Physics in Extra Dimensions

Bogdan Dobrescu (*Fermilab*)

Outline:

- **Types of extra dimensions**
- **Bosons and fermions in a compact dimension**
- **Universal extra dimensions**

February 23, 2009 - Southern Methodist University

Evidence that we live in 3 spatial dimensions:

- it is obvious! *(end of story?!?)*
- Gauss law, in $3 + $ n spatial dimensions: $V(r) \sim 1/r^{n+1}$
We observe $n = 0$ for gravity and electromagnetism.
- Standard Model agrees with the data.
- there are no renormalizable field theories in more dimensions

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- Standard Model agrees with the data.
- there are no renormalizable field theories in more dimensions

Counter-arguments:

- what's obvious may be due to preconception
  (*e.g.*, quantum mechanics is not obvious)
- Gauss law may change at short distance
- Standard Model has not been tested below 10^{-16} cm
- gravitational interactions are non-renormalizable in $D = 3 + 1$

Bosons in compact spatial dimensions

4D flat spacetime $\perp$ one dimension of size $L = \pi R$:

A scalar field in the bulk, $\phi(x^\alpha)$, $\alpha = 0, 1, ..., 4$:

$$\mathcal{L}_{5D} = (\partial^\mu \phi)^\dagger \partial_\mu \phi - \left(\partial^4 \phi\right)^\dagger \partial_4 \phi - m_0^2 \phi^\dagger \phi \ , \qquad \mu = 0, 1, 2, 3$$

$\Rightarrow$ **Equation of motion:** $\left(\partial^\mu \partial_\mu - \partial^4 \partial_4\right) \phi = m_0^2 \phi$

m_0 *is the 5D mass of* ϕ.

Neumann boundary conditions for “even” fields:

$$\frac{\partial}{\partial x^4}\phi(x^\mu, 0) = \frac{\partial}{\partial x^4}\phi(x^\mu, \pi R) = 0$$

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$$\phi(x^\mu, x^4) = \frac{1}{\sqrt{\pi R}}\left[\, \phi^{(0)}(x^\mu) + \sqrt{2}\sum_{j\geq 1} \phi^{(j)}(x^\mu)\cos\left(\frac{j x^4}{R}\right)\right]$$

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Kaluza-Klein decomposition

Zero-mode (wave function is constant along x^4)

Kaluza-Klein modes: particles of definite momentum along x^4

4D point of view: a tower of massive particles:

$$\mathcal{L}_{4D} = \int_0^{\pi R} dx^4\, \mathcal{L}_{5D} \qquad \Rightarrow \qquad m_j^2 = m_0^2 + \frac{j^2}{R^2}$$

Dirichlet boundary conditions for "odd" fields:

$$\phi(x,0) = \phi(x,\pi R) = 0$$

KK decomposition:

$$\phi(x^\mu, x^4) = \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{j\geq 1} \phi^{(j)}(x^\mu) \sin\left(\frac{j x^4}{R}\right)$$

There is no zero-mode.

The lightest KK mode is $\phi^{(1)}$, of mass $\sqrt{1/R^2 + m_0^2}$

Homework: *Check that the normalization condition for KK functions requires the factor of $\sqrt{2}$.*
Why $j < 0$ is not allowed?

Gauge bosons in 5D:

$A_\mu(x^\nu, x^4)$, $\mu, \nu = 0, 1, 2, 3$, and

$A_4(x^\nu, x^4)$ – polarization along the extra dimension.

From the point of view of the 4D theory:

$A_4(x^\nu, x^4)$ is a tower of spinless KK modes.

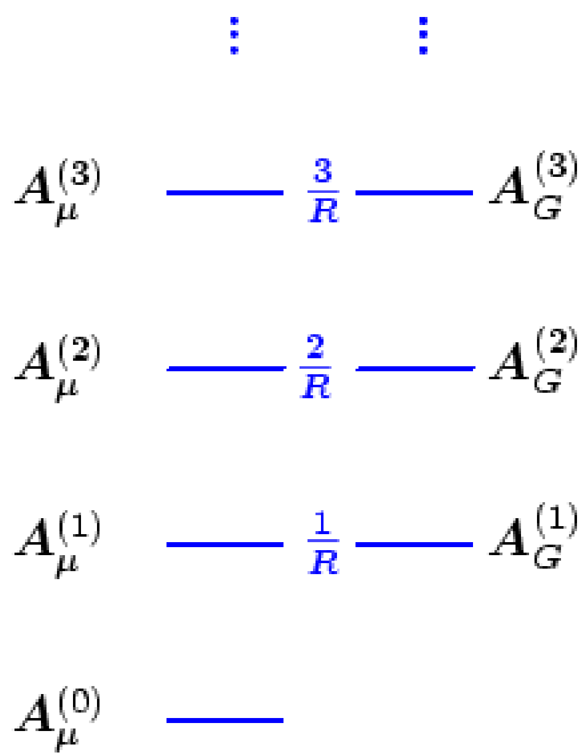
Gauge invariance requires A_μ to have a zero-mode:

$$\partial_4 A_\mu(x^\nu, 0) = \partial_4 A_\mu(x^\nu, \pi R) = 0$$

$$A_\mu(x^\nu, x^4) = \frac{1}{\sqrt{\pi R}} \left[A_\mu^{(0)}(x^\nu) + \sqrt{2} \sum_{j \geq 1} A_\mu^{(j)}(x^\nu) \cos\left(\frac{j x^4}{R}\right) \right]$$

Kaluza-Klein spectrum of gauge bosons

$A_G^{(j)}(x^\nu)$ becomes the longitudinal degree of freedom of the spin-1 KK mode $A_\mu^{(j)}(x^\nu)$.



Extra dimensions may be classified according to:

- number (1, 2, ... , 13?)
- type of compactification (*i.e.* boundary conditions)
- metric (flat, warped, ...)
- which fields propagate in the bulk (graviton, top quark, ...)
- existence of localized operators, stabilization mechanism, ...

Types of extra dimensions:

- graviton only propagates in $n \geq 2$ flat extra dimensions (ADD)
- bosons only propagate in some flat extra dimensions (DDG)
- bosons and some fermions propagate in flat extra dimensions
- all particles propagate in some flat extra dimensions (UED)
- graviton only propagates in a warped extra dimension (RS)
- all particles propagate in a warped extra dimension
- ...

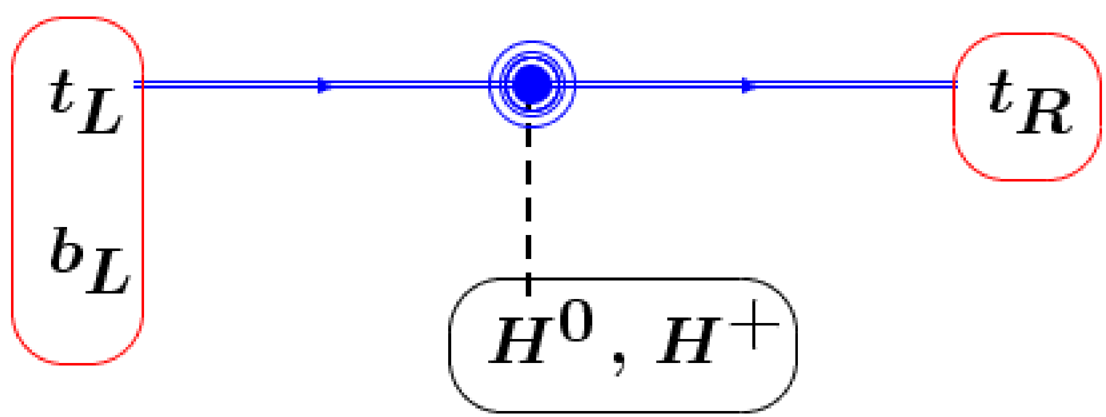
Fermions

All Standard Model fermions are chiral.

The two top quarks:

- "left-handed" top *(feels the weak interaction)*
- "right-handed" top *(no interaction with $W^{\pm}$)*

Top mass: t_L turns into t_R and vice-versa



Fermions in a compact dimension

Gamma matrices – require 5 anti-commuting matrices:

$$\gamma^\mu, \quad \mu = 0,1,2,3, \quad \text{and} \quad \gamma^4 = i\gamma_5$$

These are 4×4 matrices $\rightarrow$ 5D fermions have 4 components.

