frac{dR}{d\mu} = 0$

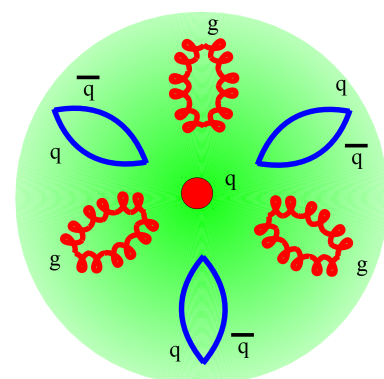
$$\frac{dR}{d\ \ Gauge} = 0$$

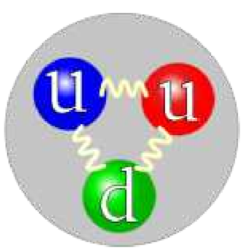
Applied to boson transverse momentum
CSS: Collins, Soper, Sterman
Nucl.Phys.B250:199,1985.

Interesting reference:
Peskin/Schroeder Text
(Renomalization ala Ken Wilson)

Scaling, and the proton structure

How do we determine the proton structure





http://en.wikipedia.org/

Quarks confined, thus we must work with hadrons & mesons

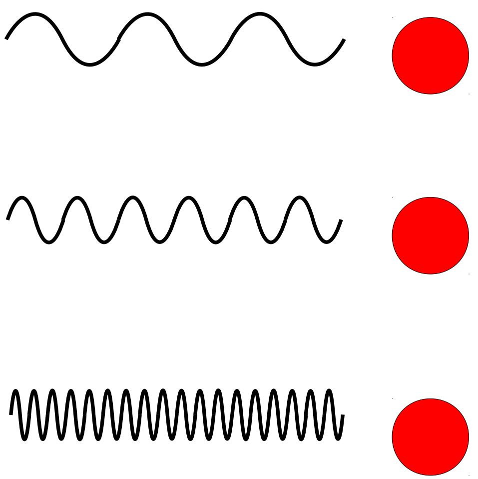
E.g, proton is a "minimal" unit

Highest energy (smallest distance) accelerators involve hadrons

E.g., HERA, TEV, LHC

We'd better learn to work with proton

What do we expect for a point like particle



$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$

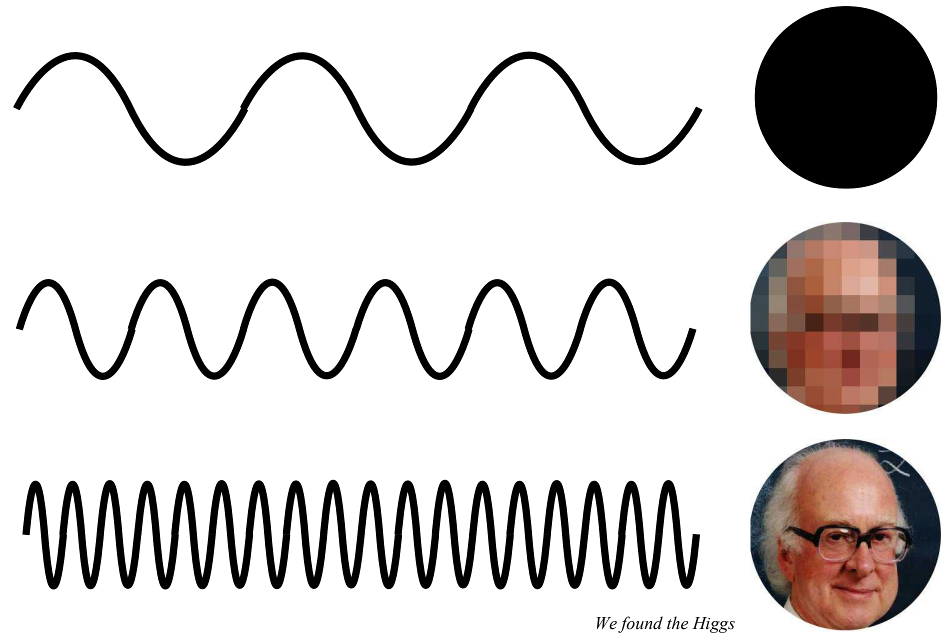
$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$

