MC Realization of IR-Improved DGLAP-CS Parton Showers:HERWIRI1.0(31)

B.F.L. Ward[a]**, S. Joseph**[a]**, S. Majhi**[a] **and S. Yost**[b]

[a] **Department of Physics, Baylor University, Waco, TX, USA**

[b] **Department of Physics, The Citadel, Charleston, SC, USA**

Outline

- **Introduction**

- **Review of Exact Amplitude-Based Resummation for QED$\otimes$QCD**

- **IR-Improved DGLAP-CS Theory: Parton Distributions, Kernels, Reduced Cross Sections with Shower/ME Matching**

- **MC Realization: IR-Improved Kernels in HERWIG6.5**

- **Conclusions**

See B.F.L.W., S. Jadach and B.F.L. Ward, S. Jadach, *et al.*,B.F.L.W. and S. Yost, MPL A 14 (1999) 491, hep-ph/0205062; *ibid.* 12 (1997) 2425; *ibid.*19 (2004) 2113; hep-ph/0503189, 0508140,0509003, 0605054, 0607198, arxiv:0704.0294, 0707.2101, 0707:3424

B. F. L. Ward   Apr. 5, 2010

Motivation

- FOR THE LHC/ILC, THE REQUIREMENTS ARE DEMANDING AND OUR $QED \otimes QCD$ SOFT n(G)-m(γ) MC RESUMMATION RESULTS WILL BE AN IMPORTANT PART OF THE NECESSARY THEORY – (YFS)RESUMMED $\mathcal{O}(\alpha_s^2)L^n, \mathcal{O}(\alpha_s\alpha)L^{n'}, \mathcal{O}(\alpha^2)L^{n''}, n = 0, 1, 2, n' = 0, 1, 2, n'' = 2, 1$, IN THE PRESENCE OF SHOWERS, ON AN EVENT-BY-EVENT BASIS, WITHOUT DOUBLE COUNTING AND WITH EXACT PHASE SPACE.

- HOW RELEVANT ARE QED HIGHER ORDER CORRECTIONS WHEN QCD IS CONTROLLED AT $\sim 1\%$ PRECISION?

- CROSS CHECK OF QCD LITERATURE:
  1. PHASE SPACE – CATANI, CATANI-SEYMOUR, ALL INITIAL PARTONS MASSLESS
  2. RESUMMATION – STERMAN, CATANI ET AL., BERGER ET AL., ...
  3. NO-GO THEOREMS–Di'Lieto et al.,Doria et al.,Catani et al.,Catani; PRD78(2008)056001
  4. IR QCD EFFECTS IN DGLAP-CS THEORY

B. F. L. Ward   Apr. 5, 2010

- CROSS CHECK OF QED-EW LITERATURE:

  1. ESTIMATES BY SPIESBERGER, STIRLING, ROTH and WEINZIERL, BLUMLEIN and KAWAMURA – FEW PER MILLE EFFECTS FROM QED CORRECTIONS TO STR. FN. EVOLUTION.

  2. WELL-KNOWN POSSIBLE ENHANCEMENT OF QED CORRECTIONS AT THRESHOLD, ESPECIALLY IN RESONANCE PRODUCTION

  3. *See for example, A. Kulesza et al., A. Denner et al., in "Proc. RADCOR07", S. Dittmaier (LP09) for large (Sudakov log, etc.) EW effects in hadron-hadron scattering – at 1TeV, W's and Z's are almost massless!*

  $\Rightarrow$ HOW TO BEST REALIZE THESE EFFECTS AT THE LHC?

- TREAT QED AND QCD SIMULTANEOUSLY IN THE (YFS) RESUMMATION TO OBTAIN THE ROLE OF THE QED-EW AND TO REALIZE AN APPROACH TO SHOWER/ME MATCHING.

- CURRENT STATE OF AFFAIRS: see N. Adam et al., JHEP **0805** (2008) 062 – Using MC@NLO and FEWZ, HORACE, PHOTOS, etc., $(4.1 \pm 0.3)\% = (1.51 \pm 0.75)\%(QCD) \oplus 3.79(PDF) \oplus 0.38 \pm 0.26(EW)\%$ accuracy on single Z to leptons at LHC was found($\sim 5.7\%$ for W, see *ibid.* 0809, 133), but no exclusive hard gluon/quark radiation phase space available – the latter are truly needed for realistic theoretical results. They are our goal, at $\lesssim 1\%$.

B. F. L. Ward   Apr. 5, 2010

PRELIMINARY STUDIES

- REPRESENTATIVE PROCESSES

  $pp \rightarrow V + m(\gamma) + n(G) + X \rightarrow \bar{\ell}\ell' + m'(\gamma) + n(G) + X,$
  where $V = W^{\pm}, Z$, and $\ell = e, \mu, \ell' = \nu_e, \nu_\mu(e, \mu)$
  respectively for $V = W^+(Z)$, and $\ell = \nu_e, \nu_\mu, \ell' = e, \mu$
  respectively for $V = W^-$.

- Realize IR-improved kernels in state-of-the-art MC environment:
  HERWIG-6.5 => HERWIRI1.0(31)

B. F. L. Ward
Apr. 5, 2010

Recapitulation of QED⊗QCD Resummation

In hep-ph/0210357(ICHEP02), Acta Phys.Polon.B33,1543-1558,2002, Phys.Rev.D52(1995)108;ibid. 66 (2002) 019903(E);PLB342 (1995) 239; Ann.Phys.323(2008)2147;Phys. Rev.D78(2008)056001; Adv. High Energy Phys. 2008 (2008): 682312, we have extended the YFS theory to QCD:

$$d\hat{\sigma}_{\exp} = \sum_n d\hat{\sigma}^n$$

$$= e^{\text{SUM}_{\text{IR}}(\text{QCD})} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{\text{QCD}}}$$

$$* \tilde{\bar{\beta}}_n(k_1,\ldots,k_n) \frac{d^3 P_2}{P_2^0} \frac{d^3 Q_2}{Q_2^0}$$

$$\tag{1}$$

where the new hard gluon residuals $\tilde{\bar{\beta}}_n(k_1,\ldots,k_n)$ defined by

$$\tilde{\bar{\beta}}_n(k_1,\ldots,k_n) = \sum_{\ell=0}^{\infty} \tilde{\bar{\beta}}_n^{(\ell)}(k_1,\ldots,k_n)$$

are free of all infrared divergences to all orders in $\alpha_s(Q)$. $\Rightarrow$

Simultaneous exponentiation of QED and QCD higher order effects, hep-ph/0404087(MPLA19(2004)2119), arXiv:0704.0294(APPB38(2007)2395), arXiv:0808.3133, 0810.0723 ,

gives

$$B_{QCD}^{nls} \to B_{QCD}^{nls} + B_{QED}^{nls} \equiv B_{QCED}^{nls},$$
$$\tilde{B}_{QCD}^{nls} \to \tilde{B}_{QCD}^{nls} + \tilde{B}_{QED}^{nls} \equiv \tilde{B}_{QCED}^{nls},$$
$$\tilde{S}_{QCD}^{nls} \to \tilde{S}_{QCD}^{nls} + \tilde{S}_{QED}^{nls} \equiv \tilde{S}_{QCED}^{nls} \tag{2}$$

which leads to

$$d\hat{\sigma}_{\exp} = e^{\text{SUM}_{\text{IR}}(\text{QCED})} \sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}}$$

$$\prod_{j_2=1}^{m} \frac{d^3 k'_{j_2}}{k'_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2})+D_{\text{QCED}}}$$

$$\bar{\tilde{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3 p_2}{p_2^0}\frac{d^3 q_2}{q_2^0}, \tag{3}$$

where the new YFS residuals
$\bar{\tilde{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$**, with** n **hard gluons and** m **hard photons,**

represent the successive application of the YFS expansion first for QCD and subsequently for QED.

The infrared functions are now

$$\mathrm{SUM_{IR}(QCED)} = 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED}$$

$$D_{\mathrm{QCED}} = \int \frac{dk}{k^0} \left(e^{-iky} - \theta(K_{max} - k^0) \right) \tilde{S}^{nls}_{QCED} \tag{4}$$

where K_{max} is a dummy parameter – here the same for QCD and QED.

Infrared Algebra(QCED):

$x_{avg}(QED) \cong \gamma(QED)/(1 + \gamma(QED))$

$x_{avg}(QCD) \cong \gamma(QCD)/(1 + \gamma(QCD))$

$\gamma(A) = \frac{2\alpha_A C_A}{\pi}(L_s - 1), A = QED, QCD$

$C_A = Q_f^2, C_F$, **respectively, for** $A = QED, QCD$

$\Rightarrow$ **QCD dominant corrections happen an order of magnitude earlier than those for QED.**

$\Rightarrow$ **Leading $\tilde{\tilde{\beta}}^{(0,0)}_{0,0}$-level gives a good estimate of the size of the effects we study:**

see arxiv.org: 0808.3133, and references therein.

Relationship to Sterman-Catani-Trentadue Soft Gluon Resummation

In Phys. Rev. D74 (2006) 074004[MADG], Abayat et al. apply the more familiar resummation for soft gluons to a general $2 \to n$ parton process [f] at hard scale Q,

$$f_1(p_1, r_1) + f_2(p_2, r_2) \to f_3(p_3, r_3) + f_4(p_4, r_4) + \cdots + f_{n+2}(p_{n+2}, r_{n+2}),$$

where the p_i, r_i label 4-momenta and color indices respectively, with all parton masses set to zero to get

$$\mathcal{M}^{[f]}_{\{r_i\}} = \sum_L^C \mathcal{M}^{[f]}_L (c_L)_{\{r_i\}}$$

$$= J^{[f]} \sum_L^C S_{LI} H^{[f]}_I (c_L)_{\{r_i\}},$$

(5)

$J^{[f]}$ **is the jet function**

S_{LI} **is the soft function which describes the exchange of soft gluons between the external lines**

$H_I^{[f]}$ **is the hard coefficient function**

infrared and collinear poles calculated to 2-loop order.