$\Rightarrow$ **5D fermions are vector-like:**

$$\chi(x^\mu, x^4) = \chi_L(x^\mu, x^4) + \chi_R(x^\mu, x^4)$$

Lorentz group in 5D, $SO(1,4)$, has a single spin-1/2 representation.

If quarks and leptons were 0-modes of 5D fermions, then they would not have been chiral ?!?

If standard model fermions propagate along the extra dimension, then the compactification must be on an interval.

(circle compactification gives vectorlike zero-modes, not compatible with the observed fermions which are chiral).

Chiral boundary conditions:

$$\chi_L(x^\mu, 0) = \chi_L(x^\mu, \pi R) = 0$$

$$\frac{\partial}{\partial x^4}\chi_R(x^\mu, 0) = \frac{\partial}{\partial x^4}\chi_R(x^\mu, \pi R) = 0$$

Dirac equation in 5D:

$$i\gamma^\mu \partial_\mu \chi_R = (\partial_4 + m_0)\chi_L$$

$$i\gamma^\mu \partial_\mu \chi_L = (-\partial_4 + m_0)\chi_R$$

Dirac equation in 5D:

$$i\gamma^\mu \partial_\mu \chi_R = (\partial_4 + m_0)\chi_L$$

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Kaluza-Klein decomposition:

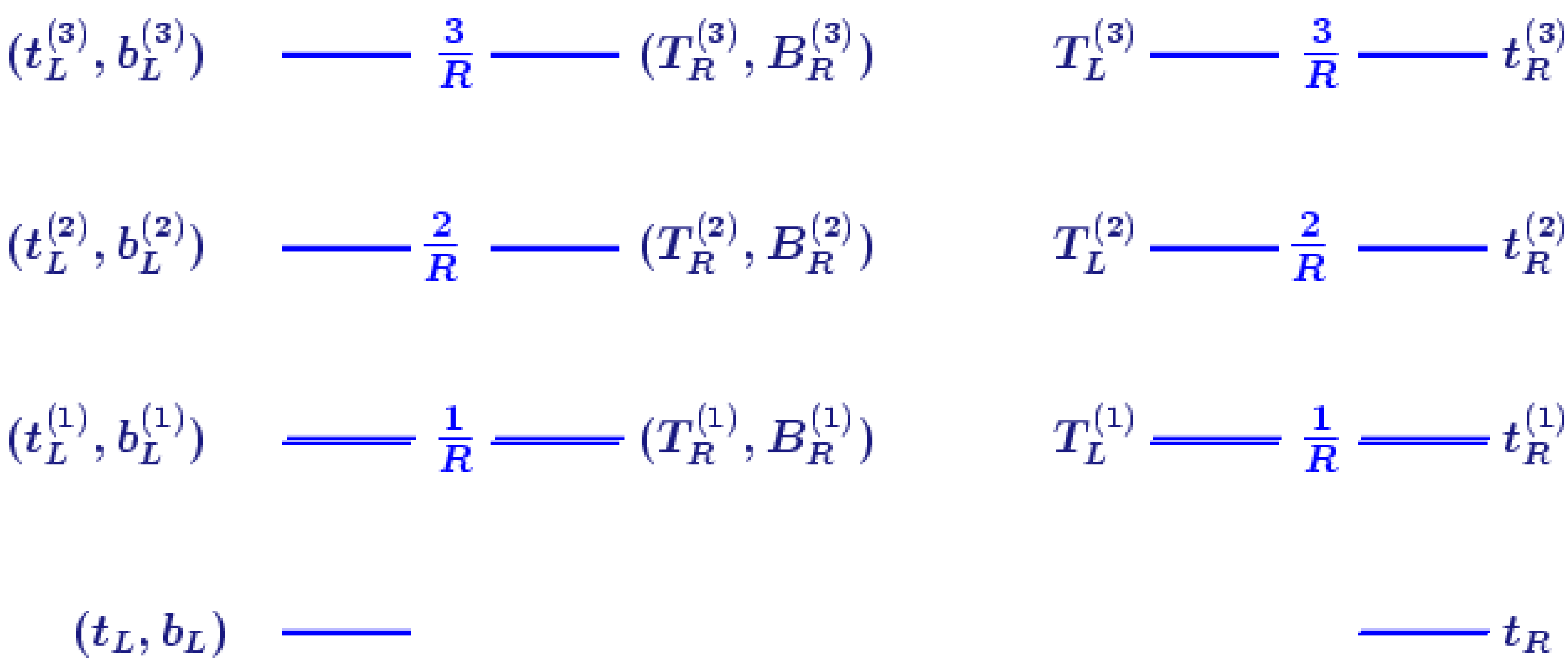
$$\chi = \frac{1}{\sqrt{\pi R}} \left\{ \chi_R^0(x^\mu) + \sqrt{2} \sum_{j \geq 1} \left[\chi_R^j(x^\mu) \cos\left(\frac{\pi j x^4}{L}\right) + \chi_L^j(x^\mu) \sin\left(\frac{\pi j x^4}{L}\right) \right] \right\}$$

0-mode is a chiral fermion!

KK modes are vectorlike fermions.

Homework: solve 5D Dirac equation when χ_R *is odd and* χ_L *is even.*

Kaluza-Klein spectrum of quarks and leptons



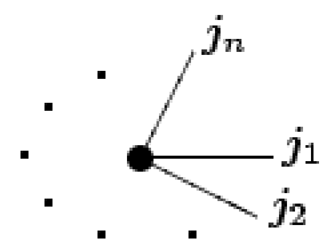
Universal Extra Dimensions

T. Appelquist, H.-C. Cheng, B. Dobrescu, Phys.Rev.D64 (2001)

All Standard Model particles propagate in $D \geq 5$ dimensions.

Momentum conservation $\longrightarrow$ KK-number conservation

$$\mathcal{L}_{4D} = \int_0^{\pi R} dx^4 \, \mathcal{L}_{5D}$$



At each interaction vertex:

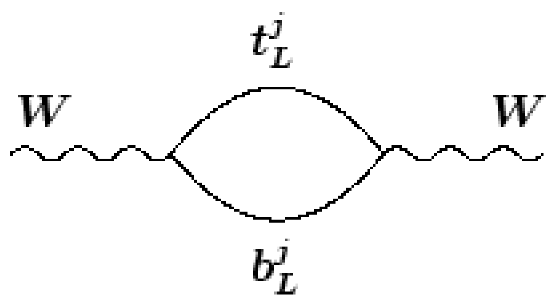
$j_1 \pm j_2 \pm ... \pm j_n = 0$ **for a certain choice of** $\pm$

In particular: $0 \pm \cdots \pm 0 \neq 1$

$\Rightarrow$ tree-level exchange of KK modes does not contribute to currently measurable quantities

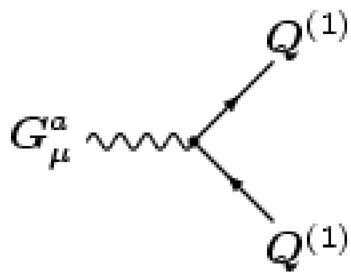
$\Rightarrow$ no single KK 1-mode production at colliders

Bounds from one-loop shifts in W and Z masses, and other observables:

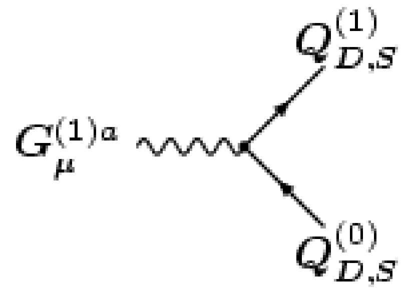


$$\frac{1}{R} \gtrsim 300 - 500 \text{ GeV}$$

Feynman rules relevant for QCD production of KK particles at hadron colliders:



$= -ig_s\gamma^\mu T^a$



$= -ig_s\gamma^\mu P_{L,R} T^a$

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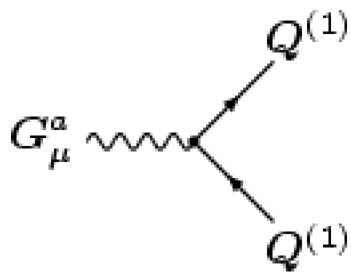

