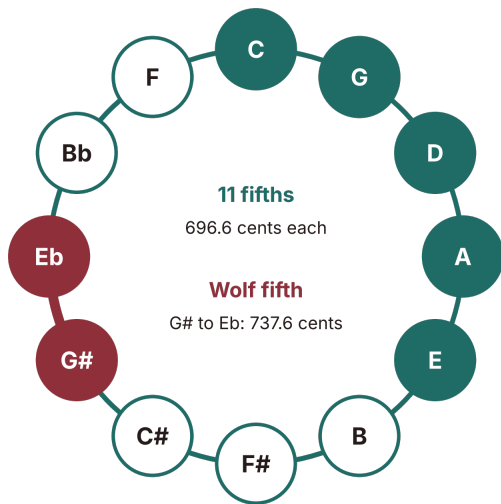


From Pure Fifths to Pure Thirds

Pythagorean tuning to quarter-comma meantone · Musical Acoustics, SMU

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Circle of fifths in quarter-comma meantone. Teal: the four fifths C-G-D-A-E that stack to a pure third. Red: the wolf fifth.

Pythagorean: the pure fifth (3:2)

Fifths are pure at 701.96 cents, ideal for one voice, as in Gregorian chant. But the major third is 407.82 cents, 21.51 cents (the syntonic comma) wider than pure, and it beats about 16 times a second on C-E.

Meantone: the pure third (5:4)

Narrow each fifth by a quarter comma (5.38 cents) to 696.58 cents. Four of them, C-G-D-A-E, stack to a pure third of 386.31 cents. The whole tone, 193.16 cents, is the mean of 9:8 and 10:9.

The price: the wolf fifth

Twelve narrowed fifths cannot close the circle. The gap lands in one interval, G# to Eb: 737.64 cents, 35.68 cents wider than pure. Stay in the home keys.

Cents at a glance

Interval (cents)	Just (pure)	Pythagorean	Meantone	Equal temperament
Fifth	701.96	701.96	696.58	700.00
Major third	386.31	407.82	386.31	400.00
Whole tone	203.91 / 182.40	203.91	193.16	200.00
Diatonic semitone	111.73	90.22	117.11	100.00
Chromatic semitone	70.67	113.69	76.05	100.00
Wolf fifth	—	678.49	737.64	none
Beating on C-E (Hz)	0	16.35	0	10.38

Cents = $1200 \times \log_2(\text{frequency ratio})$; 100 cents = one equal-tempered semitone. Beating is computed for C4 = 261.63 Hz, comparing the 4th harmonic of E with the 5th harmonic of C. Equal temperament divides the octave into 12 identical semitones of 100 cents, so no key has a wolf fifth, but no third is pure either.