To make contact with our approach, identify in $\bar{Q}'Q \to \bar{Q}'''Q'' + m(G)$ **in (1)**

$f_1 = Q, \bar{Q}', f_2 = \bar{Q}', f_3 = Q'', f_4 = \bar{Q}''', \{f_5, \cdots, f_{n+2}\} =$
$\{G_1, \cdots, G_m\}$
$\Rightarrow n = m + 2$ **here.**

Observe the following:

- **By its definition in eq.(2.23) of [MADG], the anomalous dimension of the matrix** S_{LI} **does not contain any of the diagonal effects described by our infrared functions** $\Sigma_{IR}(QCD)$ **and** D_{QCD}**.**

- **By its definition in eqs.(2.5) and (2.7) of [MADG], the jet function** $J^{[f]}$ **contains the exponential of the virtual infrared function** $\alpha_s \Re B_{QCD}$**, so that we have to take care that we do not double count when we use (5) in (1) and the equations that lead thereto.**

$\Rightarrow$

We identify $\bar{\rho}^{(m)}$ in our theory as

$$\bar{\rho}^{(m)}(p_1, q_1, p_2, q_2, k_1, \cdots, k_m) = \overline{\sum}_{colors,spin} |\mathcal{M}^{'[f]}_{\{r_i\}}|^2$$

$$\equiv \sum_{spins,\{r_i\},\{r_i'\}} \mathfrak{h}^{cs}_{\{r_i\}\{r_i'\}} |\bar{J}^{[f]}|^2 \sum_{L=1}^{C} \sum_{L'=1}^{C} S^{[f]}_{LI} H^{[f]}_{I}(c_L)_{\{r_i\}} \left(S^{[f]}_{L'I'} H^{[f]}_{I'}(c_{L'})_{\{r_i'\}} \right)^{\dagger},$$

(6)

where here we defined $\bar{J}^{[f]} = e^{-\alpha_s \Re B_{QCD}} J^{[f]}$, and we introduced the color-spin density matrix for the initial state, $\mathfrak{h}^{cs}$.

Here, we recall (see Ann.Phys.323(2008)2147;Phys. Rev.D78(2008)056001; Adv. High Energy Phys. 2008 (2008): 682312, for example) that in our theory, we have

$$d\hat{\sigma}^n = \frac{e^{2\alpha_s Re B_{QCD}}}{n!} \int \prod_{m=1}^{n} \frac{d^3 k_m}{(k_m^2 + \lambda^2)^{1/2}} \delta(p_1 + q_1 - p_2 - q_2 - \sum_{i=1}^{n} k_i)$$

$$\bar{\rho}^{(n)}(p_1, q_1, p_2, q_2, k_1, \cdots, k_n) \frac{d^3 p_2 d^3 q_2}{p_2^0 q_2^0}, \quad (7)$$

for n-gluon emission. $\Rightarrow$ **Repeat usual steps to get our formula (1), no double counting of effects - in progress.** **Today, we show platform for this progress.**

QED$\otimes$QCD RESUMMATION

$\updownarrow$

SCT RESUMMATION

$\updownarrow$(Lee & Sterman)

SCET RESUMMATION

IR-Improved DGLAP-CS Theory: Parton Distributions,

Kernels, Reduced Cross Sections with

Shower/ME Matching

IR-Improved DGLAP-CS Theory

Exponentiation of QCD higher order effects: Where to apply?

Ann.Phys.323(2008)2147;Phys. Rev.D78(2008)056001; Adv. High Energy Phys. 2008 (2008): 682312,

consider

$$\frac{dq^{NS}(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dy}{y} q^{NS}(y,t) P_{qq}(x/y) \tag{8}$$

where the well-known result for the kernel $P_{qq}(z)$ is, for $z < 1$,

$$P_{qq}(z) = C_F \frac{1+z^2}{1-z}, \tag{9}$$

$t = \ln \mu^2/\mu_0^2$ **for some reference scale μ_0.** $\Rightarrow$

Unintegrable singularity at $z = 1$, usually regularized by

$$\frac{1}{(1-z)} \rightarrow \frac{1}{(1-z)_+} \tag{10}$$

with $\frac{1}{(1-z)_+}$ such that

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z) - f(1)}{(1-z)}. \tag{11}$$

$\Rightarrow$

$$\frac{1}{(1-z)_+} = \frac{1}{(1-z)} \theta(1 - \epsilon - z) + \ln \epsilon \, \delta(1-z) \tag{12}$$

with the understanding that $\epsilon \downarrow 0$.

Require

$$\int_0^1 dz P_{qq}(z) = 0, \tag{13}$$

$\Rightarrow$ **add virtual corrections to get**

$$P_{qq}(z) = C_F \left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right). \tag{14}$$

Observations

- Smooth, divergent $1/(1-z)$ behavior as $z \to 1$ replaced with a mathematical artifact: the regime $1 - \epsilon < z < 1$ now has no probability at all; at $z = 1$ we have a large negative integrable contribution $\Rightarrow$ a finite (zero) value for the total integral of $P_{qq}(z)$

- LEP1,2 experience: such mathematical artifacts, while correct, impair precision.

**Why set $P_{qq}(z)$ to 0 for $1 - \epsilon < z < 1$ where it actually has its largest values?**

- **USE EXPERIENCE FROM LEP1,2:** $\frac{1}{(1-z)_+}$ **SHOULD BE EXPONENTIATED –SEE CERN YELLOW-BOOKS, CERN-89-08.**, **YIELDING FROM (1) THE REPLACEMENT**

$$P_{BA} = \frac{1}{2} z(1 - z) \overline{\sum_{spins}} \frac{|V_{A \to B+C}|^2}{p_\perp^2}$$

$$\Rightarrow \tag{15}$$

$$P_{BA} = \frac{1}{2} z(1 - z) \overline{\sum_{spins}} \frac{|V_{A \to B+C}|^2}{p_\perp^2} z^{\gamma_q} F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q}$$

WHERE $A = q$, $B = G$, $C = q$ AND $V_{A \to B+C}$ IS THE LOWEST ORDER AMPLITUDE FOR $q \to G(z) + q(1 - z)$.

$$G_1(\xi_1) \quad G_\ell(\xi_\ell) \quad G_n(z - \sum_{j<n} \xi_j)$$

$$\cdots \qquad \cdots$$

$$q \qquad\qquad\qquad\qquad\qquad q(1-z)$$

$$q \to q(1-z) + G_1 + \cdots + G_n$$

$\Rightarrow$

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+z^2}{1-z}(1-z)^{\gamma_q} \qquad (16)$$

where

$$\gamma_q = C_F \frac{\alpha_s}{\pi} t = \frac{4C_F}{\beta_0} \qquad (17)$$

$$\delta_q = \frac{\gamma_q}{2} + \frac{\alpha_s C_F}{\pi} \left(\frac{\pi^2}{3} - \frac{1}{2} \right) \qquad (18)$$

and

$$F_{YFS}(\gamma_q) = \frac{e^{-C_E \gamma_q}}{\Gamma(1 + \gamma_q)}. \qquad (19)$$

Note: $\int_{k_0} dz/z = C_0 - \ln k_0$

is experimentally distinguishable from

$\int_{k_0} dz/z^{1-\gamma} = C'_0 - k_0^\gamma/\gamma.$

- **NORMALIZATION CONDITION (13) $\Rightarrow$:**

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right] \quad (20)$$

  where

$$f_q(\gamma_q) = \frac{2}{\gamma_q} - \frac{2}{\gamma_q+1} + \frac{1}{\gamma_q+2}. \quad (21)$$

- **THIS IS OUR IR-IMPROVED P_{qq} DGLAP-CS KERNEL.**

$\Rightarrow$ **STANDARD DGLAP-CS THEORY:**

for $z < 1$, we have

$$P_{Gq}(z) = P_{qq}(1-z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}. \quad (22)$$

$\Rightarrow$ **TEST OF NEW THEORY – QUARK MOMENTUM SUM RULE:**

$$\int_0^1 dz\, z\, (P_{Gq}(z) + P_{qq}(z)) = 0. \quad (23)$$

⇒ **CHECK VANISHING OF**

$$I = \int_0^1 dz\, z \left(\frac{1 + (1-z)^2}{z} z^{\gamma_q} + \frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right). \quad (24)$$

NOTE,

$$\frac{z}{1-z} = \frac{z-1+1}{1-z} = -1 + \frac{1}{1-z}. \quad (25)$$

⇒

$$I = \int_0^1 dz\{(1+(1-z)^2)z^{\gamma_q} - (1+z^2)(1-z)^{\gamma_q}$$

$$+ \frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z)\}$$

$$= 0$$

QUARK MOMENTUM SUM RULE IS SATISFIED.

- **For $P_{qG}(z)$, $P_{GG}(z)$, we get, with the replacement $C_F \to C_G$ in the IR algebra, that the usual results**

$$P_{GG}(z) = 2C_G\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right)$$
$$P_{qG}(z) = \frac{1}{2}(z^2 + (1-z)^2) \tag{26}$$

become

$$P_{GG}(z) = 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G}\left\{\frac{1-z}{z}z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G}\right.$$
$$\left. + \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G)\delta(1-z)\right\}, \tag{27}$$
$$P_{qG}(z) = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G}\frac{1}{2}\left\{z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G}\right\}, \tag{28}$$

where

$$\gamma_G = C_G \frac{\alpha_s}{\pi} t = \frac{4C_G}{\beta_0} \tag{29}$$

$$\delta_G = \frac{\gamma_G}{2} + \frac{\alpha_s C_G}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right), \tag{30}$$

$$f_G(\gamma_G) = \frac{n_f}{C_G} \frac{1}{(1+\gamma_G)(2+\gamma_G)(3+\gamma_G)} + \frac{2}{\gamma_G(1+\gamma_G)(2+\gamma_G)} \tag{31}$$

$$+ \frac{1}{(1+\gamma_G)(2+\gamma_G)} + \frac{1}{2(3+\gamma_G)(4+\gamma_G)} \tag{32}$$

$$+ \frac{1}{(2+\gamma_G)(3+\gamma_G)(4+\gamma_G)}. \tag{33}$$

THE GLUON MOMENTUM SUM RULE HAS BEEN USED.

