Questions and Problems

For many years I have given a course in elementary acoustics at the University of Southern California. This course is designed to acquaint music students with the basic concepts of physics and acoustics and the application of these concepts to the production and perception of music. My book, The Acoustical Foundations of Music, is an outgrowth of my experience in teaching this course. As part of the course requirements, the students were given problem assignments designed to furnish quantitative applications of the physical and acoustical concepts discussed in the book. It was emphasized that practice by working such problems is as important to learning physics as practice on a musical instrument is important to learning to perform music.

Now that Acoustical Foundations has been published in a second edition, it seems appropriate to make available a collection of questions and problems to enhance the usefulness of the book as a classroom text. To this end, I have compiled the material presented herewith. Many of the problems given below are ones that I have used in my own course, and have thus been well tested on students over a period of years. However, the remaining ones have been composed anew for this collection, and may show deficiencies. The questions likewise have been newly composed. In constructing them, I have tried to make them somewhat challenging to a student's physical intuition, and not just queries that can be answered by finding the appropriate sentence in the book. However, they also have not been tested on students, and so can undoubtedly be improved.

It is planned to expand this collection of questions and problems into a workbook to supplement Acoustical Foundations for classroom use. To this end any suggestions for improving this material, or for new questions or problems to be added to it, will be gratefully received.

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Chapter One The Fundamental Physical Quantities

Questions

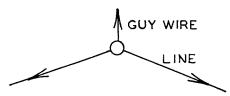
- 1-1. Name some physical quantities familiar in everyday life.
- 1-2. Name as many units of length as you can.
- 1-3. Express in simpler terms : kilomicrometer; megamillimeter; millikilometer.

The liter is defined as 10^{-3} m³. Chemists use a unit called a milliliter. ical volume does this correspond to?

The length of the year is 365,242 solar days. Why do we have leap ne present pattern is to omit leap year every exact century except isible by 400 (e.g., 1900 was not a leap year; 2000 will be). Why is

An advertisement states that a certain automobile can speed up from) miles per hour in 5 seconds. What acceleration does this correspond ne car were loaded with passengers, how would this affect the

I'wo teams engaged in a tug-of-war are at a standstill, being evenly (a) What is the resultant of the two forces exerted on the rope by teams? (b) One team gains a slight advantage and begins to pull the m along the ground at a constant speed. Now what is the resultant? When a power line turns a corner, a guy wire is used to pull on the ale as shown in the figure. Why is it used, and what direction should



A car is going around a circular curve at a constant speed. What is ion of the acceleration?

(a) A car is moving along a straight road at a constant speed. Diathe forces acting on the car. (b) The driver of the car applies the iagram the forces for this case.

What is the weight of an astronaut in a satellite vehicle orbiting the oes the astronaut have to worry about such mundane things as his head on a door frame?

Express Eq. (4), p. 14, in English units, using the pound as the unit force and mass.

In Fig. 7, p. 18, what is the resultant of all the forces on the imbject due to the pressure of the fluid?

Is physical work done by a person (a) in raising a bucket of water vell, (b) carrying it across the yard, and (c) pouring it into a tub?

The kinetic energy W in joules possessed by a mass of m kilograms ith a velocity of v meters per second is given by

$$W = \frac{1}{2}mv^2.$$

ive a reason (guided by intuition) why the velocity must be squared? Find the wattage ratings of some home appliances.

Jsing the relation 1 foot = 0.3048 meters, work out the following to ificant figures:

1 inch =	centimeters
1 meter =	feet
1 mile =	kilometers
1 centimeter =	inches
1 kilometer =	miles
1 meter =	inches
Using the above results, work out the following	ng to three sig

nificant 1-2.figures:

1 square inch =	square centimeter
1 cubic inch =	cubic centimeters
1 square mile =	square meters

1-3. Work out the following using scientific notation:

(a) $0.00000050 \times 2.500,000,000 \times 100,000,000,000$

(b)	$2,500,000 \times 3,000,000$	(c)	0.000050 X 4000 X 0.000060				
(D)	0.00000015		$30,000 \times 0.00020 \times 0.0000020$				
	0.00028		1				

/41	0.00028	(e)	1		
(d)	3,500,000 × 20,000		.00050 × 200 × 0.0000030		

1-4. Given the radius of the earth at 4.0×10^3 miles, find: (a) the radius of the earth in feet; (b) the radius in centimeters; (c) the area of the earth in square miles; (d) the area in square feet; and (e) the area in square centimeters. Use scientific notation and two significant figures. (Area of a sphere = $4\pi r^2$.)

1-5. Find the following to three significant figures:

1 km/hr =	ft/sec.
1 cm/sec =	ft/min.
1 knot =	cm/sec
60 mi/hr =	ft/sec.

1-6. If a person can walk at a speed of 8 kilometers per hour, how long would it take him to walk a distance equal to that from the earth to the sun (93 × 10⁶ miles)? Express as hours and as years.

1-7. One pound (mass) = 0.45359237 kilograms exactly. Find the following to three significant figures:

1 kilogram =	pounds
1 ounce =	grams
1 ton =	kilogran

1-8. The gallon is defined as exactly 231 in³. Find the number of quarts in one liter. (See Question 1-5.)

