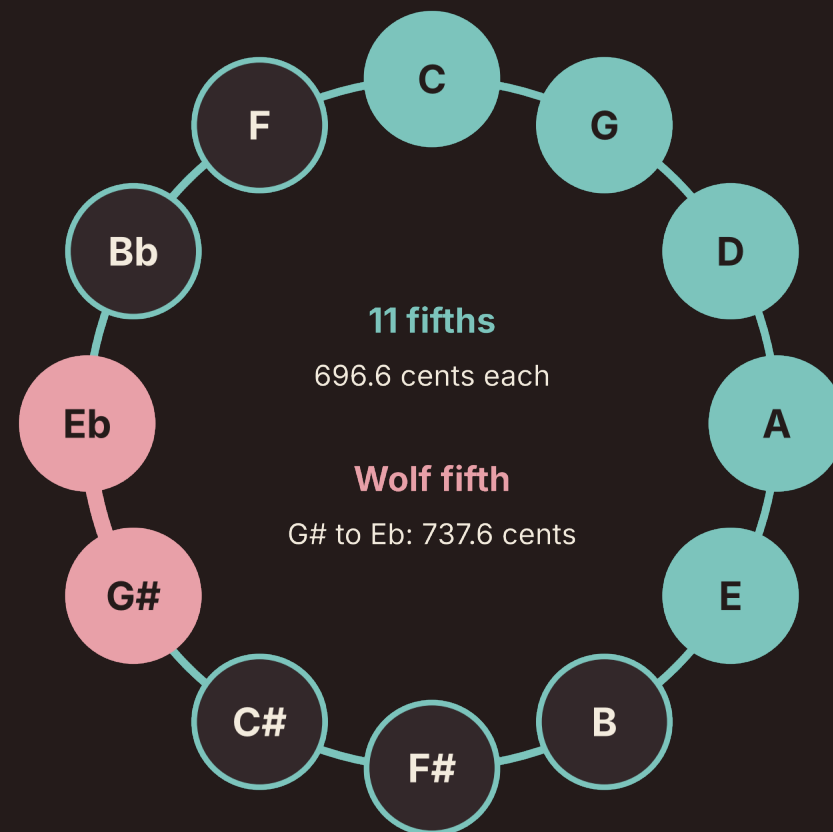


From Pure Fifths to Pure Thirds

From Pythagorean tuning to quarter-comma meantone



Listen first: two major thirds, C to E

Pythagorean

407.8

cents · ratio 81:64

Rough shimmer: about 16 beats per second.

Meantone

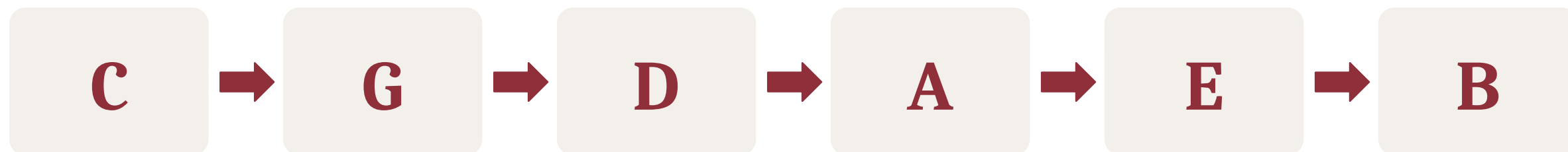
386.3

cents · ratio 5:4

Smooth and still. The third seems to glow.

The gap: 21.5 cents, about a fifth of a semitone.

One ingredient: the pure fifth



Stack fifths, then fold each note back into one octave. Everything comes from a single ratio, 3:2.

3:2

Fifth · 701.96 cents

9:8

Whole tone · 203.9 cents

4:3

Fourth · 498.0 cents

Why chant loves it

1

One voice

Chant is monophonic: a single melodic line, so no sustained chord ever exposes a clash.

2

Open fifths and fourths

Pure 3:2 and 4:3 intervals give chant a clear, open, almost architectural sound.

3

A clean melodic step

The 9:8 whole tone, 203.9 cents, is a tidy step. The theory comes down to us through Boethius.

With a single line, nothing beats.

The problem: the third

Interval	Ratio	Cents
Pythagorean third	81:64	407.8
Pure third	5:4	386.3
Syntonic comma	81:80	21.5

16.35

beats per second when C's 5th harmonic
meets the Pythagorean E's 4th.