- **THIS DEFINES THE NEW IR-IMPROVED DGLAP-CS THEORY.**

IR-IMPROVED DGLAP-CS KERNELS

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right], \qquad (34)$$

$$P_{Gq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}, \qquad (35)$$

$$P_{GG}(z) = 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \{ \frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G}$$

$$+ \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G)\delta(1-z) \}, \qquad (36)$$

$$P_{qG}(z) = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2}\{ z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G} \}. \qquad (37)$$

Higher Order DGLAP-CS Kernels

Connection with the exact $\mathcal{O}(\alpha_s^2)$, $\mathcal{O}(\alpha_s^3)$ kernel results of **Curci, Furmanski and Petronzio, Floratos et al., Moch et al., etc., is immediate:**

For example, non-singlet case, using standard notation,

$$P_{ns}^+ = P_{qq}^v + P_{q\bar{q}}^v \equiv \sum_{n=0}^{\infty} (\frac{\alpha_s}{4\pi})^{n+1} P_{ns}^{(n)+} \tag{38}$$

where at order $\mathcal{O}(\alpha_s)$ we have

$$P_{ns}^{(0)+}(z) = 2C_F\{ \frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\} \tag{39}$$

$\Rightarrow P_{ns}^{(0)+}(z)$ agrees with the unexponentiated result for P_{qq} except for an overall factor of 2. **Floratos et al., etc., have exact result for** $P_{ns}^{(1)+}(z)$**, and Moch et al. have**

exact results for $P_{ns}^{(2)+}(z)$**. Applying (1) to** $q \to q + X, \bar{q} \to q + X'$**, we get**

$$P_{ns}^{+,exp}(z) = (\frac{\alpha_s}{4\pi})2P_{qq}^{exp}(z) + F_{YFS}(\gamma_q)e^{\frac{1}{2}\delta_q}\Big[(\frac{\alpha_s}{4\pi})^2\{(1-z)^{\gamma_q}\bar{P}_{ns}^{(1)+}(z)$$

$$+ \bar{B}_2\delta(1-z)\} + (\frac{\alpha_s}{4\pi})^3\{(1-z)^{\gamma_q}\bar{P}_{ns}^{(2)+}(z) + \bar{B}_3\delta(1-z)\}\Big]$$

$$(40)$$

where $P_{qq}^{exp}(z)$ **is given above and the resummed residuals** $\bar{P}_{ns}^{(i)+}, i = 1,2$ **are related to the exact results for** $P_{ns}^{(i)+}, i = 1,2$**, as follows:**

$$\bar{P}_{ns}^{(i)+}(z) = P_{ns}^{(i)+}(z) - B_{1+i}\delta(1-z) + \Delta_{ns}^{(i)+}(z) \qquad (41)$$

where

$$\Delta_{ns}^{(1)+}(z) = -4C_F\pi\delta_1\{\frac{1+z^2}{1-z} - f_q\delta(1-z)\}$$

$$\Delta_{ns}^{(2)+}(z) = -4C_F(\pi\delta_1)^2\{\frac{1+z^2}{1-z} - f_q\delta(1-z)\} \qquad (42)$$

$$- 2\pi\delta_1\bar{P}_{ns}^{(1)+}(z)$$

and

$$\bar{B}_2 = B_2 + 4C_F\pi\delta_1 f_q$$

$$\bar{B}_3 = B_3 + 4C_F(\pi\delta_1)^2 f_q - 2\pi\delta_1\bar{B}_2.$$

(43)

The constants B_i, $i = 2, 3$ are given by

$$B_2 = 4C_G C_F\left(\frac{17}{24} + \frac{11}{3}\zeta_2 - 3\zeta_3\right) - 4C_F n_f\left(\frac{1}{12} + \frac{2}{3}\zeta_2\right) + 4C_F^2\left(\frac{3}{8} - 3\zeta_2 + 6\zeta_3\right)$$

$$B_3 = 16C_G C_F n_f\left(\frac{5}{4} - \frac{167}{54}\zeta_2 + \frac{1}{20}\zeta_2^2 + \frac{25}{18}\zeta_3\right)$$

$$+ 16C_G C_F^2\left(\frac{151}{64} + \zeta_2\zeta_3 - \frac{205}{24}\zeta_2 - \frac{247}{60}\zeta_2^2 + \frac{211}{12}\zeta_3 + \frac{15}{2}\zeta_5\right)$$

$$+ 16C_G^2 C_F\left(-\frac{1657}{576} + \frac{281}{27}\zeta_2 - \frac{1}{8}\zeta_2^2 - \frac{97}{9}\zeta_3 + \frac{5}{2}\zeta_5\right)$$

$$+ 16C_F n_F^2\left(-\frac{17}{144} + \frac{5}{27}\zeta_2 - \frac{1}{9}\zeta_3\right)$$

$$+ 16C_F^2 n_F\left(-\frac{23}{16} + \frac{5}{12}\zeta_2 + \frac{29}{30}\zeta_2^2 - \frac{17}{6}\zeta_3\right)$$

$$+ 16C_F^3\left(\frac{29}{32} - 2\zeta_2\zeta_3 + \frac{9}{8}\zeta_2 + \frac{18}{5}\zeta_2^2 + \frac{17}{4}\zeta_3 - 15\zeta_5\right).$$

(44)

Contact with Wilson Expansion

N-th moment of the invariants $T_{i,\ell}$, $i = L, 2, 3$, $\ell = q, G$, of the forward Compton amplitude in DIS:(Gorishni et al.)

$$\mathcal{P}_N \equiv \left[\frac{q^{\{\mu_1} \cdots q^{\mu_N\}}}{N!} \frac{\partial^N}{\partial p^{\mu_1} \cdots \partial p^{\mu_N}} \right] \big|_{p=0}, \qquad (45)$$

$x_{Bj} = Q^2/(2qp)$ **in the standard DIS notation – Projects the coefficient of** $1/(2x_{Bj})^N$**. Terms which we resum here** $\Leftrightarrow$ **Formally** γ_q**-dependent anomalous dimensions associated with the respective coefficient, not in Wilson's expansion by usual definition.:**

LARGE λ NOT ALL ON TIP OF LIGHTCONE.

COMMENTS

($*$) **IRI-DGLAP-CS RESUMS IR SINGULAR ISR;BY FACTORIZATION THIS IS NOT CONTAINED IN ANY RESUMMATION OF HARD SHORT-DISTANCE COEFFICIENT FN CORRECTIONS AS IN THE STERMAN, CATANI-TRENTADUE, COLLINS ET AL. FORMULAS**

($**$) **WE DO NOT CHANGE THE PREDICTED HADRON CROSS SECTION:**

$$\sigma = \sum_{i,j} \int dx_1 dx_2 F_i(x_1) F_j(x_2) \hat{\sigma}(x_1 x_2 s)$$
$$= \sum_{i,j} \int dx_1 dx_2 F'_i(x_1) F'_j(x_2) \hat{\sigma}'(x_1 x_2 s)$$

(46)

ORDER BY ORDER IN PERTURBATION THEORY.

$\{P^{exp}\}$ **factorize** $\hat{\sigma}_{\text{unfactorized}} \Rightarrow \hat{\sigma}' $**– NEW SCHEME**

$\{P\}$ **factorize** $\hat{\sigma}_{\text{unfactorized}} \Rightarrow \hat{\sigma}$

($***$) **QUARK NUMBER CONSERVATION AND CANCELLATION OF IR SINGULARITIES IN XSECTS: Quaranteed by fundamental quantum field theoretic principles: Global Gauge Invariance, Unitarity – Everybody may use these principles.**

B. F. L. Ward Apr. 5, 2010

Effects on Parton Distributions

Moments of kernels $\Leftrightarrow$ Logarithmic exponents for evolution

$$\frac{dM_n^{NS}(t)}{dt} = \frac{\alpha_s(t)}{2\pi} A_n^{NS} M_n^{NS}(t) \qquad (47)$$

where

$$M_n^{NS}(t) = \int_0^1 dz\, z^{n-1} q^{NS}(z,t) \qquad (48)$$

and the quantity A_n^{NS} is given by

$$A_n^{NS} = \int_0^1 dz\, z^{n-1} P_{qq}(z),$$

$$= C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} [B(n,\gamma_q) + B(n+2,\gamma_q) - f_q(\gamma_q)] \qquad (49)$$

where $B(x,y)$ is the beta function given by

$$B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$$

.

Compare the usual result

$$A_n^{NS^o} \equiv C_F \left[-\frac{1}{2} + \frac{1}{n(n+1)} - 2 \sum_{j=2}^{n} \frac{1}{j} \right]. \tag{50}$$

- **ASYMPTOTIC BEHAVIOR: IR-improved goes to a multiple of $-f_q$,**
  consistent with
  $\lim_{n \to \infty} z^{n-1} = 0$ **for** $0 \leq z < 1$;
  usual result diverges as $-2C_F \ln n$.