- 3. The distance from the earth to the moon, in round numbers, is 00 miles. How long will it take a flash of light to leave the earth, reflect the moon, and return to the earth, if light travels at a speed of 93,500 meters per second? (Use scientific notation and the proper for significant figures.)
- 10. One newton equals how many pounds force?
- 11. The formula $D = S \times t$ is valid for consistent units of distance. and time. Find a formula which will give the distance D in feet ed in a time t minutes by a car going S miles/hr.
- o significant figures are sufficient for the following:
- 12. A motorcycle starts from rest and accelerates uniformly to a speed miles per hour in 8 seconds. Find its acceleration (a) in miles per hour 1; (b) in feet per second squared; (c) in feet per hour second; and (d) in er second hour. (e) How fast will it be traveling 12 seconds after startsuming that it maintains the same acceleration?
- 3. A motorcycle is traveling 30 miles per hour. If the brakes are d, causing it to decelerate at a rate of 8.8 feet/second squared, (a) how rill it take to stop, and (b) how many feet will it go while stopping?
- .4. A force of 10 newtons is exerted on a 20 kilogram mass initially at f it acts for 12 seconds, how fast will the mass then be moving?
- .5. A force of 10 newtons acts on a 5 kilogram mass originally at rest. ong must it act to give the mass a speed of 20 meters per second?
- .6. A golf ball of mass 0.10 kilograms is struck by a club and given a of 80 meters per second in 0.005 seconds. Find (a) the acceleration of Il. and (b) the force exerted on it by the club.
- .7. A ball of mass 0.20 kilogram is thrown vertically downward onto a loor. It strikes the floor with a speed of 20 meters per second, and nds upward with the same speed. If it is in contact with the floor for seconds, find (a) its acceleration, and (b) the force exerted on it while contact with the floor.
- .8. A man has a mass of 72 kilograms. (a) What is his weight on the e of the earth? (b) What would be his weight on the moon?
- .9. What is the mass of the air in a vertical column of one square meter section and extending past the top of the atmosphere?
- 10. (a) At what depth under water is the pressure due to the water to one atmosphere? (b) What will be the total pressure at this depth? density of water is 1 kilogram per liter, and one atmosphere = 1.01 X ewtons/square meter.)
- 21. A cylindrical can contains 8 kilograms of water. If the bottom is and 12 inches in diameter, what is the pressure on it in newtons per
- 22. If the can of Problem 1-21 is emptied of water, closed off with a id the air in it evacuated, what will be the force on the lid due to atmospressure?
- 23. A force of 10 newtons is exerted on a 20 kilogram mass initially at t acts for 12 seconds, and moves the mass a distance of 36 meters. (a) the work in joules done by the force, and (b) find the power expended force while acting.
- 24. A man throws a ball of mass 0.6 kilograms, giving it a speed of 5 3 per second in 0.25 seconds. (a) Find the acceleration of the ball; and d the force exerted on it.
- 25. (See Question 1-16.) In Problem 1-23, (a) find the speed of the after the force has been acting on it. (b) Using this, find the kinetic

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energy of the mass and see if it is the same as the work that was done by the

- 1-26. Similarly, in Problem 1-24 (a) find the kinetic energy of the ball after it has been thrown; (b) from this, find the distance over which the man exerted the force when throwing the ball; and (c) find the power expended while throwing it.
- 1-27. A 5 kilogram mass is raised 20.4 meters off the ground. (a) How much work is done on it? (b) It is then released so that it falls and strikes the ground. How fast is it moving when it hits? (c) For how long does it fall?
- 1-28. A 60 watt lamp is lit for 24 hours. (a) Find the energy expended in it in joules; and (b) find how far this energy would lift a mass of one ton.
- 1-29. If the energy available in one ton of water descending a distance of 1000 feet is used to light a 100 watt lamp, for how many hours will the lamp
- 1-30. Refer to Fig. 3(b), p. 12. If a push of 25 newtons is required to give the block a constant speed of 6 meters per second, what power in watts is being expended to keep the block in motion?

Answers to Problems

- 1-1. 2.54 cm; 3.28 ft; 1.61 km; 0.394 in; 0.621 mi; 39.4 in.
- 1-2. 6.45 cm^2 ; 16.4 cm^3 ; $2.59 \times 10^6 \text{ m}^2$.
- 1-3. (a) 1.25×10^{14} ; (b) 5×10^{19} ; (c) 1; (d) 4×10^{-15} ; (e) 3.3×10^{6} .
- 1-4. (a) 2.1×10^7 ft; (b) 6.4×10^8 cm; (c) 2.0×10^8 mi²; (d) 5.6×10^{15} ft²; (e) 5.2×10^{18} cm².
- 1-5. 0.911 ft/sec; 1.97 ft/min; 51.5 cm/sec; 88 ft/sec.
- 1-6. 1.9×10^7 hr; 2100 years.
- 1-7. 2,20 lb; 28,3 gm; 907 kg.
- 1-8. 1.057.
- 1-9. 2.57 sec.
- 1-10. 0.224 lb.
- 1-11. $D_{ft} = 88 S_{mi/hr} \times t_{min}$.
- 1-12. (a) 7.5 mi/hr sec; (b) 11 ft/sec²; (c) 4.0×10^4 ft/hr sec; (d) 4.0×10^4 ft/sec hr; (e) 90 mi/hr or 132 ft/sec.
- 1-13. (a) 5 sec.; (b) 110 ft.
- 1-14. 6 m/sec.
- 1-15. 10 sec.
- 1-16. (a) 1.6×10^4 m/sec²; (b) 1.6×10^3 N.
- 1-17. (a) 2×10^4 m/sec²; (b) 4×10^3 N.
- 1-18. 706 N; 118 N.
- $1-19. \ 1.0 \times 10^4 \ \text{kg}.$
- 1-20. (a) 10 m; (b) 2.0×10^5 N/m².
- 1-21. $1.1 \times 10^3 \text{ N/m}^2$.
- 1-22. 7.4×10^3 N.
- 1-23. (a) 360 J; (b) 30 W.
- 1-24. (a) 20 m/sec^2 ; (b) 12 N.
- 1-25. (a) 6 m/sec; (b) 360 J.
- 1-26. (a) 7.5 J; (b) 0.63 m; (c) 30 W.
- 1-27. (a) 1000 J; (b) 20 m/sec; (c) 2.0 sec.
- 1-28. (a) 5.2×10^6 J; (b) 580 m.
- 1-29. 2.7×10^4 sec. or 7.5 hr.
- 1-30. 150 W.

Chapter Two Simple Vibrating Systems

ons

. Give some examples of periodic motion.

. A boy bounces a tennis ball on a racquet so that it moves vertically es to the same height each bounce. Is the motion of the ball simple nic? Does the period depend on the amplitude of the motion, i.e. the to which the ball rises on each bounce?

. Referring to Question 2, sketch a graph of the displacement of the ball above the racquet as a function of time.

. Again referring to Question 2, sketch a graph of the speed of the ball as a function of time. Do the same for the acceleration.

. A mass m hung on a spring of force constant k is moving vertically mple harmonic motion. The amplitude of the motion is A. How could dculate the velocity of the mass as it goes through the equilibrium n^2

. When the amplitude of the motion of Question 2-5 is reduced to half ial value by frictional forces, what fraction of the mechanical energy of tem has disappeared? Where has it gone?

. Find the broadcast frequencies in hertz of some local radio and telestations

. Give some musical examples of damped vibrations.