$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$

Dimensional considerations

Structure Function

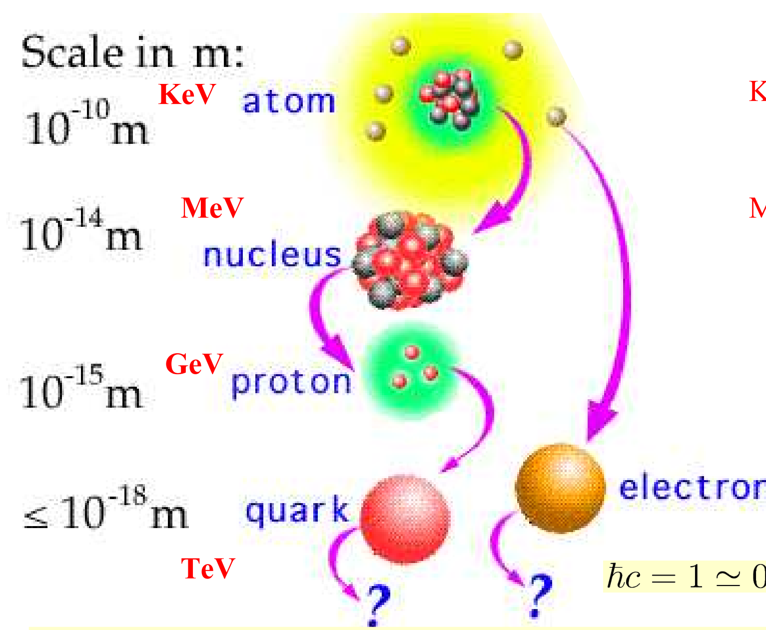
Is this a point like particle ???



We found the Higgs

Scaling, and the proton structure

Relative Sizes



KeV $\sim 10^{-9}$

MeV $\sim 10^{-12}$

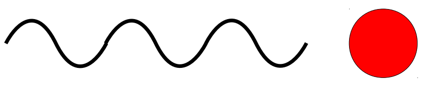
GeV $\sim 10^{-15}$

TeV $\sim 10^{-18}$

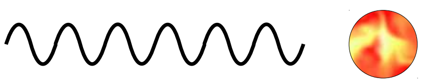
$\hbar c = 1 \simeq 0.2$ GeV fm

Going to smaller scale, we get to simpler, more fundamental objects

Structure of the Proton

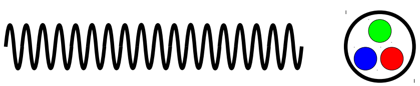


$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$$



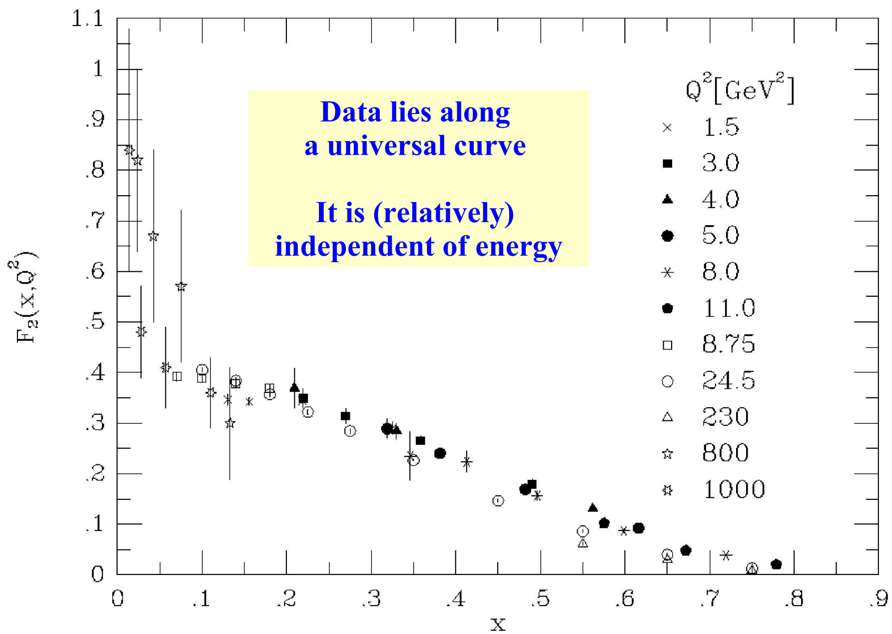
$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times F\left(\frac{Q^2}{\Lambda^2}\right)$$