- **Different for finite n as well: for** $n = 2$ **we get, for example, for** $\alpha_s \cong .118$,

$$A_2^{NS} = \begin{cases} C_F(-1.33) & , \quad \text{un-IR-improved} \\ C_F(-0.966) & , \quad \text{IR-improved} \end{cases} \tag{51}$$

- **For completeness we note**

$$M_n^{NS}(t) = M_n^{NS}(t_0) e^{\int_{t_0}^{t} dt' \frac{\alpha_s(t')}{2\pi} A_n^{NS}(t')}$$

$$= M_n^{NS}(t_0) e^{\bar{a}_n [Ei(\frac{1}{2}\delta_1\alpha_s(t_0)) - Ei(\frac{1}{2}\delta_1\alpha_s(t))]} \tag{52}$$

$$\underset{t,t_0 \text{ large with } t>>t_0}{\Longrightarrow} M_n^{NS}(t_0) \left(\frac{\alpha_s(t_0)}{\alpha_s(t)} \right)^{\bar{a}'_n}$$

where $Ei(x) = \int_{-\infty}^{x} dr e^r / r$ **is the exponential integral function,**

$$\bar{a}_n = \frac{2C_F}{\beta_0} F_{YFS}(\gamma_q) e^{\frac{\gamma_q}{4}} [B(n, \gamma_q) + B(n+2, \gamma_q) - f_q(\gamma_q)]$$

$$\bar{a}'_n = \bar{a}_n \left(1 + \frac{\delta_1}{2} \frac{(\alpha_s(t_0) - \alpha_s(t))}{\ln(\alpha_s(t_0)/\alpha_s(t))} \right) \tag{53}$$

with

$\delta_1 = \frac{C_F}{\pi} \left(\frac{\pi^2}{3} - \frac{1}{2} \right).$

Compare with un-IR-improved result where last line in eq.(52) holds exactly with $\bar{a}'_n = 2A_n^{NS^o}/\beta_0$.

- **For $n = 2$, taking $Q_0 = 2$GeV and evolving to $Q = 100$GeV, with $\Lambda_{QCD} \cong .2 GeV$ and $n_f = 5$ for illustration, (52,53) $\Rightarrow$ a shift of evolved NS moment by $\sim 5\%$,** of some interest in view of the expected HERA precision (see for example, T. Carli et al., Proc. HERA-LHC Wkshp, 2005).

ANOTHER EXAMPLE: THRESHOLD CORRECTIONS

We have applied the new simultaneous QED$\otimes$QCD exponentiation calculus to the single Z production with leptonic decay at the LHC (and at FNAL) to focus on the ISR alone, for definiteness. See also the work of Baur *et al.*, Dittmaier and Kramer, Zykunov for exact $\mathcal{O}(\alpha)$ results and Hamberg *et al.*, van Neerven and Matsuura and Anastasiou *et al.* for exact $\mathcal{O}(\alpha_s^2)$ results.

For the basic formula

$$d\sigma_{exp}(pp \to V+X \to \bar{\ell}\ell'+X') = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s),$$

(54)

we use the result in (3) here with semi-analytical methods and structure functions from Martin *et al.*. A MC realization will appear, but see below.

SHOWER/ME MATCHING

– Note the following: In (54) WE **DO NOT** ATTEMPT *HERE* TO REPLACE **HERWIG and/or PYTHIA** – WE INTEND *HERE* TO COMBINE OUR EXACT YFS CALCULUS, $d\hat{\sigma}_{exp}(x_i x_j s)$, WITH **HERWIG and/or PYTHIA** BY USING THEM/IT TO GENERATE A PARTON SHOWER STARTING FROM (x_1, x_2) AT FACTORIZATION SCALE μ AFTER THIS POINT IS PROVIDED BY $\{F_i\}$: THERE ARE TWO APPROACHES TO THE MATCHING UNDER STUDY, ONE BASED ON p_T-MATCHING AND ONE BASED ON SHOWER-SUBTRACTED RESIDUALS $\{\hat{\bar{\bar{\beta}}}_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m)\}$, WHEREIN THE SHOWER FORMULA AND THE $QED \otimes QCD$ EXPONENTIATION FORMULA CAN BE EXPANDED IN PRODUCT AND REQUIRED TO MATCH THE GIVEN EXACT RESULT TO THE SPECIFIED ORDER – SEE hep-ph/0509003.

– THIS COMBINATION OF THEORETICAL CONSTRUCTS CAN BE SYSTEMATICALLY IMPROVED WITH EXACT RESULTS ORDER-BY-ORDER IN α_s, α, WITH EXACT PHASE SPACE.

– THE RECENT ALTERNATIVE PARTON EVOLUTION ALGORITHM BY JADACH and SKRZYPEK,A P P B35, 745 (2004),CPC175(2006)511, CAN ALSO BE USED.

– LACK OF COLOR COHERENCE $\Rightarrow$ ISAJET NOT CONSIDERED HERE.

With this said, we compute , with and without QED, the ratio

$$r_{exp} = \sigma_{exp}/\sigma_{Born}$$

to get the results (We stress that we *do not* use the narrow resonance approximation here.)

$$r_{exp} = \begin{cases} 1.1901 & , \textbf{QCED} \equiv \textbf{QCD+QED, LHC} \\ 1.1872 & , \textbf{QCD, LHC} \\ 1.1911 & , \textbf{QCED} \equiv \textbf{QCD+QED, Tevatron} \\ 1.1879 & , \textbf{QCD, Tevatron} \end{cases} \tag{55}$$

$\Rightarrow$

∗*QED IS AT .3% AT BOTH LHC and FNAL.*

∗*THIS IS STABLE UNDER SCALE VARIATIONS.*

∗*WE AGREE WITH BAUR ET AL., HAMBERG ET AL., van NEERVEN and ZIJLSTRA–NOTE THAT AS WE HAVE AN EXPONENTITED FORMULA, IT MAKES SENSE TO COMPARE WITH THE LATTER.*

∗*QED EFFECT SIMILAR IN SIZE TO STR. FN. RESULTS.*

∗*DGLAP-CS SYNTHESIZATION HAS NOT COMPROMISED THE NORMALIZATION.*

QUARK MASSES and RESUMMATION in PRECISION QCD THEORY

(PHYS. REV.D78(2008)056001)

- Di'Lieto et al.(NPB183(1981)223), Doria et al.(*ibid.*168(1980)93), Catani et al.(*ibid.*264(1986)588;Catani(ZPC37(1988)357): IN ISR, BLOCH-NORDSIECK CANCELLATION FAILS AT $\mathcal{O}(\alpha_s^2)$ for $m_q \neq 0$.

- FOR $q + q' \rightarrow q'' + q''' + V + X$, THEY GET

$$\text{flux } \frac{d\sigma}{d^3Q} = \frac{-g^4 \bar{H}}{(d-4)32\pi^2} \left(\frac{1-\beta}{\beta} \right) \left(\frac{1}{\beta} \ln(\frac{1+\beta}{1-\beta}) - 2 \right) \qquad (56)$$

- HERE, $\bar{H}$ IS THE HARD PROCESS DRESSED AS

$$F_1 = C_2(G) H_{ab}^{\alpha\beta} (T_i)^{\beta\alpha} (T_i)_b a \qquad (57)$$

FOR

$$f_{ijk} f_{ijl} = C_2(G) \delta_{kl}$$

$$(T_i T_i)_{ab} = C_2(F) I_{ab}.$$

THEY EVALUATE THE GRAPHS IN FIG.1 USING MUELLER'S THM.
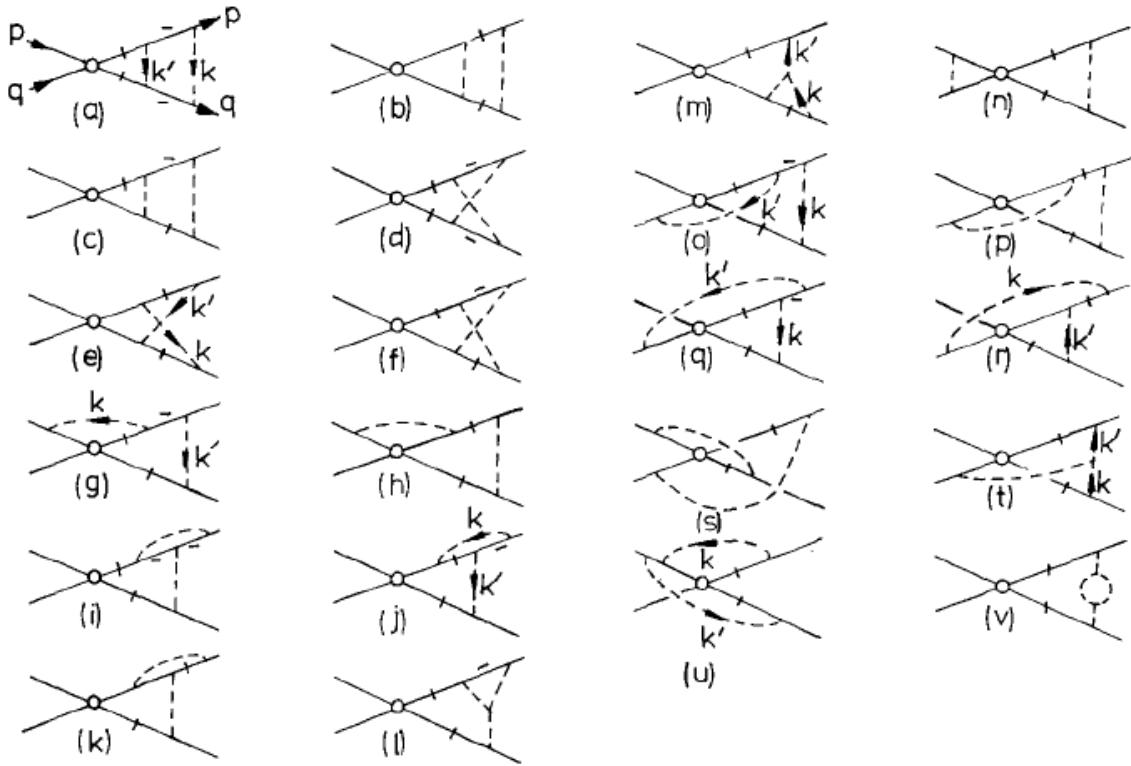
Figure 1. Graphs evaluated in Ref. [2] (see the first paper therein especially) in arriving at the result in (3) using Mueller's theorem for the respective cross section. The usual Landau-Bjorken-Cutkosky (LBC) [10] rules obtain so that a slash puts the line on-shell and a dash changes the iε-prescription; and, graphs that have cancelled or whose contributions are implied by those in the figure are not shown explicitly.