. What will be the effect of a layer of tape wrapped around one prong ning fork?

ns

. A student driving to school travels as follows: he travels 20 miles/hr first three minutes; he then speeds up to go 60 miles/hr for the next les; the next two miles require three minutes; then he gets caught in a jam and goes nowhere for 1 minute; and the last two miles to school the can do is 15 miles/hr. (a) Make a table showing his distance from it the end of each of these intervals and his speed during that interval. the a graph of his distance from home as a function of the time in s since he left. (c) On the same graph plot his speed as a function of minutes. Assume that no time is required to change speed.

A stone is dropped from a tower. The distance D in meters it has seconds after being dropped is given by the formula

$$D = 4.9t^2$$

ce a table showing the distance fallen at the end of each second up to 4 s. (b) Plot this as a graph of distance fallen vs. time.

If a ball is thrown vertically upward with a speed of 80 feet per , its height h in feet above the ground t seconds after being thrown is y the formula

$$h = 80t - 16t^2$$

t a graph showing the height of the ball above the ground at the end of —etc. seconds after being thrown. (b) When does it reach its highest (c) How high does it go? (d) When does it hit the ground?

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- 2-4. A certain spring requires a force of 5 newtons to stretch it 40 centimeters. (a) How much work in joules is required to stretch it this distance? (b) How much work will be required to stretch it 60 centimeters?
- 2-5. (a) If a mass of 0.5 kilogram is hung on the spring of Problem 2-4, what will be the frequency of oscillation? (b) What should the mass be to halve the frequency?
- 2-6. A certain spring requires 2 joules of work to stretch it 0.5 meter. (a) What force is required to stretch it this distance? (b) How much work will be required to stretch it one meter?

2-7. A mass hung on a particular spring stretches it 2.45 centimeters. What will be the oscillation frequency of this system?

2-8. The mass of 0.5 kilogram hung on the spring of Problem 2-4 is pulled 40 centimeters down from its equilibrium position and then released. What will be its speed when it passes through the equilibrium position?

2-9. Refer to Fig. 3, p. 28. The vibrating system has a certain amount of mechanical energy remaining at the beginning of a vibration cycle. Estimate from the figure the percentage of this energy that is lost during the subsequent cycle.

Answers to Problems

2-1.	(a)	Time-min	3	8	11	12	20	
		Distance-mi	1	6	8	8	10	
		Speed-mi/hr	20	60	40	0	15	
2-2.	(a)	Time-sec	0	1	2	3	4	
		Distance-m	0	4.9	19.6	44	78	
2-3.	(b) 2.	5 sec; (c) 100:	ft; (d)	5 sec.			2-4.	(a) 1.0 J; (b) 2.25 J.
2-5.	(a) 0.8	80 Hz; (b) 2.0	kg.				2-6.	(a) 8N; (b) 8J.
2-7.	3.2 H	z.					2-8.	2 m/sec.
2-9	About	t. 45%						

Chapter Three Waves and Wave Propagation

Questions

- 3-1. Chemists generally express the densities of solids in terms of grams per cubic centimeter. How can this be changed to kilograms per cubic meter?
- 3-2. To look ahead a bit, which musical instruments might you expect to use transverse waves to produce sound? Which ones would you expect to use longitudinal waves?
 - 3-3. Is water a compressible medium? Is steel?
- 3-4. When a freight train starts moving, the locomotive starts pulling the first car with an audible "clank"; the first car starts pulling the second car with another "clank", and so on. This sound travels the length of the train in a very short time, even though by that time the train is barely moving. Explain how this can be. Is the "clank" a transverse or longitudinal impulse?
- 3-5. Waves in deep water have a speed that depends on the wavelength, being greater for longer waves. Does Eq. 2, p. 41, still hold for these waves? For these waves, would you expect the wavelength to be halved if the frequency is doubled?

- 6. A sound source is on one end of a railroad flatcar moving along the Will a listener on the other end of the car hear a frequency shift because : Doppler effect?
- 7. Waves in water move more slowly in shallow water than in deep . Why do ocean waves coming into a beach appear to have their crests ally parallel to the shoreline?
- 8. Ocean waves are not reflected from a sandy beach. Why?
- 9. What is the absorption coefficient of a rigid wall? Of an open w?

ems

- 1. A wire has a linear density of 10 grams per meter length. How y must it be stretched to provide a wave speed of 150 meters per d?
- 2. A clothesline has a total mass of 5 kilograms and is 20 meters long. stretched between poles to a tension of 400 newtons. (a) What is the of a wave on the line? (b) How long will it take an impulse to go from ole to the other?
- 3. What will be the speed of sound waves in air at 10°C: (a) in meters cond: (b) in feet per second?
- 4. What is the air temperature (°C and °F) if the speed of sound is 350 's per second?
- 5. (a) What is the wavelength of a sound wave of frequency 200 hertz ing in hydrogen gas at 0° centigrade? (b) What is the wavelength in air at entigrade? (c) If the air temperature is lowered to 0° centigrade and the ength is kept the same, what will be the new frequency?
- 6. (a) If a sound wave in carbon dioxide gas at 0°C has a wavelength of teters, what is its frequency? (b) If the wavelength is 0.5 meters in air at what is the frequency? (c) If the temperature of the air is increased to and the frequency is kept the same as in part (b) above, what will be w wavelength?
- 7. From Eq. (3), p. 43, and the data given in the text, calculate the y of air at 0°C.
- 3. The speed of sound may be taken as 1100 feet per second for many ations. (a) To what temperature in degrees Fahrenheit does 1100 feet econd correspond to? (b) What temperature in degrees Fahrenheit would juired to give a sound speed of 1200 feet per second?
- 9. Refer to Fig. 6, p. 30. If the tuning fork vibrates at a frequency of lertz and is moved in the direction shown in the figure at a speed of 36 neters per second, how far apart are the individual peaks inscribed on the
- 10. Refer to Fig. 6, p. 30. If the tuning fork is moved across the plate a speed of 24 centimeters per second, and inscribes peaks on the plate 2 neters apart, what is the frequency of the tuning fork?
- 11. Radio waves travel with the speed of light, 3.0×10^8 meters per d. (a) What is the wavelength of the signal broadcast by a radio station ; frequency is 150 megahertz? (b) What is the frequency of a signal wavelength is 60 centimeters?
- 12. (a) An "A" tuning fork (440 hertz) is sounded in air at 20°C. What wavelength of the sound waves produced; (1) in meters? (2) in feet? (b) ound waves from the 440 cycle fork enter water at 15°C. What is their ength in the water, in meters and in feet?