Λ of order of the proton mass scale



$$d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times \sum_i e_i^2$$

The Scaling of the Proton Structure Function



The Scaling of the Proton Structure Function

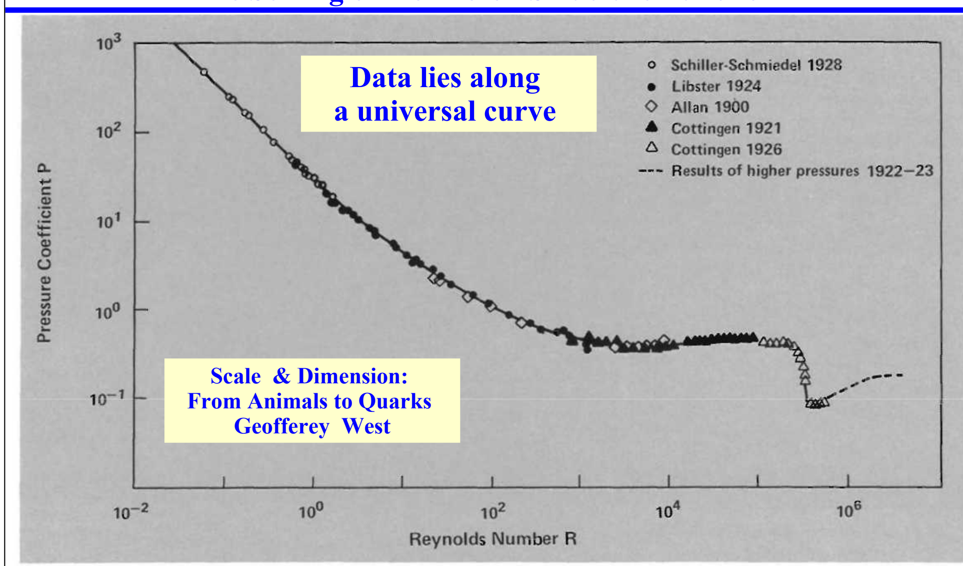


Fig. 5. The scaling curve for the motion of a sphere through a fluid that results when data from a variety of experiments are plotted in terms of two dimensionless variables: the pressure or drag coefficent P *versus Reynolds number* R. *(Figure adapted from* AIP Handbook of Physics, *2nd edition (1963):section II, p. 253.)*

End of lecture 1: Recap

QCD is just like QED, ... only different

QCD is non-Abelian, Quarks are confined,

Running coupling $\alpha_s(\mu)$ tells how interaction changes with distance

β-function: logarithmic derivative of $\alpha_s(\mu)$

We can compute: Negative for QCD, positive for QED

$\alpha_s(\mu)$ is **not** a physical quantity

Discontinuous at NNLO

New physics can influence $\alpha_s(\mu)$

Unification of couplings at GUT scale

Running of $\alpha_s(\mu)$ can help us “resum” perturbation theory

Scaling and Dimensional Analysis are useful tools

END OF LECTURE 1