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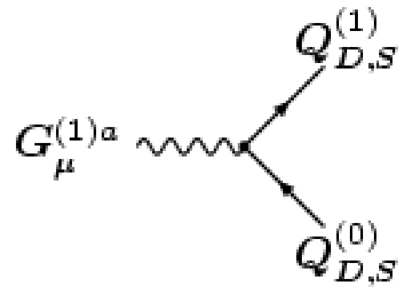

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Interaction of a level-1 quark with a level-1 gluon is chiral

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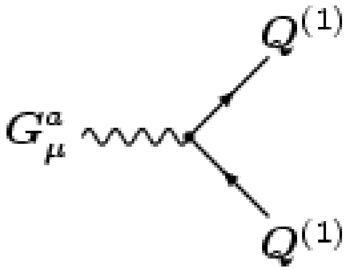

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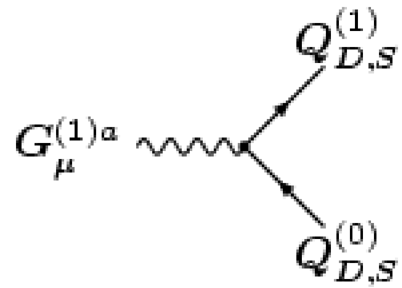
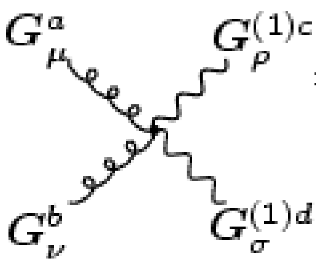


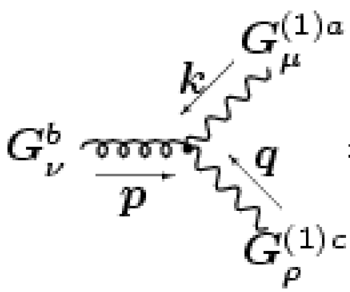
$= -ig_s\gamma^\mu P_{L,R}T^a$

Interaction of a level-1 quark with a level-1 gluon is chiral

Feynman rules for interactions of standard-model gluons with KK modes are fixed by gauge invariance:



$$= -ig_s^2[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})]$$

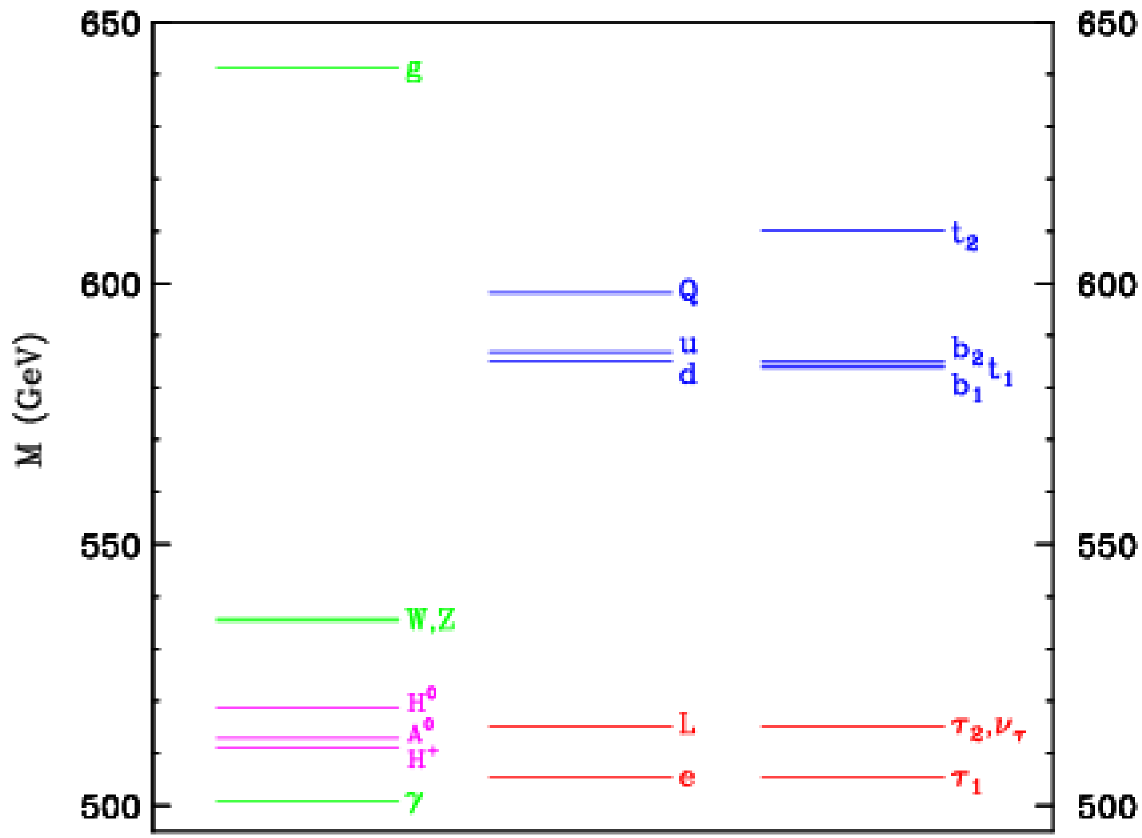


$$= g_s f^{abc}[(k-p)_\lambda g_{\mu\nu} + (p-q)_\mu g_{\nu\rho} + (q-k)_\nu g_{\mu\rho}]$$

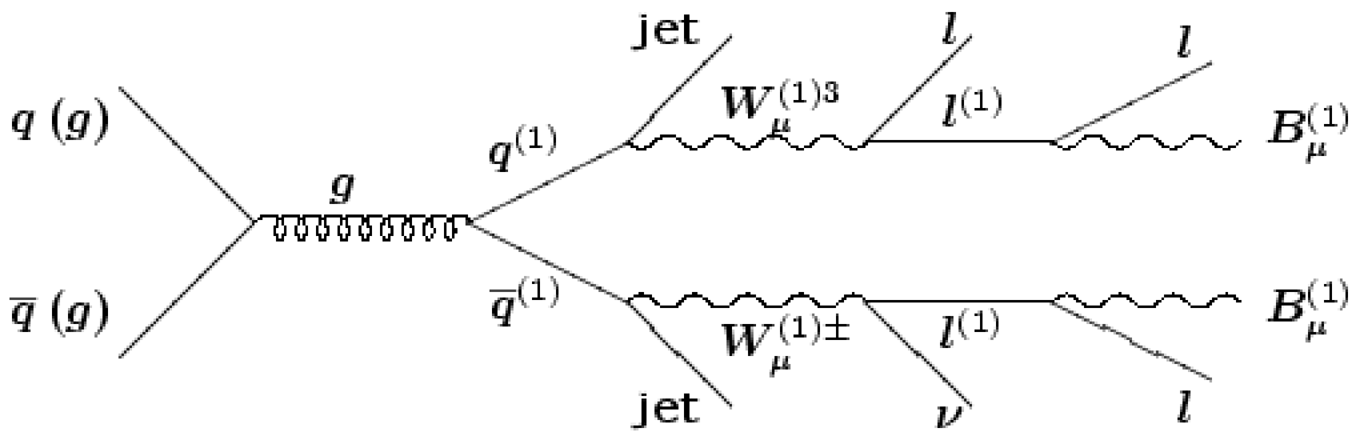
(1) modes have a tree-level mass of $1/R$, and KK parity $-$.

One-loop contributions (and electroweak symmetry breaking) split the spectrum *(Cheng, Matchev, Schmaltz, hep-ph/0204342)*

Mass spectrum of the (1) level:



Pair production of (1) modes at hadron colliders:



Look for: **2 hard leptons (~ 100 GeV)**

+ 1 soft lepton (~ 10 GeV)

+ 2 jets (~ 50 GeV)

+ $\not{E}_T$

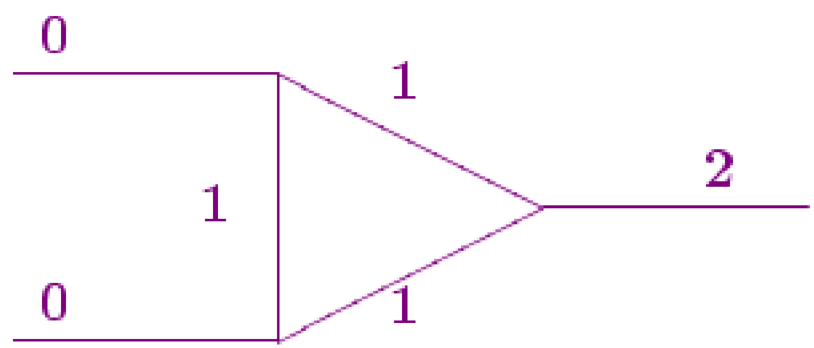
(Cheng, Matchev, Schmaltz, hep-ph/0205314; ...)