3-13. (a) A man claps his hands and hears an echo 0.2 seconds later. How many meters distant is the reflecting surface? (Temperature is 20°C). (b) He claps his hands in front of another wall 200 meters distant. How many seconds later will he hear the echo?

3-14. A man claps his hands in front of a flight of steps, and hears a momentary musical sound of 275 hertz due to successive reflections from the

steps. How wide are the steps?

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3-15. If the steps of Problem 3-14 are 15 inches wide, what is the fre-

quency of the reflected sound?

3-16. The man claps his hands while standing midway between two reflecting walls 20 feet apart. The sound impulse produced reflects back and forth between the walls, so that the man hears a sound consisting of a succession of impulses. What is the frequency of this sound?

3-17. A bell at a railroad crossing emits a sound of frequency 400 hertz. A passenger in a train approaching the crossing hears an apparent frequency of 420 hertz. (a) How fast is the train approaching the crossing, in feet per second? (b) What apparent frequency will the passenger hear after the train has passed the crossing? (Refer to Fig. 14(a), p. 45.)

3-18. A bell at a railroad crossing emits a sound of frequency 500 hertz. A train is approaching the crossing at 60 miles per hour. (a) What apparent frequency does a passenger in the train hear? (b) What will be the apparent frequency after the train passes the crossing?

3-19. A train moving 30 miles per hour is approaching a man standing on the track. The train blows its whistle, producing a sound of 240 hertz. What frequency does the man hear? (Refer to Fig. 14(b), p. 45.)

3-20. A clarinet player is sounding a sustained tone of 147 hertz. A second player sounds what he intends to be the same note, but hears 3 beats per second. What frequency is he playing?

3-21. A tuning fork produces a sound of 440 hertz. A second fork sounded with it produces 3 beats per second. Some tape is wrapped around one of the prongs of the second fork, and it is observed to still produce 3 beats per second when sounded with the first fork. What is the original frequency of the second fork, without tape.

3-22. A plane sound wave is reflected back and forth between parallel walls. After how many reflections will the wave be reduced in intensity to 35% of its original value: (a) If the absorption coefficient of the material of the walls is 0.41, (b) if it is 0.10?

3-23. A clarinet is producing the sound power output given in Table II, p. 52. (a) If its efficiency is one percent, what power is the clarinet player expending? (b) Find the sound intensity produced by the clarinet at a distance of 4 meters, assuming that the instrument is in a region where there are no reflecting surfaces and that it radiates equally in all directions. (c) Find the sound intensity produced at 8 meters.

3-24. How many tubas, each blasting out the maximum sound power given in Table II, p. 52, would be required to give a sound intensity of 8×10^{-4} watts per square meter at a distance of 30 meters? Assume that the instruments radiate uniformly in all directions and that there are no reflections.

3-25. If a tuba player must expend 25 watts to produce the sound power given in Table II, p. 52, what is the efficiency of the instrument?

swers to Problems

1. 225 N. 3. (a) 338 m/sec; (b) 1109 ft/sec. 5. (a) 6.35 m; (b) 1.72 m; (c) 193 6. (a) 516 Hz; (b) 663 Hz; (c) 0.5	Hz.
7. 1.29 kg/m ³ .	3-8. (a) 44°F; (b) 135°F.
9. 0.30 cm.	3-10. 120 Hz.
1. (a) 2.0 m; (b) 5.0× 10 ⁸ Hz.	
2. (a) 0.78 m, 2.56 ft; (b) 3.3 m,	10.7 ft.
	3-14. 2 ft.
5. 440 Hz.	3-16. 55 Hz
7. (a) 55 ft/sec; (b) 380 Hz.	3-18. (a) 540 Hz: (b) 460 Hz
9. 250 Hz.	3-20. 144 or 150 Hz.
1. 443 Hz.	
2. (a) 2 reflections; (b) 10 reflections	ons
3. (a) $5W$; (b) 2.5×10^{-4} W/m ² ; (c)	6.3×10 ⁻⁵ W/m ²
1. 45.	3-25. 0.8%.

Chapter Four Complex Vibrations and Resonance

tions

- -1. In Fig. 1, p. 57, suppose that one of the traveling waves, say the one ng to the right, has a somewhat larger amplitude than the other. What d be the appearance of the resulting combination of the two waves?
- -2. The vibration frequency of a mass hung on a spring depends on the , while the vibration frequency of a pendulum formed by a mass hung on ng does not depend on the mass. Why?
- -3. A small mass (of negligible dimensions) is fastened to the center of a stretched between supports. For example, suppose a small beetle to be ing tenaciously to the center of a cello string. What effect will this mass on the modes of vibration of the string?
- 4. A hole is drilled in the closed end of the closed tube of Fig. 9, p. 66. will this affect the frequencies of the modes of vibration of the tube?
- 5. A small piece is cut off the tip of a cone and the cut end left open. will the frequencies of the modes of vibration of the truncated cone are with those of the original cone?
- 6. The tip of the cone is cut off as in Problem 4-5 and the cut end is i. How will the frequencies of the modes of vibration compare with of the original cone?
- 7. Can a square membrane vibrate with a nodal line along a diagonal of uare?
- 8. Referring to Fig. 22, p. 81, can you think of an expression which will 1 measure of what might be called the "sharpness" of the resonance
- 3. Where is zero frequency on the scale of Fig. 25(b), p. 83?