- SINCE BN VIOLATION VANISHES FOR $m_q \to 0$, MUST SET $m_q = 0$ IN ISR for $\mathcal{O}(\alpha_s^n)$, $n \geq 2$: NOTE, $m_b \cong 5$GeV.

- SOURCE OF BN-VIOLATION: LOOK AT CONTRIBUTION OF DIAGRAMS (q-o) IN FIG.1:

$$A_{q-o} = \frac{1}{\beta^2} \int \frac{d^3k \, d^3k' \, 2k_z}{(k_z + k'_z + i\epsilon)(\beta^2 k_z^2 - \mathbf{k}^2)(\beta^2 k_z^2 - \mathbf{k}'^2 + i\epsilon)(k_z^2 + \epsilon^2)} \tag{58}$$

UV-REGULATED RESULT: USE THE REGULATOR $e^{-\mathbf{k}^2/\Lambda^2}$,

$\Rightarrow$

$$A_{q-o}|_{UV-reg} = \frac{4\pi^{n+1}(\Lambda^2)^{n-3}}{\beta^2} \left\{ \frac{1}{(n-3)^2} + \frac{1}{2(n-3)} \ln\left(\frac{1+\beta}{1-\beta}\right) \right\}. \tag{59}$$

$\Rightarrow$

$$F_{nbn} = \frac{(1-\beta)(\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\beta)}{\ln\left(\frac{1+\beta}{1-\beta}\right)} \tag{60}$$

IS FRACTION OF SINGLE-POLE TERM UN-CANCELLED.

- ANALYSIS IN PRD 78 (2008) 056001 $\Rightarrow$ REAL EMISSION IN A_{q-o} SATURATES SINGLE IR POLE.

- THUS, WE WRITE

$$\text{flux } \frac{d\sigma}{d^3Q} = \frac{-g^4\bar{H}}{64\pi^6} F_{nbn} A_{q-o}|_{\mathfrak{R},\text{real rad., IR pole part}}, \tag{61}$$

WHERE FROM PRD78(2008)056001 WE HAVE

$$A_{q-o}|_{\mathfrak{R},\text{real rad., IR pole part}} = \frac{4\pi^4}{\beta^2}\left(\frac{1}{2(n-3)}\ln\left(\frac{1+\beta}{1-\beta}\right)\right). \tag{62}$$

- APPLY QCD RESUMMATION TO REAL EMISSION IN $A_{q-o}|_{\mathfrak{R}}$: APPLY IT TO THE FRACTION F_{nbn}; REMAINING $1 - F_{nbn}$ CANCELLED BY VIRTUAL CORRECTIONS

- USING

$$d\hat{\sigma}_{\exp} = e^{\text{SUM}_{\text{IR}}(\text{QCD})} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-p_X-\sum k_j)}$$

$$* e^{D_{\text{QCD}}} \bar{\bar{\tilde{\beta}}}_n(k_1,\ldots,k_n) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0} \frac{d^3 p_X}{p_X^0}$$

(63)

WE GET

$$F_{nbn} A_{q-o}\big|_{\Re,\text{real rad., resummed}} = F_{nbn} \Re \frac{-i\pi^2}{\beta^2} \int d^2 k_\perp \int_0^{\sqrt{\epsilon}} dk_z F_{YFS}(\bar{\gamma}_q) e^{\bar{\delta}_q/2}$$

$$(\beta k_z)^{\bar{\gamma}_q} \left(-\ln(k_z + i\epsilon - \beta k_z) + \ln(k_z + i\epsilon + \beta k_z)\right)$$

$$\frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2},$$

(64)

WHERE WE HAVE DEFINED

$$\bar{\gamma}_q = 2C_F \frac{\alpha_s(Q^2)}{\pi}(\ln(s/m^2) - 1) \tag{65}$$

$$\bar{\delta}_q = \frac{\bar{\gamma}_q}{2} + \frac{2\alpha_s C_F}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right). \tag{66}$$

- USING THE SUBSTITUTION $k_z = \sqrt{\epsilon}\,\bar{k}_z$, WE HAVE

$$F_{nbn} A_{q-o}\big|_{\Re,\text{real rad., resummed}} = F_{nbn} \Re \frac{-i\pi^2 \epsilon^{\frac{\bar{\gamma}_q}{2}}}{\beta^2} \int d^2 k_\perp \int_0^1 d\bar{k}_z F_{YFS}(\bar{\gamma}_q) e^{\bar{\delta}_q/2}$$

$$(\beta\bar{k}_z)^{\gamma_q} \left(-\ln(\bar{k}_z + i\sqrt{\epsilon} - \beta\bar{k}_z) + \ln(\bar{k}_z + i\sqrt{\epsilon} + \beta\bar{k}_z)\right)$$

$$\frac{1}{-(1-\beta^2)\epsilon\bar{k}_z^2 - \mathbf{k}_\perp{}^2} \frac{2\bar{k}_z}{\bar{k}_z^2 + \epsilon}. \tag{67}$$

THE RHS OF THIS LAST EQUATION VANISHES AS $\epsilon \to 0$, REMOVING THE VIOLATION OF BLOCH-NORDSIECK CANCELLATION IN (56).

CONCLUSION:

- RESUMMATION CURES LACK OF BN CANCELLATION IN MASSIVE QCD

MC Realization: IR-Improved Kernels in HERWIG6.5

- Approach:

  – Modify the kernels in the HWBRAN and Related Modules - (BW,MS)

  $$\text{DGLAP-CS } P_{AB} \Rightarrow \text{IR-I DGLAP-CS } P_{AB}^{exp} \qquad (68)$$

  – Leave Hard Processes Alone for the Moment:

  In progress (SY,BFLW,VH,MH,SM,SJ)– include YFS synthesized EW modules

  from Jadach et al. MC's for HERWIG6.5,++ hard processes.

  – ISSUE: CTEQ and MRST BEST(after 2007) P.Dstrbn. Fns DO NOT INCLUDE

  PRECISION EW HO CORR.

Implementation Illustration

Probability that no branching occurs above virtuality cutoff Q_0^2 is $\Delta_a(Q^2, Q_0^2)$

$\Rightarrow$

$$d\Delta_a(t, Q_0^2) = \frac{-dt}{t} \Delta(t, Q_o^2) \sum_b \int dz \frac{\alpha_s}{2\pi} P_{ba}(z), \qquad (69)$$

$\Rightarrow$

$$\Delta_a(Q^2, Q_0^2) = \exp\left[-\int_{Q_0^2}^{Q^2} \frac{dt}{t} \sum_b \int dz \frac{\alpha_s}{2\pi} P_{ba}(z) \right]. \qquad (70)$$

Non-branching probability appearing in the evolution equation is

$$\Delta(Q^2, t) = \frac{\Delta_a(Q^2, Q_o^2)}{\Delta_a(t, Q_o^2)}, \quad t = k_a^2 \quad \text{the virtuality of gluon } a. \qquad (71)$$

Virtuality of parton a is generated with

$$\Delta_a(Q^2, t) = R, \qquad (72)$$

where R is a random number uniformly distributed in $[0, 1]$.

With

$$\alpha_s(Q) = \frac{2\pi}{b_0 \log\left(\frac{Q}{\Lambda}\right)}, \tag{73}$$

we get

$$\int_0^1 dz \frac{\alpha_s(Q^2)}{2\pi} P_{qG}(z) = \frac{4\pi}{2\pi b_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)} \int_0^1 dz \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

$$= \frac{2}{3} \frac{1}{b_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)}. \tag{74}$$

$\Rightarrow$

$$I = \int_{Q_0^2}^{Q^2} \frac{1}{3} \frac{dt}{t} \frac{2}{b_0 \ln\left(\frac{t}{\Lambda^2}\right)},$$

$$I = \frac{2}{3b_0} \ln \ln \frac{t}{\Lambda^2}\Big|_{Q_0^2}^{Q^2}$$

$$= \frac{2}{3b_0} \left[\ln \left(\frac{\ln\left(\frac{Q^2}{\Lambda^2}\right)}{\ln\left(\frac{Q_0^2}{\Lambda^2}\right)} \right) \right]. \tag{75}$$

Finally

$$\Delta_a(Q^2, Q_0^2) = \exp\left[-\frac{2}{3b_0} \ln \left(\frac{\ln\left(\frac{Q^2}{\Lambda^2}\right)}{\ln\left(\frac{Q_0^2}{\Lambda^2}\right)} \right) \right]$$

$$= \left[\frac{\ln\left(\frac{Q^2}{\Lambda^2}\right)}{\ln\left(\frac{Q_0^2}{\Lambda^2}\right)} \right]^{-\frac{2}{3b_0}}. \tag{76}$$

Let $\Delta_a(Q^2, t) = R$, then

$$\left[\frac{\ln\left(\frac{t}{\Lambda^2}\right)}{\ln\left(\frac{Q^2}{\Lambda^2}\right)} \right]^{\frac{2}{3b_0}} = R \tag{77}$$

$\Rightarrow$

$$t = \Lambda^2 \left(\frac{Q^2}{\Lambda^2}\right)^{R^{\frac{3b_0}{2}}}. \tag{78}$$