Homework: draw other diagrams which contribute to this signal.

CDF analysis of $3l + \not{E}_T$: $1/R > 280$ GeV (Run I)

At one-loop level: $j_1 \pm j_2 \pm \ldots \pm j_n =$ **even**

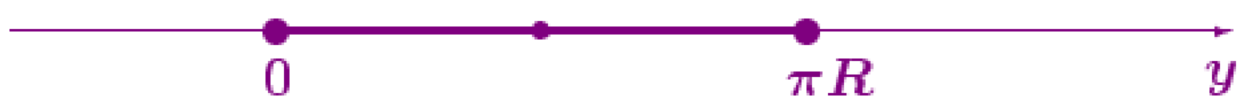
At colliders: s**-channel production of the** 2**-modes**



Kaluza-Klein parity: invariance under reflections with respect to the center of the compact dimension.

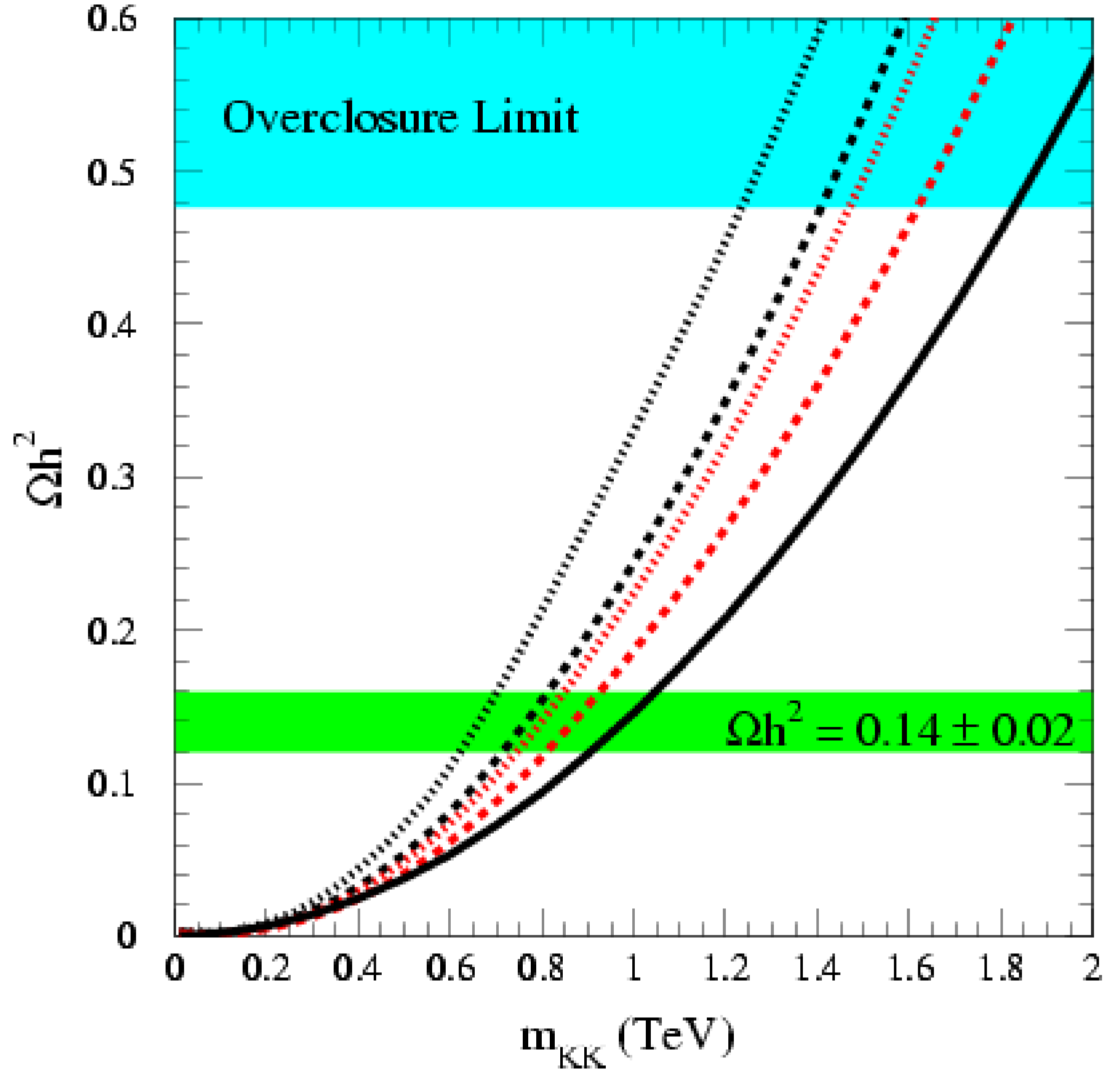
KK parity $(-1)^j$ **is conserved** $\Rightarrow$ **lightest KK-odd particle is stable.**

(only KK modes with odd j are odd under KK parity)



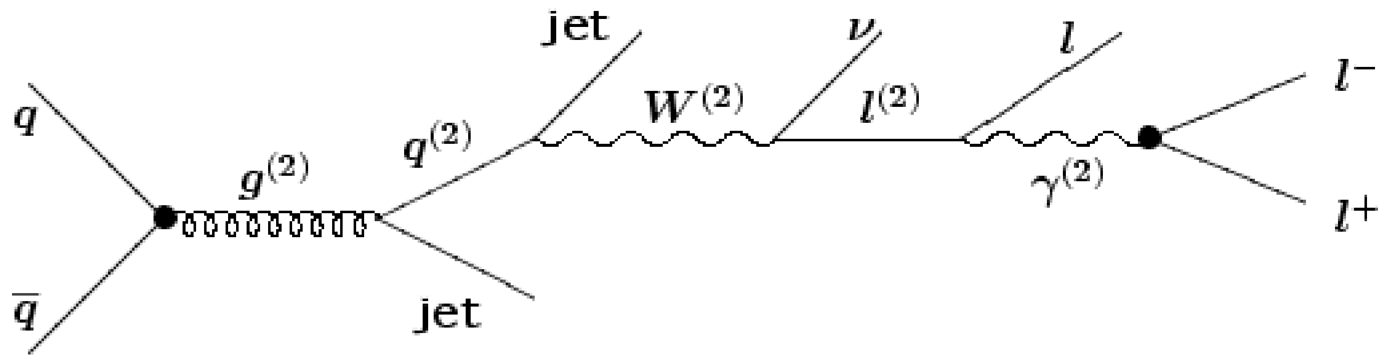
Lightest KK particle is stable in UED: $\gamma^{(1)}$ is a viable dark matter candidate

(from Servant, Tait, hep-ph/0206071)



Level-(2) masses: $\sim 2/R$.