Problems

Ouestions and Problems

- 4-1. A string is stretched between supports 3 meters apart, the tension in it being such that the velocity of a wave on it is 150 meters per second. Find the frequencies of the first four vibration modes.
- 4-2. A string of density 0.01 kilograms/meter is stretched between supports 2 meters apart, the tension in it being 400 newtons. Find the frequencies of the first four vibration modes.
- 4-3. (a) A spring 3 feet long is hung from a support with the bottom end free. If the longitudinal wave velocity is 18 feet per second, find the frequencies of the first four vibration modes. (b) The same spring has the bottom end fixed. Find the frequencies of the first four vibration modes.
- 4-4. A tube one meter long, closed at both ends, is filled with carbon dioxide gas at 0°C. Find the frequencies of the first four vibration modes.
- 4-5. A tube 2 meters long, open at both ends, is filled with air at 20°C. Find the frequencies of the first four vibration modes. (Neglect end effects.)
- 4-6. A tube 1.2 meters long, closed at one end and open at the other, is filled with hydrogen gas at 0°C. Find the frequencies of the first four vibration modes. (Neglect end effects.)
- 4-7. A steel bar 2.5 meters long is held at the center. Find the frequencies of the first four longitudinal vibration modes.
- 4-8. Find the lowest frequency for longitudinal vibrations of a steel bar four feet long when it is held one foot from one end.
- 4-9. If the tuning fork on the box shown in Fig. 18, p. 76, has a frequency of 256 hertz, how long should the box be to resonate with the fork? (Assume the box is a closed tube, and neglect end effects.)
- 4-10. In Fig. 19, p. 78, if the fork has a frequency of 320 hertz, what air column lengths will resonate with it? (Assume the glass tube to be one meter long.)
- 4-11. (a) Taking the speed of sound as 332 meters per second at 0° centigrade, find the first four vibration frequencies of a tube 100 centimeters long, closed at one end and open at the other, containing air at 0°C. (b) Find the fundamental frequency of the same tube at room temperature, 20° centigrade. (c) Find how much the tube must be lengthened to have the same fundamental frequency at 20°C as the original 100 centimeter tube had at 0°C.
- 4-12. A string of density of 0.01 kilograms per meter length is stretched between supports 0.667 meters apart: (a) If the tension in the string is 100 newtons, what will be its fundamental frequency? (b) What tension would be needed to give double the frequency in part (a)? (c) What density would be required to give double the frequency, if the tension is 100 newtons?
- 4-13. The string of Problem 4-12, part (a), is sounded with another string of the same density and length. What must be the tension in the second string to give three beats per second in the resulting sound?
- 4-14. How long must an open tube be if its fundamental is to have the same frequency as the second mode of a closed tube 120 centimeters long? (Neglect end effects.)
- 4-15. The two tubes of Problem 4-14 have inside diameters of 3 centimeters. Do Problem 4-14 taking end effects into account.

Answers to Problems

4-1, 25, 50, 75, 100 Hz. 4-2, 50, 100, 150, 200 Hz. 4-3. (a) 1.5, 4.5, 7.5, 10.5 Hz; (b) 3.0, 6.0, 9.0, 12.0 Hz.

l. 129, 258, 387, 516 Hz.

4-5, 86, 172, 258, 344 Hz.

3. 265, 795, 1325, 1855 Hz.

4-7. 1000, 3000, 5000, 7000 Hz.

3. 4100 Hz.

4-9. 0.34 m.

-). 27 cm, 81cm.
- L. (a) 83, 249, 415, 581 Hz; (b) 86 Hz; (c) 3.6 cm.
- 2. (a) 75 Hz; (b) 400 N; (c) 0.0025 kg/m.
- 3. 92 or 108 N.

4-14. 80 cm.

5. 78.8 cm.

Chapter Five The Ear: Intensity and Loudness Levels

:tions

- -1. In a lever system, a small force acting over a large distance produces ge force acting over a small distance, and conversely. Justify this statet from the standpoint of conservation of energy.
- -2. Why can we hear in the left ear sounds coming to us from the right?
- -3. If the intensity of a sound is increased by a factor of 1000, by how v decibels is the intensity level increased?
- -4. If the sound pressure of a sound is increased by a factor of 1000, by many decibels is the sound pressure level increased?
- -5. Doubling the sound pressure of a sound will increase the sound sure level by how many decibels?
- -6. How many violins would be required to sound four times as loud as violin?
- -7. A tone of frequency 1200 hertz has a level of 80 decibels. What must ne level of a 1600 hertz tone in order to be barely audible in the presence e other?
- -8. What must be the level of an 800 hertz tone in order to be heard in resence of the 1200 hertz 80 decibel tone?

lems

- -1. A speaking voice was measured as giving an output of 8.8 x 10⁻⁵ s. Find the intensity level in decibels produced by such a voice output at eters distance, assuming the voice is completely isolated so that there are eflections.
- -2. A sound source gives an intensity level of 66 decibels at a distance of neters in an anechoic room. What is the power output of the sound
- -3. What sound pressure level will be produced by the voice of Problem the same distance?
- -4. A trumpet player sounds a tone of intensity level 60 decibels at a in distance. The player then increases the intensity level to 75 decibels. 'hat factor has he increased the sound power output of the trumpet?
- -5. (a) Find the rms sound pressure corresponding to a unidirectional d wave of intensity level 120 decibels. (b) How does this pressure compare atmospheric pressure?

5-6. If the rms pressure in a unidirectional sound wave is one-tenth the atmospheric pressure, what sound level is produced?

5-7. A sound source of frequency 70 hertz is producing a sound level of 60 decibels. By what percentage must the power output of the source be

increased in order to just hear it as louder?

5-8. A violin playing at a certain distance is observed to produce an average intensity level of 64 decibels. Find the intensity level produced by (a) two similarly played violins together; (b) four violins; (c) 10 violins; (d) 40 violins.

5-9. How many similarly played violins must be added to the 40 violins of Problem 5-8 to produce a just noticeable increase in intensity level? Assume a

playing frequency of 200 hertz.

Questions and Problems

5-10. Find the intensity level of a barely audible tone of (a) 1000 hertz, (b) 100 hertz, and (c) 50 hertz.

5-11. Three sounds of frequency 1000 hertz have intensities of 70, 73, and 77 decibels when measured separately. What will be the intensity level of the three sounded together?

5-12. A loudspeaker is producing a sound of loudness level 60 phons at 1000 hertz. (a) In order to produce the same loudness level at 100 hertz, find its output in decibels. (b) By what factor is its power output increased at the lower frequency?

5-13. Repeat Problem 5-12 for a frequency of 50 hertz.

5-14. A sound of loudness level 75 phons and frequency 100 hertz is added to one of 80 phons and the same frequency. Find (a) the intensity level of each sound, (b) the intensity level of the sum, and (c) the loudness level of

5-15. A sound of intensity 2×10^{-5} watts per square meter and another sound of intensity 4 x 10⁻⁵ watts per square meter are added together. (a) If the sounds have a frequency of 50 hertz, find the intensity level in decibels of each sound and the intensity level of the two added together. (b) Find the loudness level in phons of each sound and the loudness level of the two added together.

5-16. (a) Find the loudness in sones of two sounds whose loudness levels are 64 phons and 74 phons, respectively, of different frequencies. (b) Find the resulting loudness level if the two sounds are added together.