Recall

$$\begin{aligned}
b_0 &= \left(\frac{11}{3} n_c - \frac{2}{3} n_f\right) \\
&= \frac{1}{3}\left(11 n_c - 10\right), \quad n_f = 5 \\
&= \frac{2}{3}\mathbf{BETAF}. \tag{79}
\end{aligned}$$

The momentum available after a $q\bar{q}$ split in HERWIG is given by

$$QQBAR = QCDL3 \left(\frac{QLST}{QCDL3} \right)^{R^{BETAF}}. \tag{80}$$

Let us now repeat the above calculation for the IR-Improved kernels.

$$P_{qG}(z)^{exp} = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2} \left[z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G} \right] \tag{81}$$

so

$$\int_0^1 dz \frac{\alpha_s\left(Q^2\right)}{2\pi} P_{qG}(z)^{exp} = \frac{4F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G}}{b_0 \ln\left(\frac{Q^2}{\Lambda^2} \right) (\gamma_G + 1)(\gamma_G + 2)(\gamma_G + 3)}. \tag{82}$$

$\Rightarrow$

$$I = \int_{Q_0^2}^{Q^2} \frac{dt}{t} \frac{4F_{YFS}(\gamma_G)e^{\frac{1}{2}\delta_G}}{b_0 \ln\left(\frac{t}{\Lambda^2}\right)(\gamma_G + 1)(\gamma_G + 2)(\gamma_G + 3)},$$

$$I = \left. \frac{4F_{YFS}(\gamma_G)e^{0.25\gamma_G}}{b_0(\gamma_G + 1)(\gamma_G + 2)(\gamma_G + 3)} Ei\left(1, \frac{8.369604402}{b_0 \ln\left(\frac{t}{\Lambda^2}\right)}\right) \right|_{Q_0^2}^{Q^2} \tag{83}$$

Where we have used

$$\delta_G = \frac{\gamma_G}{2} + \frac{\alpha_s C_G}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right), \tag{84}$$

with $C_G = 3$ the gluon quadratic Casimir invariant. So finally

$$\Delta_a(Q^2, t) = \exp\left[-\left(F\left(Q^2\right) - F\left(t\right)\right)\right], \tag{85}$$

where

$$F(Q^2) = \frac{4F_{YFS}(\gamma_G)e^{0.25\gamma_G}}{b_0(\gamma_G + 1)(\gamma_G + 2)(\gamma_G + 3)} Ei\left(1, \frac{8.369604402}{b_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)}\right), \tag{86}$$
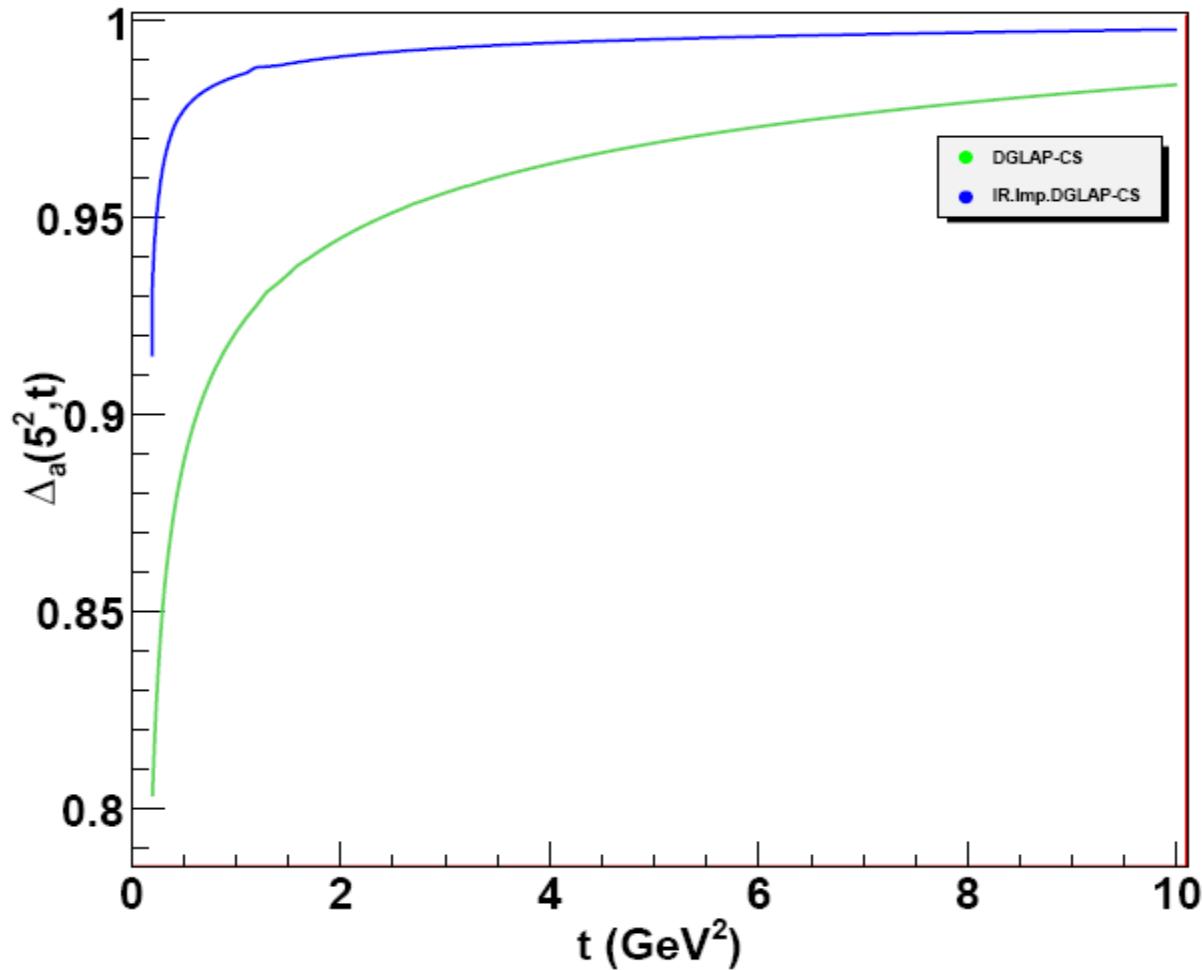
Figure 1. Graph of $\Delta_a(Q^2,t)$ for the DGLAP-CS and IR-Imp. DGLAP-CS kernels (76,85)

B.F.L. Ward   **Apr. 5, 2010**

RESULTS

We have illustrative results on IR-Improved Showers in HERWIG6.5:

we compare the z-distributions, p_T-dist. etc., of the IR-Improved and

usual DGLAP-CS showers in the following figs.

NOTE: SIMILAR RESULTS FOR PYTHIA and MC@NLO IN PROGRESS.

- First, 2→2 hard processes at LHC

Energy fraction of parton shower constituents for QCD 2→2 hard parton scattering.
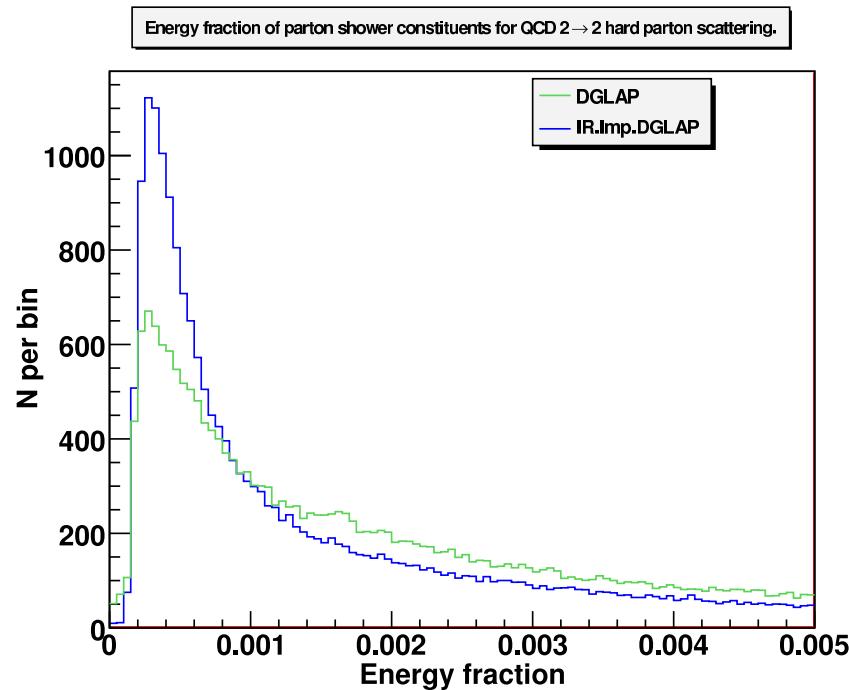
Figure 2: The z-distribution(ISR parton energy fraction) shower comparison in HERWIG6.5.