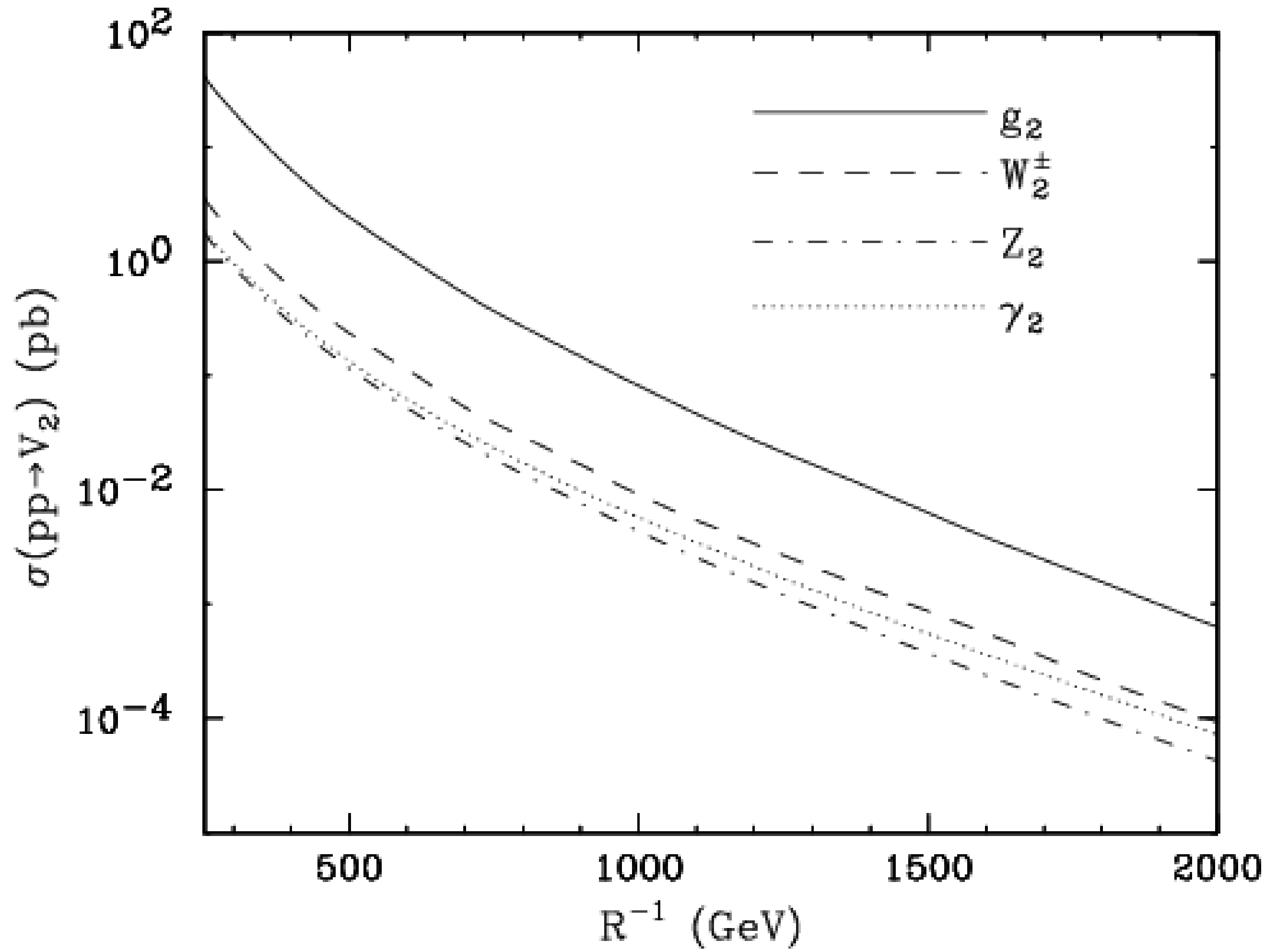
Cascade decay of the 2-mode is followed by $\gamma^{(2)}$ decay into hard leptons:



Particularly useful at the LHC (A. Datta, K. Kong, K. Matchev, hep-ph/0509246)

$\longrightarrow$ *would allow discrimination of UED & MSSM.*

Cross section for s-channel production of a level-2 boson (of mass $2/R$ + corrections) at the LHC:



(*A. Datta, K. Kong, K. Matchev, hep-ph/0509246*)

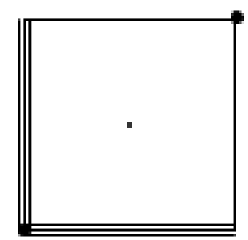
Two Universal Extra Dimensions

hep-ph/0601186, hep-ph/0703231

(G. Burdman, E. Ponton, KC Kong, R. Mahbubani, ...)

All Standard Model particles propagate in $D = 6$ dimensions.

Two dimensions are compactified on a square.



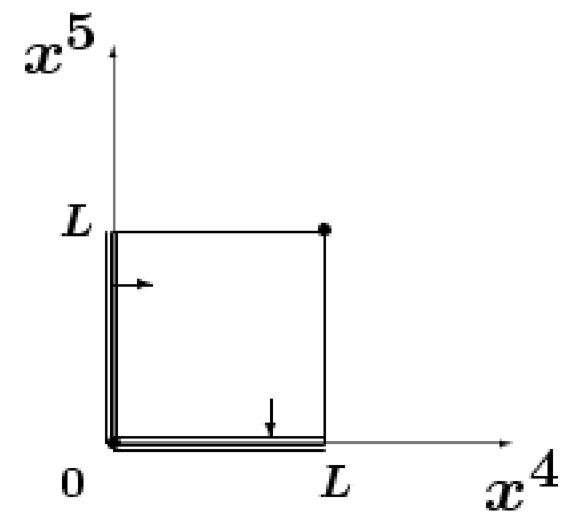
Kaluza-Klein particles are states of definite momenta along the two compact dimensions, labelled by two integers (j, k).

Tree-level masses: $\sqrt{j^2 + k^2}/R$

Momentum conservation $\longrightarrow$ KK-parity given by $j + k$

$\Rightarrow$ (1,0) particles are produced only in pairs at colliders

Chiral boundary conditions on a square



Identify pairs of adjacent sides:

$$\mathcal{L}(x^\mu, y, 0) = \mathcal{L}(x^\mu, 0, y)$$

$$\mathcal{L}(x^\mu, y, L) = \mathcal{L}(x^\mu, L, y)$$

$\Phi(y, 0) = e^{i\theta}\Phi(0, y), \ \ldots$

$\Rightarrow \ \theta = n\pi/2$

$$\partial_5\Phi|_{(x^4,x^5)=(y,0)} = -e^{in\pi/2}\,\partial_4\Phi|_{(x^4,x^5)=(0,y)}$$

Complete sets of functions satisfying the boundary conditions:

$$f_{0,2}^{(j,k)}(x^4, x^5) = \frac{1}{1+\delta_{j,0}} \left[\cos\left(\frac{jx^4 + kx^5}{R}\right) \pm \cos\left(\frac{kx^4 - jx^5}{R}\right) \right]$$

$$f_{1,3}^{(j,k)}(x^4, x^5) = i \sin\left(\frac{jx^4 + kx^5}{R}\right) \mp \sin\left(\frac{kx^4 - jx^5}{R}\right)$$

Spectrum of KK modes:

(j, k)	(1,0)	(1,1)	(2,0)	$(2,1)$ $(1,2)$	(2,2)	(3,0)	$(3,1)$ $(1,3)$
$M_{j,k}R$	1	$\sqrt{2}$	2	$\sqrt{5}$	$2\sqrt{2}$	3	$\sqrt{10}$

KK decomposition of the gauge fields:

$$A_\mu(x^\nu, x^4, x^5) = \frac{1}{L}\left[A_\mu^{(0,0)}(x^\nu) + \sum_{j\geq 1}\sum_{k\geq 0} f_0^{(j,k)}(x^4, x^5) A_\mu^{(j,k)}(x^\nu)\right]$$

$$A_4 + iA_5 \equiv A_+(x^\nu, x^4, x^5) = -\frac{1}{L}\sum_{j\geq 1}\sum_{k\geq 0} f_3^{(j,k)}(x^4, x^5) A_+^{(j,k)}(x^\nu)$$

$$A_4 - iA_5 \equiv A_-(x^\nu, x^4, x^5) = \frac{1}{L}\sum_{j\geq 1}\sum_{k\geq 0} f_1^{(j,k)}(x^4, x^5) A_-^{(j,k)}(x^\nu)$$

Physical degrees of freedom:

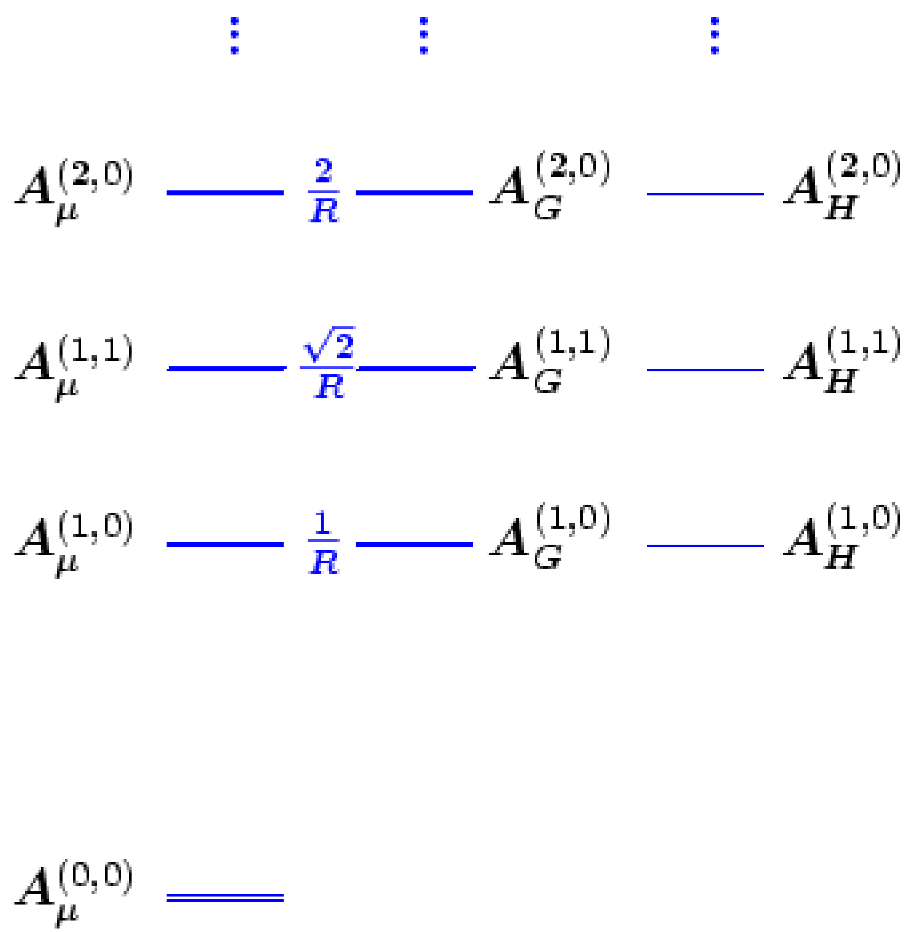
$$A_\pm^{(j,k)} = \frac{j + ik}{\sqrt{j^2 + k^2}}\left(A_H^{(j,k)} \mp iA_G^{(j,k)}\right)$$

$A_G^{(j,k)}$ is the longitudinal polarization of $A_\mu^{(j,k)}$

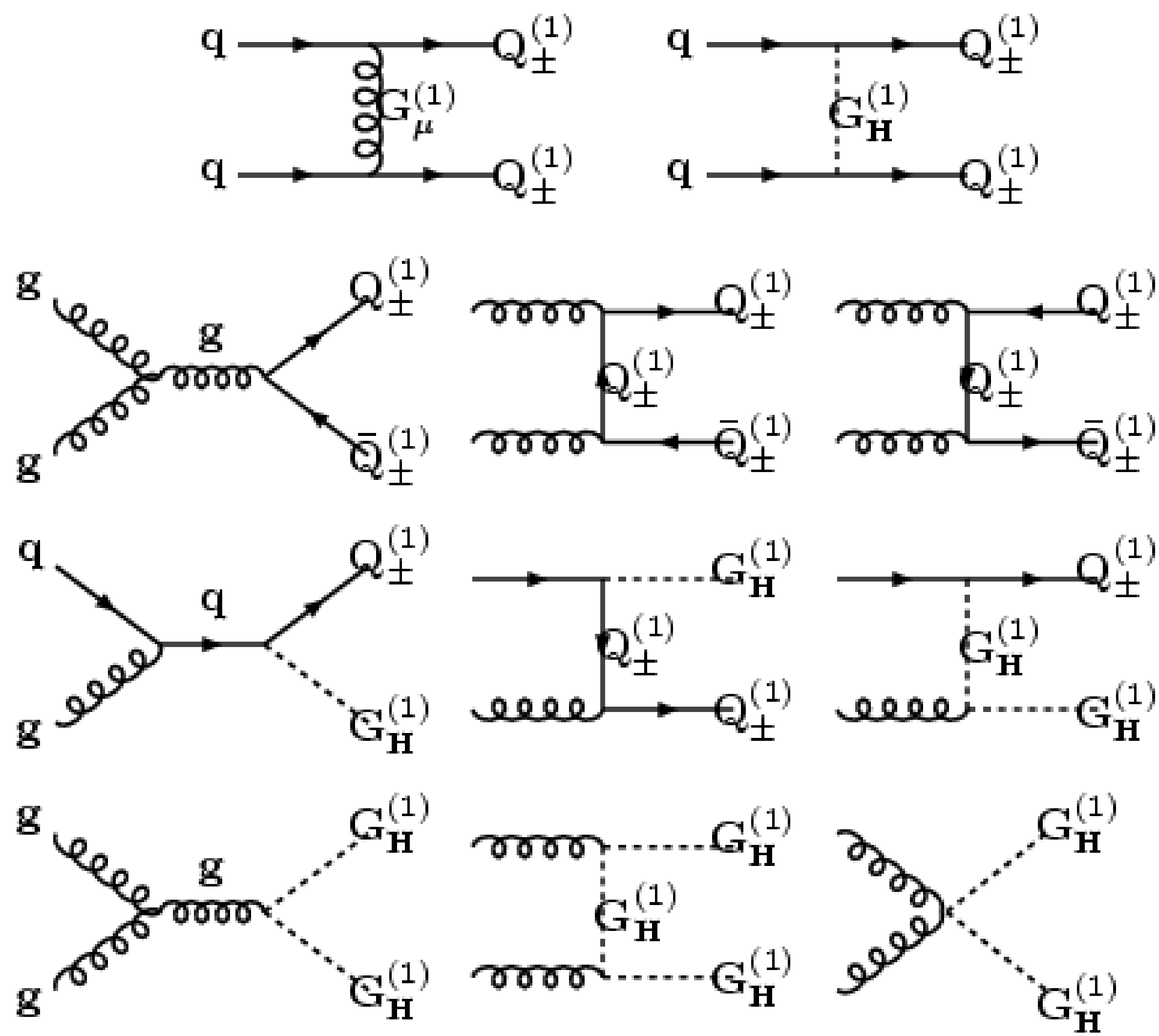
$A_H^{(j,k)}$ is a real scalar field ("spinless adjoint")

Kaluza-Klein spectrum of gauge bosons

$A_G^{(j,k)}(x^\nu)$ *becomes the longitudinal degree of freedom of the spin-1 KK mode* $A_\mu^{(j,k)}(x^\nu)$.



Production of (1,0) particles at the LHC

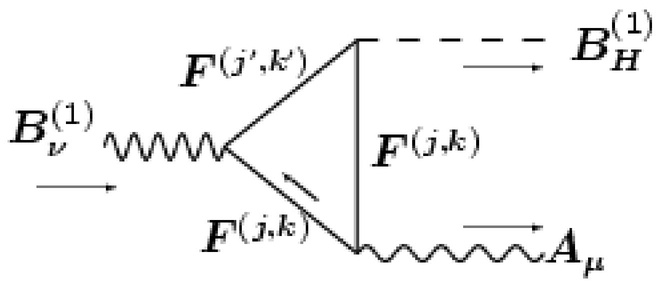


Use CalcHEP to compute cross section for (1,0) pair production.