Answers to Problems

Albweis to I loutello	
5-1. 59 dB.	$5-2. 7.2 \times 10^{-3} \text{ W}.$
$5-3. 1.8 \times 10^{-2} \text{ N/m}^2$.	5-4. 32 times.
5-5. (a) 20 N/m ² ; (b) 2×10^{-4}	4 of atmospheric pressure.
5-6. 174 dB.	5-7 . 15%.
5-8. (a) 67 dB; (b) 70 dB; (c)	74 dB; (d) 80 dB.
5-9. 4 violins.	5-10. (a) 6 dB; (b) 25 dB; (c) 40 dB.
5-11. 79 dB.	5-12. (a) 68 dB; (b) 6.3 times.
5-13. (a) 78 dB; (b) 63 times.	
5-14. (a) about 79 and 84 dB;	(b) about 85 dB; (c) about 81 phons.
5-15. (a) 73 dB, 76 dB, and 78	dB; (b) about 53, 57, and 60 phons.
5-16. (a) 5 and 10 sones; (b) 80) phons.

Chapter Six Tone Quality

uestions

- 6-1. A tone of frequency 100 hertz is given a vibrato such that the freiency ranges over 3 percent above and below the average frequency. Give e actual frequency range in hertz,
- 6-2. Repeat Question 6-1 for a tone of frequency 2000 hertz.
- 6-3. A spring is wound so that adjacent turns are normally touching. What ll the graph of displacement verus force, Fig. 9(b), p. 123, look like?
- 6-4. A sinusoidal force is applied to the spring of Question 6-3. What will a resulting displacement, Fig. 9(c), p. 123, look like?

oblems

- 6-1. A tone is composed of partials of frequencies 240, 360, and 480 rtz. At what frequency does the wave pattern repeat?
- 6-2. A tone of 160 hertz is added to the combination of Problem 6-1. At at frequency does the wave pattern repeat?
- 6-3. Draw the spectrum for the wave of Fig. 3(c), p. 111.
- 6-4. The third harmonic (dotted curve in Fig. 3(a), p. 111) is shifted in ase by a half cycle. Draw the resulting waveform.
- 6-5. Sketch the result of adding one wave to another wave of 3 times the quency and 1/9 the amplitude. The relative phase is such that the two ves are maximum at the same instant.
- 6-6. A hypothetical instrument has a format region extending from 350 tz to 550 hertz. In this region partials pass through undiminished in amude; outside it, amplitudes of the partials are reduced to 20 percent of ir origianl values. A sawtooth wave, Fig. 6(a), p. 115 of frequency 100 tz, is impressed on the instrument. Draw the spectrum of the output.
- 5-7. Repeat Problem 6-6 for a square wave of 100 hertz.
- 5-8. Two clarinets are sounding tones of 165 hertz and 167 hertz respecly. How many beats will there be between the third harmonics?
- 3-9. Two tones of frequencies 200 hertz and 350 hertz are sounded ether. Write down the frequencies of some combination tones below 500 tz.
- 3-10. Repeat Problem 6-9 for two tones of 200 hertz and 500 hertz.

wers to Problems

1. 120 Hz.

6-2. 40 Hz.

8. 6.

6-9. 50, 100, 150, 300 Hz.

0. 100, 200, 300, 400 Hz.

Chapter Seven Frequency and Pitch

Questions

- 7-1. How many cycles of a tone of frequency 100 hertz are required to give a sensation of pitch?
- 7-2. How many cycles of a tone of frequency 1000 hertz are required to give a sensation of pitch?

Problems

- 7-1. (a) By what percentage must a 250 hertz tone be increased in frequency in order to sound just noticeably sharper? (b) By how many cycles per second must a 2000 hertz tone be increased in order to sound just noticeably sharper?
- 7-2. What is the frequency of the periodicity of a tone having partials of 180, 270, and 360 hertz?
- 7-3. What is the frequency of the periodicity of a tone having partials of 180, 240, and 360 hertz?

Answers to Problems

7-1. (a) 1.2%; (b) 10 Hz.

7-2. 90 Hz.

7-3, 60 Hz.

Chapter Eight Intervals, Scales, Tuning, and Temperament

Questions

- 8-1. Given a tone of frequency 240 hertz, find the frequency of a tone (a) a fourth higher, (b) a fifth higher, and (c) an octave higher.
- 8-2. Given the tone of 240 hertz, find the frequency of a tone (a) a fourth lower, (b) a fifth lower, and (c) an octave lower.
- 8-3. In the Pythagorean scale, what note makes an interval of ratio 9/4 with C?
- 8-4. Given a tone of frequency 240 hertz, find the frequency of a tone (a) a just minor third higher, and (b) a just major third higher.
- 8-5. Given a tone of frequency 240 hertz, find the frequency of a tone (a) a just minor third lower, and (b) a just major third lower.
 - 8-6. What triad is represented by the proportion 1/6:1/5:1/4?

Problem

8-1. Find the frequencies of the notes of a Pythagorean diatonic scale starting on a C of frequency 256 hertz.

- -2. Calculate the frequency of F# in the scale of Problem 1.
- -3. Calculate the frequency of G# in the scale of Problem 1.
- -4. Given a C of frequency 256 hertz, find the number of beats per nd that would occur between the note a just major third higher and the a Pythagorean major third higher.
- -5. Find the frequency ratio of the one-quarter syntonic comma.
- -6. Starting with a C of 256 hertz, find the frequencies of the notes of a quarter comma meantone diatonic scale.
- -7. Build a just diatonic scale starting with a frequency of 240 hertz.
- -8. Designating the frequency of 240 hertz in Problem 8-7 as C₄, find chords corresponding to each of the following vibration proportions: (a) 4; (b) 3:4:5; (c) 4:5:8; (d) 5:6:8; (e) 10:12:15. The lowest note of a d is to be in the octave starting with C_4 , and the scale may be extended ard as far as necessary. (There are no sharps or flats.)
- -9. A string 120 centimeters long gives the pitch C3 when bowed near end. At what distances from the bowed end of the string should frets be ed so as to allow the just diatonic scale to be played?
- -10. Given an organ tuned to the tempered scale with $C_4 = 261.63$ hertz. the number of beats per second between the following:
 - (a) 3rd harmonic of C4 and 2nd harmonic of G4
 - (b) 3rd harmonic of F₄ and 2nd harmonic of C₅
 - (c) 5th harmonic of C4 and 4th harmonic of E4

the scale frequencies given on p. 153; five significant figures are necessary.) -11. Calculate the number of cents in the interval of the major sixth .) in (a) the Pythagorean scale; and (b) the just scale.