Histogram of P_T^2 for QCD parton shower in Herwig6.5 for 2→2 hard parton scattering.
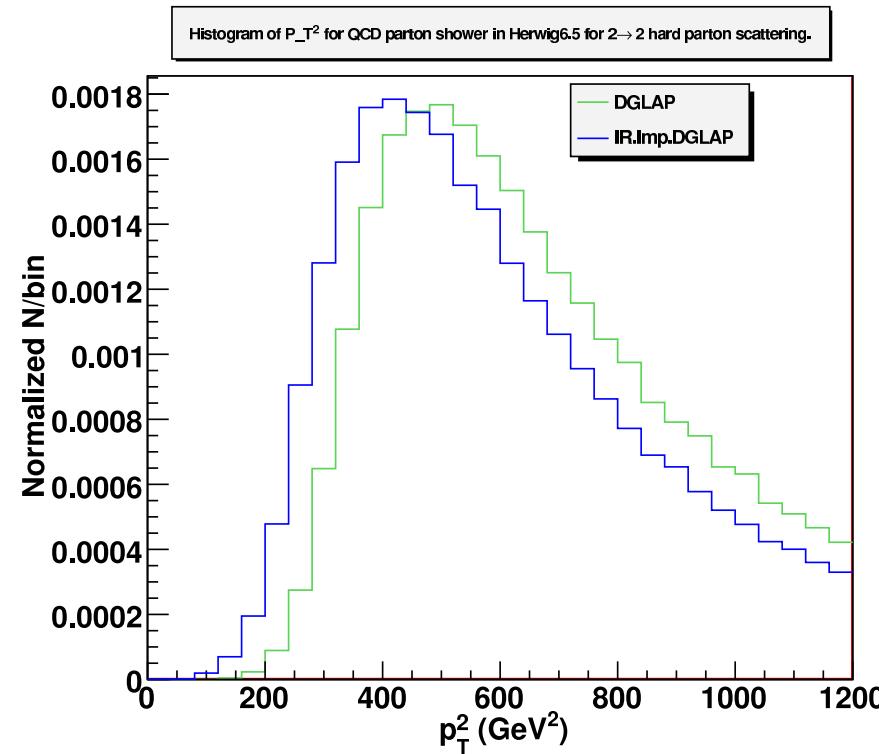
Figure 3: The P_T^2-distribution (ISR parton) shower comparison in HERWIG6.5.

Energy fraction of π^+ in HERWIG6.5 for QCD $2 \rightarrow 2$ hard parton scattering.

Figure 4: The π^+ energy fraction distribution shower comparison in HERWIG6.5.

Histogram of P_T^2 for π^+ for QCD $2 \to 2$ hard parton scattering.

Figure 5: The π^+ P_T^2-distribution shower comparison in HERWIG6.5.

- **Single Z-production at LHC**

IV-14

Energy fraction distribution of parton shower for single Z production.
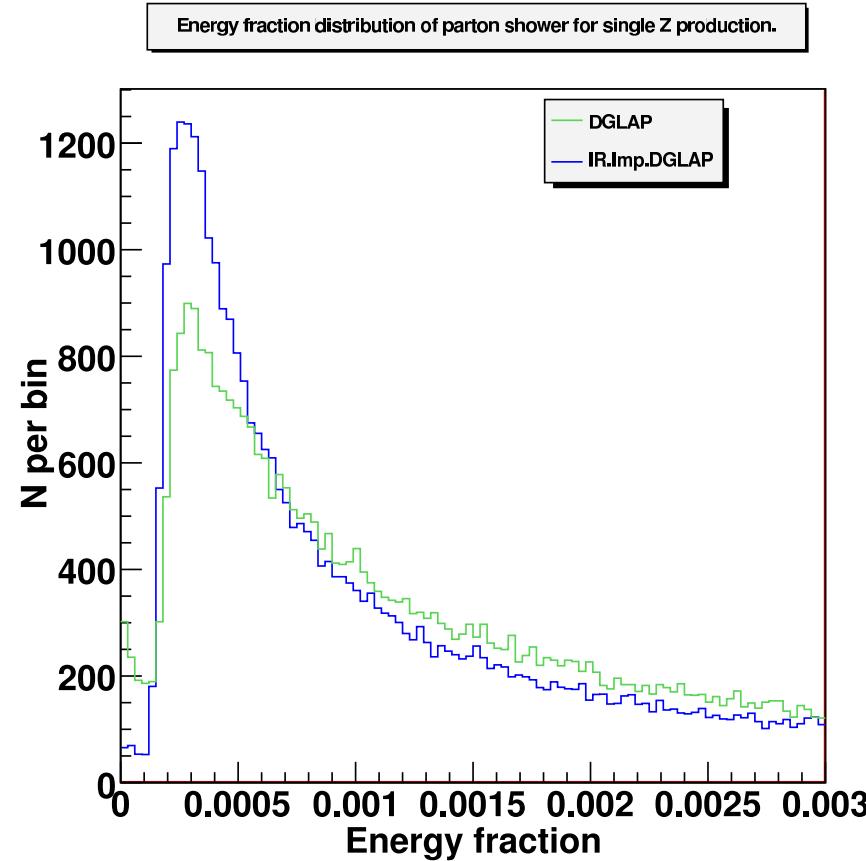
Figure 6: The z-distribution(ISR parton energy fraction) shower comparison in HERWIG6.5.

Generated Z transverse momentum.
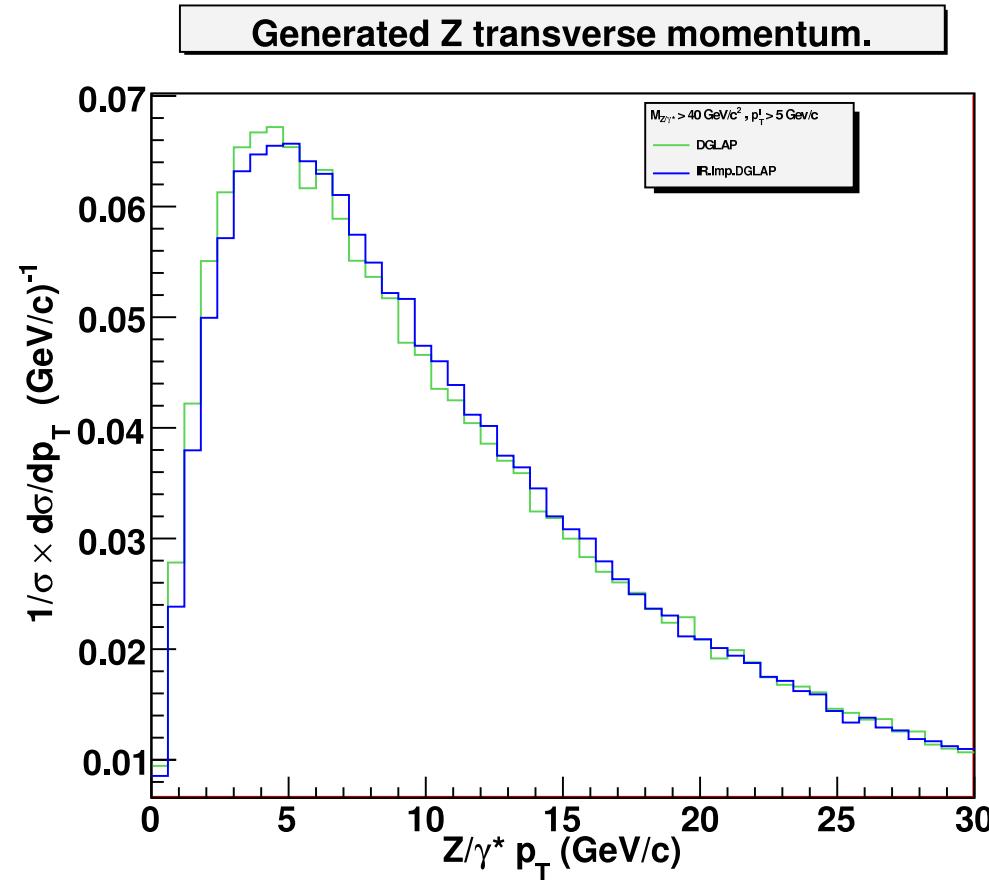
Figure 7: The Z p_T-distribution(ISR parton shower effect) comparison in HER-WIG6.5.
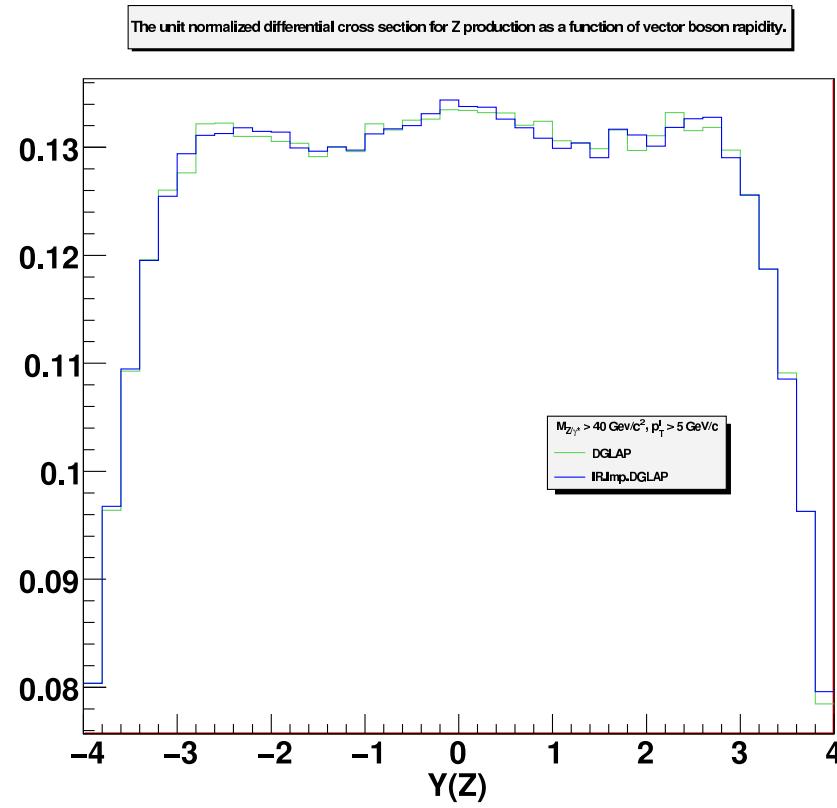
Figure 8: **The Z rapidity-distribution(ISR parton shower) comparison in HER-WIG6.5.**