Four stacked fifths overshoot the pure 5:4 third by the syntonic comma, 21.5 cents.

When the third became the star

13th–15th century

Thirds turn sweet

English singers begin treating thirds and sixths as consonances, not tense dissonances.



Polyphony grows

Full triadic harmony

Chords of sustained thirds put the third center stage.



1523

Aron writes it down

Pietro Aron describes tuning the keyboard to favor pure thirds.

A tuning built for fifths was serving the wrong lead actor.

The fix: quarter-comma meantone

701.96

Pure fifth, in cents

—

5.38

A quarter of the 21.5-cent
comma

=

696.58

Meantone fifth, in cents

Four narrowed fifths, C–G–D–A–E, stack to 2786.3 cents: two octaves plus a pure 5:4 third of 386.3 cents.

A slightly flat fifth beats slowly and gently. A bad third beats fast and rough.

Why it is called mean-tone

Large tone, 9:8

203.9

cents

Meantone tone, the mean

193.2

cents

Small tone, 10:9

182.4

cents

117.1

Diatonic semitone, cents

76.0

Chromatic semitone, cents

Uneven semitones mean split
black keys: D-sharp and E-flat are
different notes.

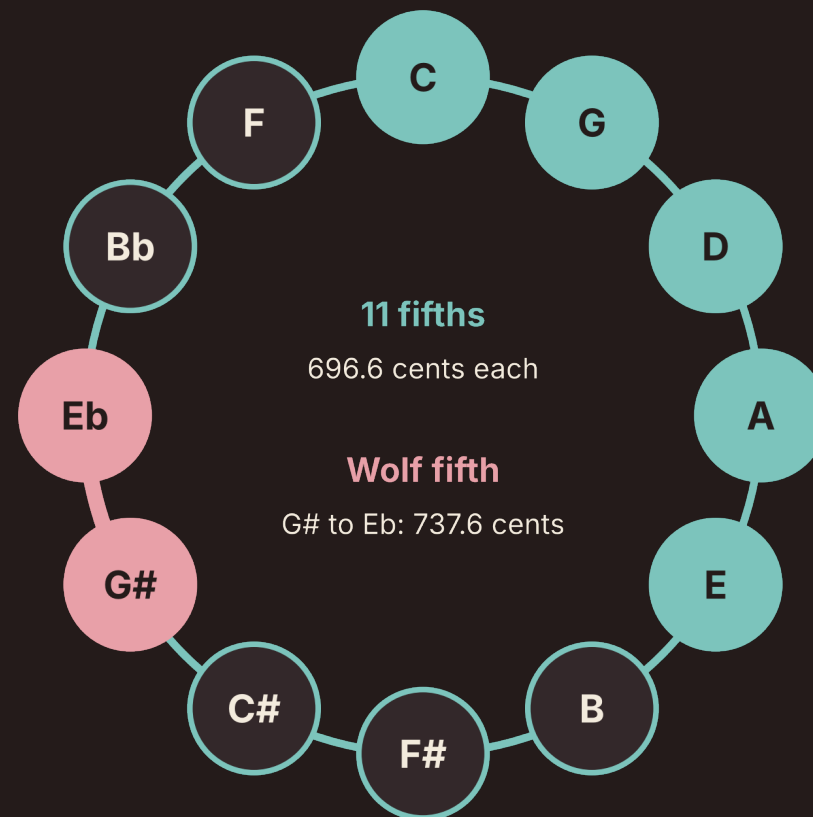
The price: the wolf fifth

737.6

cents, which is 35.7 cents wider than a pure fifth

Twelve narrowed fifths fall short of seven octaves. The whole gap lands in one interval, typically G-sharp to E-flat, and it howls.

Stay home: play the keys that sound great.



Takeaway

Pythagorean

Optimizes the fifth, the interval of a single melodic line.

Meantone

Optimizes the third, the interval of harmony.

Every tuning is a trade-off: you cannot have everything, so you choose what the music needs.

Next: well temperament and equal temperament, where every key becomes usable.

Reference numbers

Interval	Ratio or construction	Cents	Equal temperament
Pure fifth	3:2	701.96	700.00
Pythagorean major third	81:64	407.82	400.00
Pure major third	5:4	386.31	400.00
Syntonic comma	81:80	21.51	none
Meantone fifth	Pure fifth minus a quarter comma	696.58	700.00
Meantone whole tone	Mean of 9:8 and 10:9	193.16	200.00
Wolf fifth	7 octaves minus 11 meantone fifths	737.64	none