- ·12. The seventh harmonic of the note C2 is flat as compared with the the tempered scale starting with C₄. Find the difference in cents.
- ·13. Comparing the present standard of pitch with Handel's standard. is the difference in cents?

vers to Problems 8-14 and 8-15 are to be expressed in the subscript ion for musical pitch. Actual frequencies are not necessary.

- 14. (a) Find the pitches of the first six partial tones of a pipe open at ends and sounding C₃ as its fundamental. (b) Find the pitches of the six partial tones of a pipe open at both ends and sounding C3 as its harmonic. (c) A pipe closed at one end and open at the other sounds Be fifth harmonic. Find the pitches of the lower harmonics.
- 15. (a) A string is plucked at the center and sounds C₄ as its third harc. What pitches below the seventh harmonic will be present? (Assume harmonics having a node at the point where the string is plucked will not 1.) (b) A string 250 centimeters long is plucked at a point 100 centimeters one end and sounds G₃ as its second harmonic. What pitches below the th harmonic will be present?

ers to Problems

256, 288, 324, 341.3, 384, 432, 486, 512 Hz. 364.5 Hz. 8-3. 410.1 Hz. 4 beats/second. 8-5. 1.00311. 256, 286.2, 320, 342.4, 382.8, 428.0, 478.5, 512 Hz. 240, 270, 300, 320, 360, 400, 450, 480 Hz.

- 8-8. (a) C_4 G_4 C_5 , F_4 C_5 F_5 , A_4 E_5 A_5 , E_4 B_4 E_5 , G_4 D_5 G_5 ;
 - (b) G₄ C₅ E₅, D₄ G₄ B₄, C₄ F₄ A₄;
 - (c) C₄ E₄ C₅, F₄ A₄ F₅, G₄ B₄ G₅; (d) $A_4 C_5 F_5$, $E_4 G_4 C_5$, $B_4 D_5 G_5$;
 - (e) A₄ C₅ E₅, E₄ G₄ B₄, not D₄ F₄ A₄.
- 8-9. D₃, 106.7 cm; E₃, 96.0 cm; F₃, 90.0 cm; G₃, 80.0 cm; A₃, 72.0 cm; B_3 , 64.0 cm; C_4 , 60.0 cm.
- 8-10. (a) 0.84 beats/sec; (b) 1.19 beats/sec; (c) 10.37 beats/sec.
- 8-11. (a) 906 cents; (b) 884 cents.

Questions and Problems

- 8-12. 31 cents.
- 8-13. 70 cents.
- $8-14. \ \ (a) \ C_3 \ , C_4 \ , G_4 \ , C_5 \ , E_5 \ , G_5 \ ; (b) \ F_1 \ , F_2 \ , C_3 \ , F_3 \ , A_3 \ , C_4 \ ; \ (c) \ G_3 \ , D_5 \ , B_5 \ .$
- 8-15. (a) F_2 , C_4 , A_4 ; (b) G_2 , G_3 , D_4 , G_4 , D_5 .

Chapter Nine

Auditorium and Room Acoustics

Questions

- 9-1. In a room with a reverberation time T seconds, how long will it take the reverberant sound (from an impulse sound source) to fall to 10⁻³ of its original intensity?
- 9-2. We have available material of absorption coefficient 0.50, and another material of absorption coefficient 0.33. How much of each will be needed to get 100 absorption units?
- 9-3. An auditorium has a volume of 200,000 cubic feet. Find the optimum reverberation time for various kinds of music. What average value would you use?
- 9-4. What is the approximate absorption coefficient of an individual in an audience?

Problems

- 9-1. Check the statement on p. 163 that doubling the distance from a sound source in the open will reduce the intensity level by 6 decibels.
- 9-2. A certain hall has a length of 250 feet as measured from the front of the stage to the back wall. For a listener sitting halfway between the stage and the back wall, what is the time interval between the direct sound and the first reflection? (Assume the first reflection to come from the back wall.) Temperature is 68°F.
- 9-3. Calculate the reverberation time of the room discussed in the example on p. 170.
- 9-4. A certain auditorium is approximately 70 feet long, 50 feet wide, 40 feet high, and has 450 upholstered seats. The side walls and back are covered with a cellulose tile, the back of the stage and the ceiling and plaster, and the floor is cork tile. Assume the cellulose tile to have an absorption coefficient of 0.7, and the plaster and cork tile to have a coefficient of 0.06. Calculate the reverberation time at 500 hertz for this hall (a) with all the seats empty, and (b) with the seats all occupied. (c) What would be the optimum reverberation time for this hall for chamber music? (d) What would be the optimum reverberation time at 100 hertz?

9-5. A trumpet is sounded in the empty auditorium of Prob. 9-4 and duces a continuous sound power output as given in Table II, p. 52. What be the intensity level of the reverberant sound in the auditorium?

3-6. A certain rehearsal hall is 50 feet long, 40 feet wide and 20 feet high. floor is hardwood with an absorption coefficient of 0.04, the ceiling is ter with an absorption coefficient of 0.10, and side walls and end walls brick with an absorption coefficient of 0.20, all measured at 500 hertz. Find the optimum reverberation time for this room for chamber music. Calculate the actual reverberation time for this room. (c) Find how large audience in this room (seated in upholstered seats) will bring its reveration time down to the optimum value found above. (d) Find how many are feet of material of absorption coefficient 0.60 must be put on the side is to give the optimum reverberation time, without an audience.

1-7. Given the room of Prob. 9-6 with the absorption necessary to get the imum reverberation time, what power output of a sound source will be aired to give a reverberant sound level of 76 decibels?

wers to Problems

. 0.22 sec.

- 9-3, 0.54 sec.
- . (a) 0.89 sec; (b) 0.80 sec; (c) 1.3 sec; (d) 1.8 sec.
- . 86 dB.
- . (a) 1.1 sec; (b) 2.0 sec; (c) 130; (d) 2000 ft².
- 6.7×10^{-3} W.

Chapter Ten The String Instruments

stions

- 0-1. Sketch a graph of the lateral force of the bowed violin string on the ge during a vibration cycle.
- 0-2. On the same time axis, sketch a graph of the force of the bowed n string on the nut during a vibration cycle.
- 0-3. From Fig. 4, p. 194, sketch a graph of the force of the bowed string he bow during a vibration cycle.
- 0-4. How many resonances below 1000 hertz are there in the response e shown in Fig. 9, p. 205?