**(a)**   **(b)**
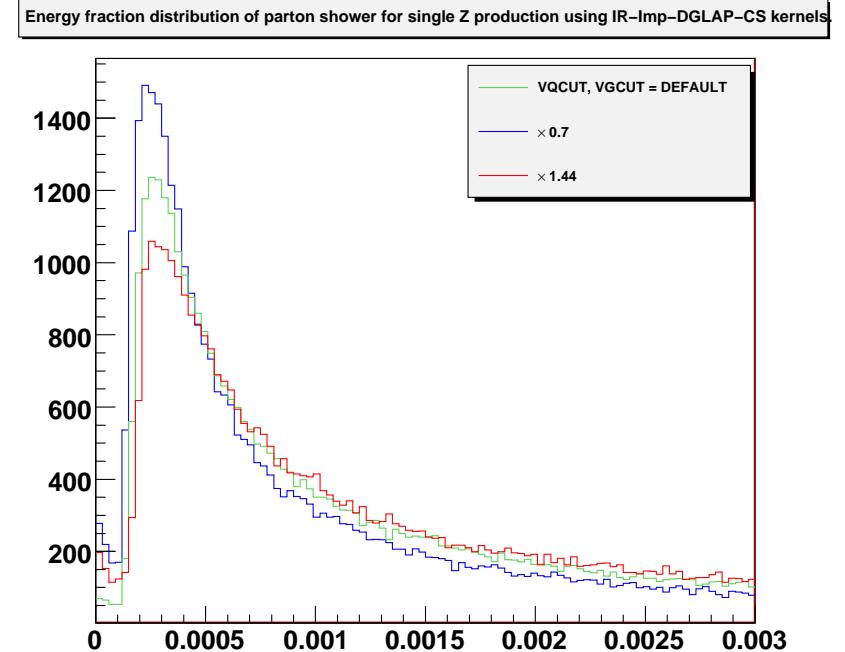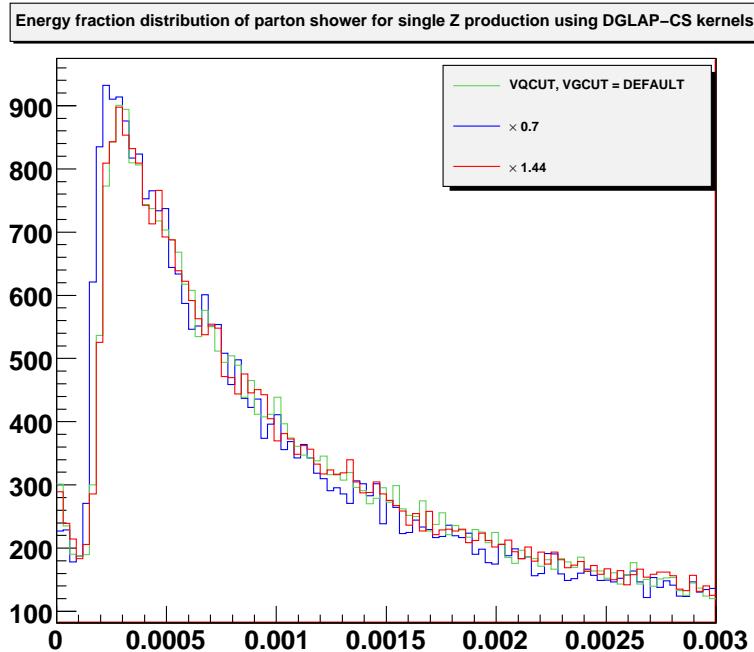
Figure 9: IR-cut-off sensitivity in z-distributions of the ISR parton energy fraction: (a), DGLAP-CS (b), IR-I DGLAP-CS – for the single Z hard subprocess in HERWIG-6.5 environment.

COMPARISON WITH DATA NOW FOLLOWS

(Galea, Proc. DIS 2008; Abasov et al., PRL100, 102002 (2008).)
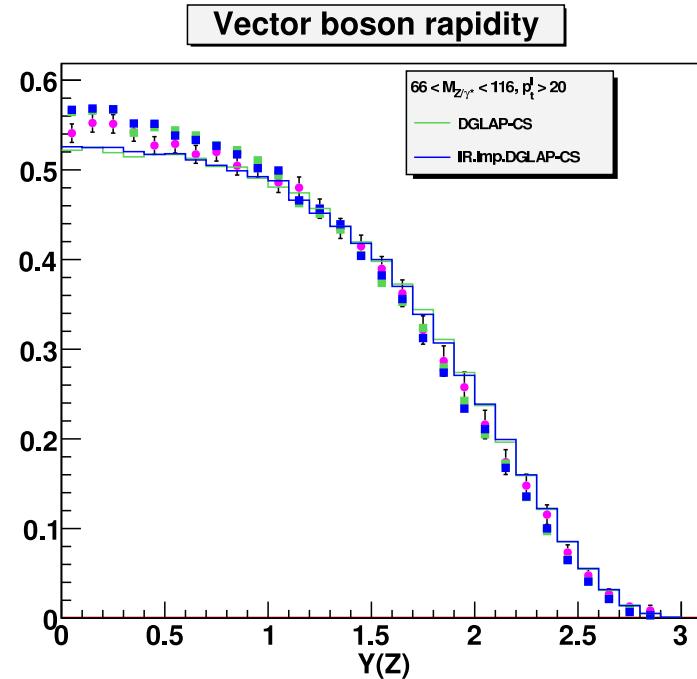
(a)   **(b)**
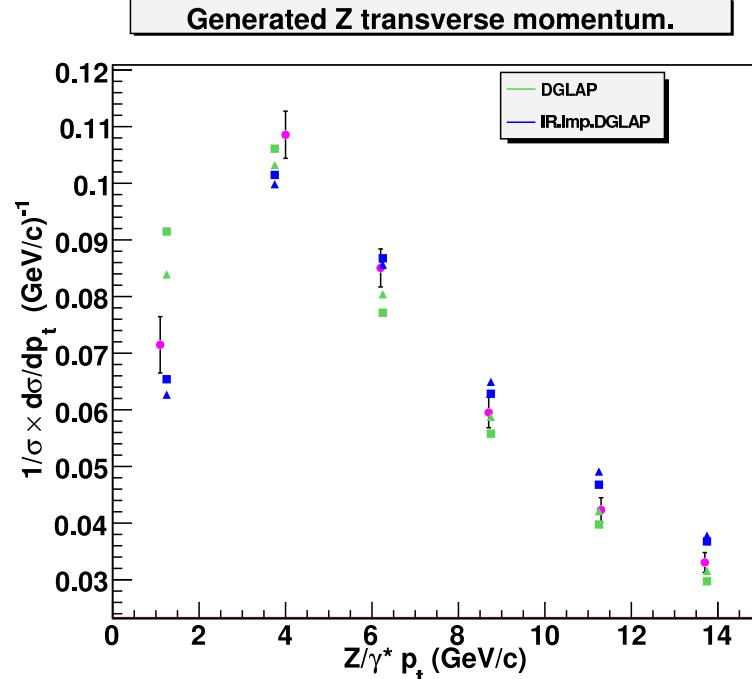
Figure 10: Comparison with FNAL data: (a), CDF rapidity data on (Z/γ^*) production to e^+e^- pairs, the circular dots are the data, the green(blue) lines are HERWIG6.510(HERWIRI1.031); (b), D0 p_T spectrum data on (Z/γ^*) production to e^+e^- pairs, the circular dots are the data, the blue triangles are HERWIRI1.031, the green triangles are HERWIG6.510 – in both (a) and (b) the blue squares are MC@NLO/HERWIRI1.031, and the green squares are MC@NLO/HERWIG6.510. These are untuned theoretical results.

For the D0 p_T data, we see that HERWIRI1.031 gives a better fit to the data compared to HERWIG6.5 for low p_T, (for $p_T < 12.5$GeV, the χ^2/d.o.f. are $\sim$ 2.5 and 3.3 respectively if we add the statistical and systematic errors), showing that the IR-improvement makes a better representation of QCD in the soft regime for a given fixed order in perturbation theory. We see that the MC@NLO $\mathcal{O}(\alpha_s)$ corrections improves χ^2/d.o.f. where the MC@NLO/HERWIRI1.031 has better agreement in soft regime.

Conclusions

YFS-TYPE METHODS (EEX AND CEEX) EXTEND TO NON-ABELIAN GAUGE THEORY AND ALLOW SIMULTANEOUS RESMN OF QED AND QCD WITH PROPER SHOWER/ME MATCHING BUILT-IN.
FOR QED$\otimes$QCD

- **FULL MC EVENT GENERATOR REALIZATION OPEN.**

- **WE HAVE FIRST PHASE OF FULL MC REALIZATION: IR-IMPROVED HERWIG6.5 (HERWIG6.5-YFS)**

- **COMPARISON WITH THEORY ENCOURAGING: SOFTER SPECTRA, MORE ROBUSTNESS TO CUTS, ETC. – $\Delta\sigma_{Shower}$ IN PLAY**

- **COMPARISON WITH DATA IMMINENT – EXPERIMENTALISTS WELCOME**

- **IMPLEMENTATION IN PYTHIA, HERWIG++ IN PROGRESS**

- **IMPLEMENTAION OF PRECISION EW MODULES (FROM JADACH ET AL.) IN HERWIG ALSO IN PROGRESS.**

- **A FIRM BASIS FOR THE COMPLETE $\mathcal{O}(\alpha_s^2, \alpha\alpha_s, \alpha^2)$ MC RESULTS NEEDED FOR THE PRECISION FNAL/LHC/RHIC/ILC PHYSICS HAS BEEN DEMONSTRATED AND ALL THE LATTER IS IN PROGRESS, WITH M. Kalmykov, S. Majhi, S. Yost and S. Joseph.– SEE JHEP0702(2007)040,arxiv:0707.3654,0708.0803, 0810.3238, 0901.4716, 0902.1352, NEW RESULTS FOR HO F-Int's,etc. –no time to discuss here**