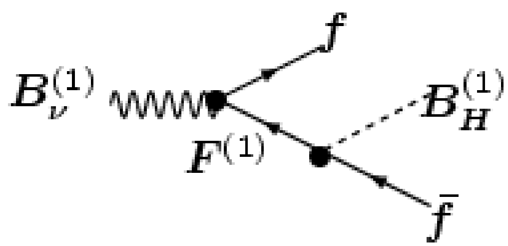
http://theory.fnal.gov/people/kckong/6d/

At one loop: $\frac{c}{R^{-1}} B_H^{(1,0)} B_{\mu\nu}^{(1,0)} \tilde{F}^{\mu\nu}$

Competition between 1-loop induced 2-body decays and tree-level 3-body decays of the (1,0) bosons.



$$Br\left(B_\mu^{(1,0)} \to B_H^{(1,0)}\gamma\right) \approx 30\%$$

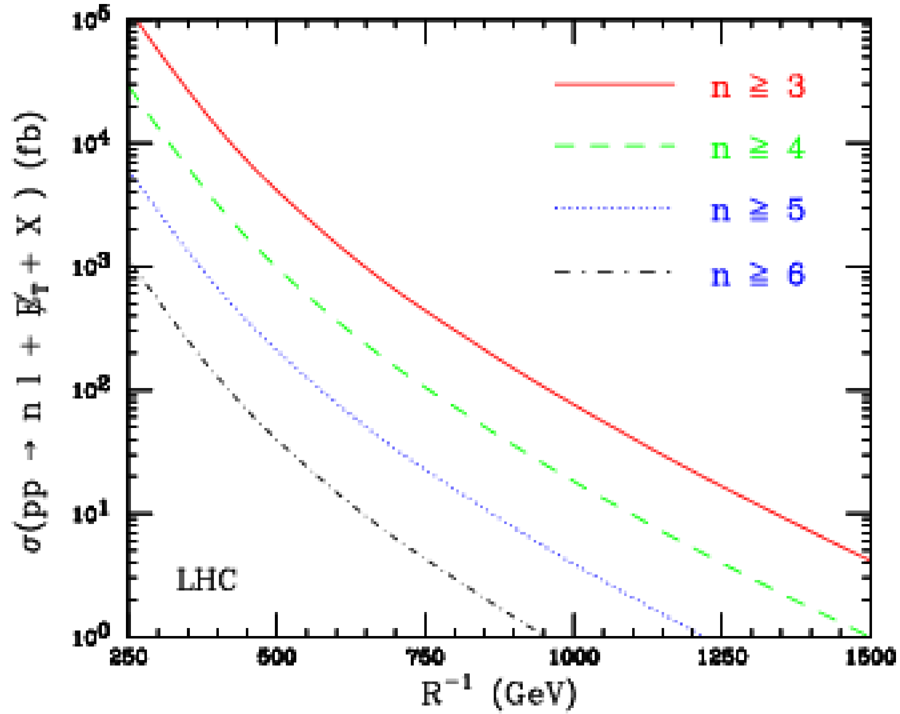


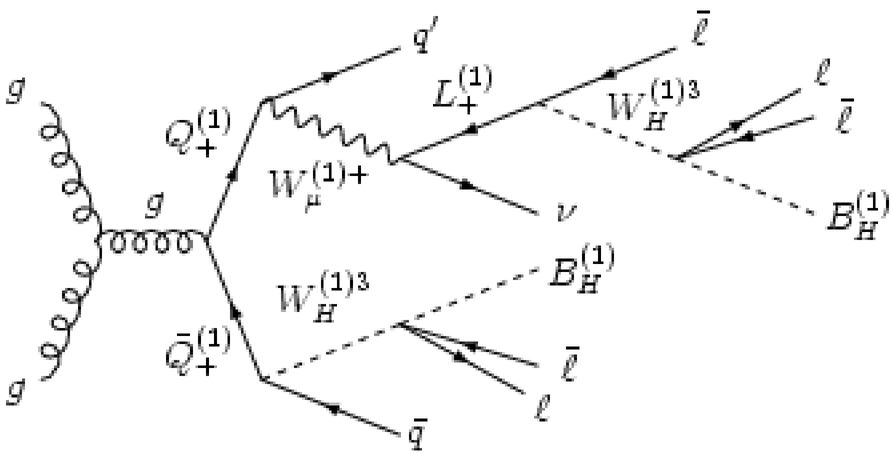
$$Br\left(B_\mu^{(1,0)} \to B_H^{(1,0)}\ell^+\ell^-\right) \approx 23\%$$

→ **Events with leptons, photons and missing E_T.**

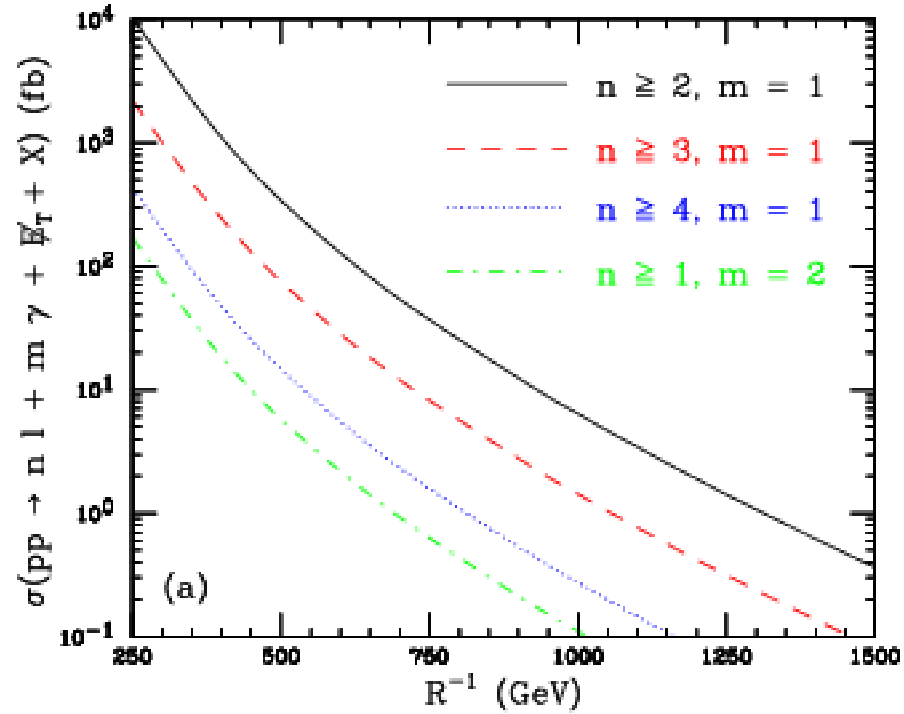
Work with K.C. Kong and Rakhi Mahbubani (hep-ph/0703231).

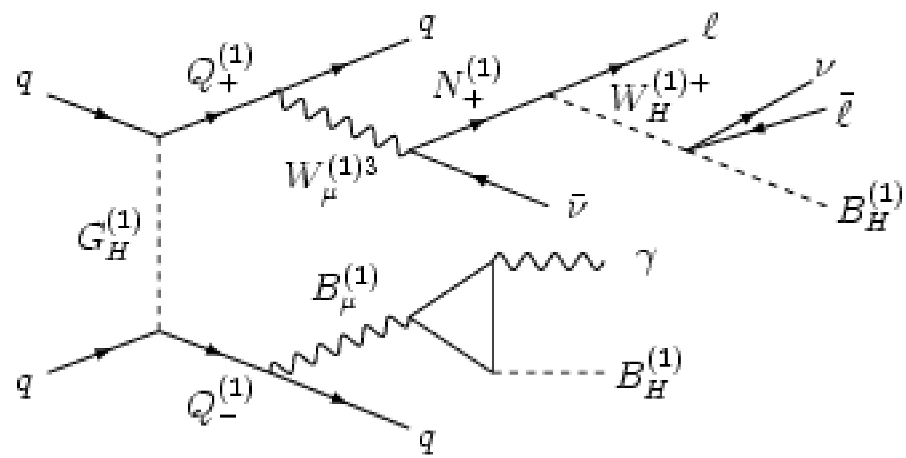
Multi-lepton signal at the LHC:





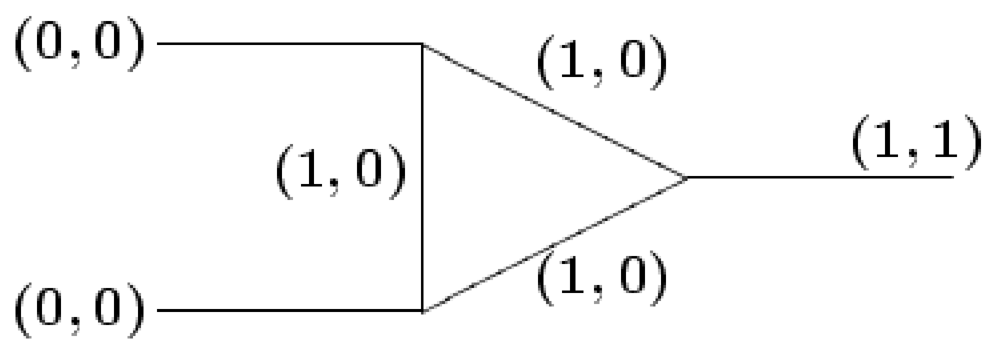
Leptons + photons at the LHC:





KK parity is conserved: $(-1)^{j+k}$

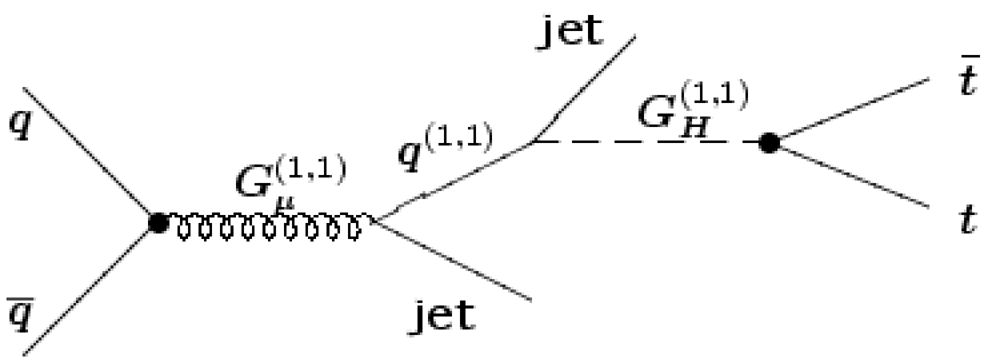
At colliders: s-channel production of the even-modes at 1-loop



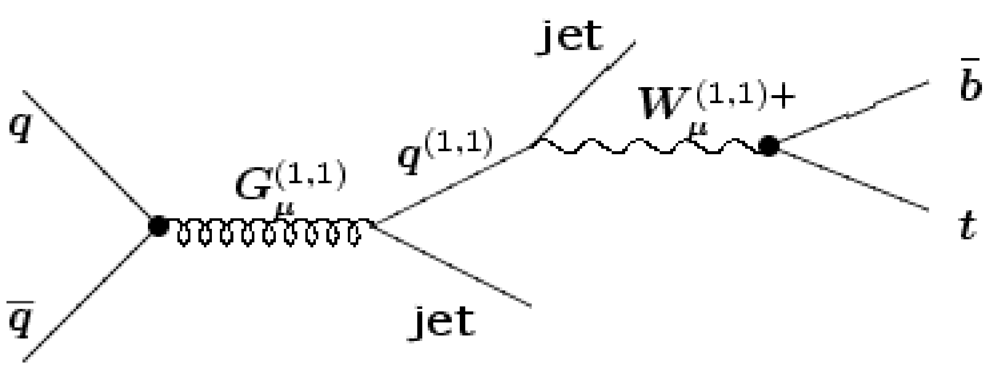
Signals of (1,1) particles at the Tevatron and LHC:

1. s-channel production of a (1,1) gluon of mass $\sim \sqrt{2}/R\,(1+\alpha_s)$

$\rightarrow$ $t\bar{t}$ resonance + 2 jets ($\sim 50 - 100$ GeV):

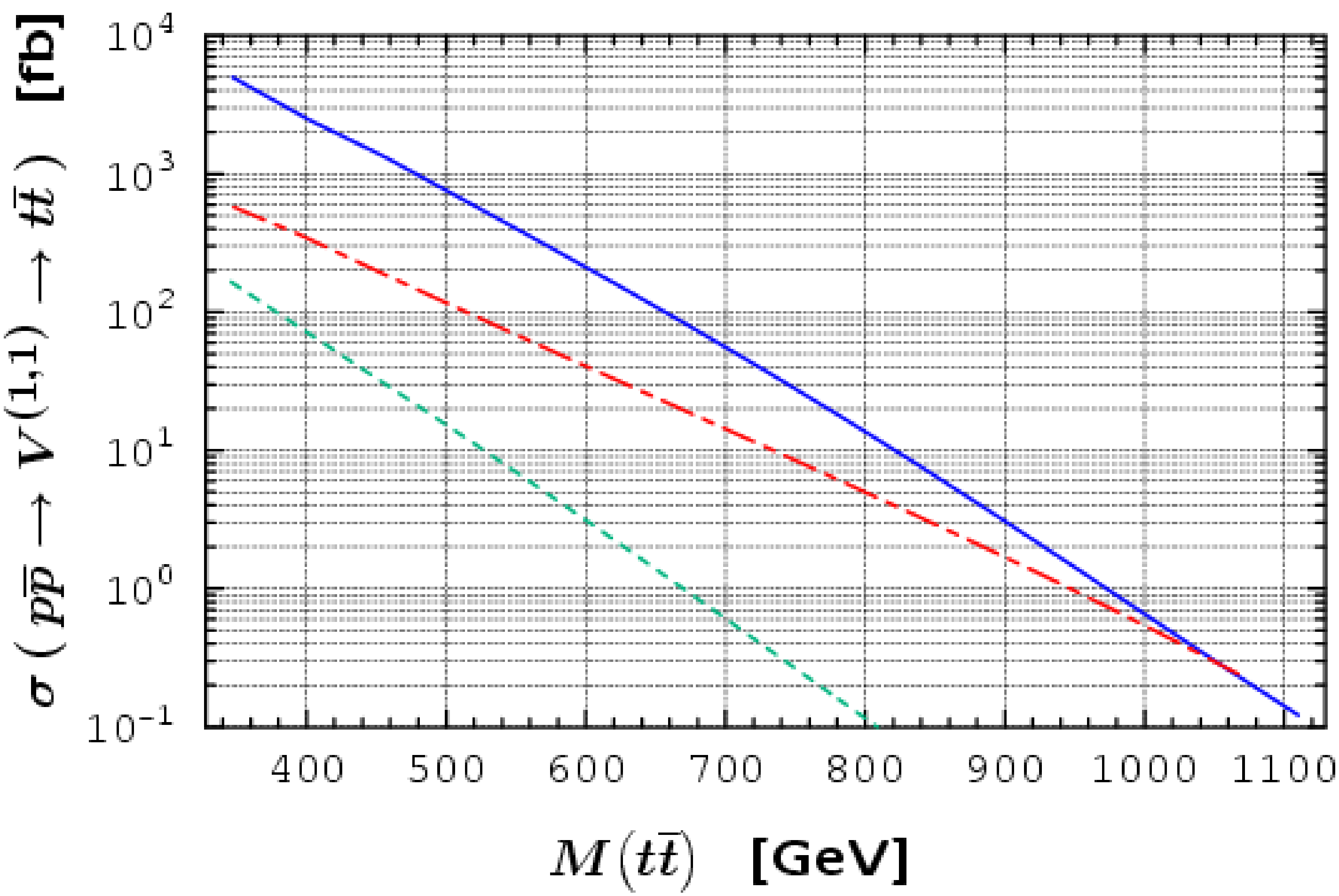


$\rightarrow$ $t\bar{b}$ resonance + 2 jets ($\sim 50 - 100$ GeV):



Production of $t\bar{t}$ pairs at the Tevatron from mass peaks at:

- $G_H^{(1,1)} + W_\mu^{(1,1)3}$ (solid blue line) $\quad M_{t\bar{t}} \simeq 1.10\,\sqrt{2}/R$
- $W_H^{(1,1)3} + B_\mu^{(1,1)}$ (red long-dashed line) $\quad M_{t\bar{t}} \simeq 0.96\,\sqrt{2}/R$
- $B_H^{(1,1)}$ (green dashed line) $\quad M_{t\bar{t}} \simeq 0.87\,\sqrt{2}/R$



Conclusions

- Any particle that propagates in $D \geq 5$ would appear in experiments as a tower of heavy 4D particles. There are purely 4D theories with similar spectra and interactions, which are interesting whether or not extra dimensions exist in nature!

- Universal Extra Dimensions

– compactification scale can be as low as ~ 300 GeV.

– lightest KK mode is a dark matter candidate

- Look for Kaluza-Klein modes at the Tevatron and the LHC:

– 3 or 4 leptons + jets + $\not{E}_T$

– series of narrow $\ell^+\ell^-$ or $t\bar{t}$ resonances due to level-2 particles.