)lems

- 0-1. A violin string is bowed to sound G₃. It is bowed 1/5 the way along string from the bridge to the nut. What is the "flyback" time, i.e. the time ired for the string to move from configuration (a) to configuration (c)?
- 0-2. If the amplitude A_B of the string motion under the bow in Fig.4, p. is one millimeter, what is the speed of the bow?
- 0-3. Verify that the lowest plate resonances given in Fig. 6, p. 199, are
- 0-4. If a double bass is designed to have its main wood resonance at D₂, many times larger than the violin must it be according to the linear gement principle?

10-5. How many times larger than the violin would the double bass be according to the scaling curve of Fig. 11, p. 211?

10-6. A guitar string has a length of 63 centimeters from bridge to nut. At what distances in centimeters from the nut must frets be placed to play a tempered chromatic scale up to and including the octave?

Answers to Problems

Questions and Problems

10-1. 1.0 millisec.

10-2. 2.5 cm/sec.

10-4. 6 times.

10-5. 3.5 times.

10-6. 3.54, 6.87, 10.02, 13.00, 15.80, 18.45, 20.95, 23.31, 25.54, 27.64, 29.63, 31.50 cm.

Chapter Eleven

The Woodwind Instruments, and Others

Questions

- 11-1. Make a sketch of the pressure amplitudes of the first two vibration modes in the air column of a clarinet showing where the vent-hole should be theoretically located for best favoring of the second mode.
- 11-2. Check the result of Question 11-1 against the clarinet dimensions given in Ch. 11. How well do they check?
 - 11-3. Repeat Question 11-1 for the conical air column of an oboe.
- 11-4. Why must the vent-hole in the bassoon move up along the instrument as the scale is ascended?

Problems

- 11-1. Verify the statement on p. 224 that the length of the vibrating air column in the flute corresponds to a vibration frequency closer to C#4 than to C₄.
- 11-2. What should be the theoretical length (from the fipple hole to the far end) of the alto recorder?
- 11-3. The actual length of the alto recorder from the fipple hole to the far end is about 42 centimeters. Explain the discrepancy between this length and the theoretical length calculated in Problem 11-2.
- 11-4. Check the statement on p. 237 that an air column of length about 60 centimeters will give a resonance frequency corresponding to the lowest note on a clarinet.
- 11-5. Find the lowest frequency corresponding to the length (254 centimeters) of the bassoon air column. Compare with the actual frequency of the lowest note on the bassoon.

Answers to Problems

- 11-2. 49.3 cm.
- 11-3. The fipple hole has an acoustical mass equivalent to 7 centimeters length of the recorder bore.
- 11-5. 67.7 Hz; $B_1 = 58.3$ Hz, 260 cents lower.

Chapter Twelve The Brass Instruments

iestions

12-1. What do the French horn and the bass trombone have in common? 12-2. A large perforated rubber stopper (e.g. one of about a 6 centimeter ameter with a 2 centimeter diameter hole) is pushed into the bell of a impet. How will this affect the modes of the trumpet?

oblems

- $12_71.$ Calculate the length of an open tube that will give the same freency (Bb2) as the trombone with the slide in.
- 12-2. Using the result of Problem 12-1, calculate the lengths of tubing ded by pulling the slide out to positions 2, 3, and 4.
- 12-3. The valve slides for the first two valves on a trumpet are adjusted so it the trumpet is in tune with the tempered scale when the valves are used gly. What will be the discrepancy in cents when the two valves are used gether?

swers to Problems

-1. 295 cm.

12-2. 17.6 cm, 36.2 cm, 55.9 cm.

-3. 11 cents sharp.

Chapter Thirteen The Piano, and Others

stions

13-1. Can the tone of a note played on a harpsichord be affected by the mer of striking the key?

blems

- .3-1. The C_4 strings on a particular piano have a length of 0.64 meters and ass of 3.8×10^{-3} kilograms. To what tension must they be stretched, in tons? In pounds?
- .3-2. Check the statement on p. 290 that a difference of two cents been two strings for C_4 will give about one beat in three seconds.

wers to Problems

3-1. 666 N or 150 lb.

Chapter Fourteen The Percussion Instruments

Problems

14-1. If a kettledrum can cover the range F_2 to B_2 , by what factor does the tension in the head increase in going from the lowest note to the highest? 14-2. A 28-inch kettledrum has a head whose density is 0.03 grams per square centimeter. What must be the tension in the head (newtons per meter) if it is to sound F_2 ?

Answers to Problems

14-1. 2 times.

14-2. 777 N/m.

Chapter Fifteen The Electronic Production of Sound

Problems

15-1. A pure tone of 10 kilohertz is recorded on a disc record at the standard speed of 33-1/3 revolutions per minute, being recorded as a wavy groove. If the groove is toward the center of the record and has a diameter of 4 inches, what is the wavelength of the groove, in inches?

Answers to Problems

15-1. 7×10^{-4} in.

Appendix

Powers of Ten and Simple Logarithms

oblems-

-1. Express as powers of ten, as for example

$$31.6 = 10 \times 3.16 = 10 \times 10^{0.5} = 10^{1.5}$$

-) $3160; 3.16 \times 10^6; 0.00316$
- (b) 21.6; 683 (= 31.6×21.6). (d) 12.6; 1.26×10^{-4} .

) 1.46; 1460.

- -2. Find the logarithms of the following numbers:
-) 2;4;8;10.

-) $150;400;2000;5 \times 10^6$.
- (b) 15; 20; 25; 100.(d) $0.2; 0.004; 10^{-6}; 5 \times 10^{-6}.$

iswers to Problems

- -1. (a) $10^{3.5}$; $10^{6.5}$; $10^{-2.5}$. (c) $10^{0.17}$; $10^{3.17}$.

- -1. (a) $10^{3.5}$; $10^{6.5}$; $10^{-2.5}$. (b) $10^{1.33}$; $10^{2.83}$. (c) $10^{0.17}$; $10^{3.17}$. (d) $10^{1.10}$; $10^{-3.90}$. -2. (a) 0.301; 0.602; 0.903; 1.000. (b) 1.178; 1.301; 1.398; 2.000.
 - (c) 2.176; 2.602; 3.301; 6.000. (d) -0.699; -2.398; -6.000; -5.301.