## Pythagorean Tuning

All tones found by pure 5ths

## Relationship in ratios:

OCTAVE: $2 / 1$
FIFTH: $\quad 3 / 2$
FOURTH: 4/3


## Pythagorean Third: C-E

$\mathrm{C} \times 3 / 2=\mathrm{G} \rightarrow \mathrm{G} \times 3 / 2=\mathrm{D} \rightarrow \mathrm{D} \times 1 / 2=\mathrm{D} \rightarrow \mathrm{D} \times 3 / 2=\mathrm{A} \rightarrow \mathrm{A} \times 3 / 2=\mathrm{E} \rightarrow \mathrm{E} \times 1 / 2=\mathrm{E}$ Calculated out:
$3 / 2 \times 3 / 2=9 / 4 \rightarrow 9 / 4 \times 1 / 2=9 / 8 \rightarrow 9 / 8 \times 3 / 2=27 / 16 \rightarrow 27 / 16 \times 3 / 2=81 / 32 \rightarrow 81 / 32 \times 1 / 2=81 / 64$
$E=81 / 64$

Relationship in cents:


## Pythagorean Comma:

Difference between B\# arrived at by twelve 5ths (702фf) and C arrived at by seven octaves (1200ф).
Comma $=24 \varnothing$

## Just Temperament

Based on pure $5^{\text {th }}$ plus pure $3^{\text {rd }}$

## Relationship in ratios:

OCTAVE: $\quad 2 / 1$
FIFTH: $\quad 3 / 2$
FOURTH: $\quad 4 / 3$
THIRD: 5/4
MAJ SECOND: 9/8 (less preferred 10/9)
MIN SECOND: 16/15


Major Second: G-A


## Minor second: E-F

$\overbrace{5 / 4}^{\text {D }^{4 / 3}} \mathbf{E} \quad$ F
$\overbrace{5 / 4}^{\text {D }^{4 / 3}} \mathbf{E} \quad$ F
$4 / 3-5 / 4=4 / 3 \rightarrow 4 / 3 \times 4 / 5=16 / 15$ 16/15
0
E F

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## Mean Tone Tuning

Based on altering successive $5^{\text {ths }}$ to get pure 3 rd

## Finding Pure $3^{\text {rd }}$ :

- Spreading $1 / 4$ Didymean Comma ( $80 / 81$ or 22 cents) over each of the four $5^{\text {h }}$
- $1 / 4$ of 22 cents is $51 / 2$ cents
- So each $5^{\text {th }}$ is $6961 / 2$ cents instead of 702 cents

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Pythagorean: \(\mathrm{C}^{702} \mathrm{G} \quad{ }^{702}\) D \({ }^{702}\) A \({ }^{702}\) E \(=2808\)
Mean tone: \(\quad 6961 / 2 \quad 6961 / 2 \quad 6961 / 2 \quad 6961 / 2 \quad=2786\)
    \(2768-2400\) (two octaves) \(=386 \quad\) (Note: \(2808-2786=22\) cents too wide)
        Pure \(3^{\text {rd }}=386 \varnothing\)
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OCTAVE: 1200ф
FIFTH: $\quad 69612 \not 2 \downarrow$
THIRD: 386 d
SECOND: $\quad 386 / 2$ (meantone) $*=193 \phi$


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