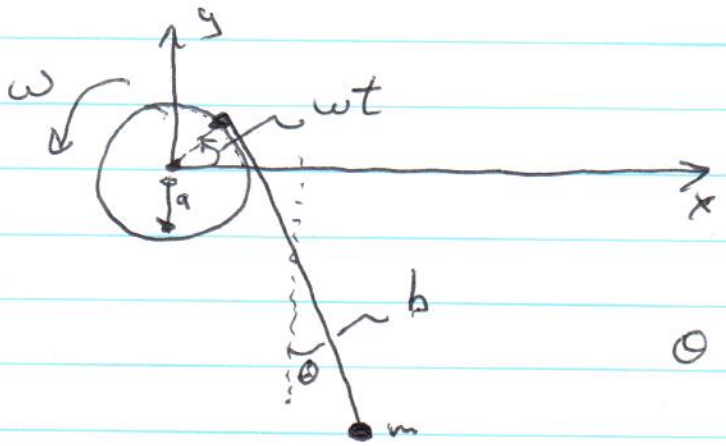
LAGRANGIAN EXAMPLE

PENDULUM ON A ROTATING RIM



ω = const

θ wrt VERTICAL

$L = T - U$ FIND EQN OF MOTION

FIRST, FIND T

$$x = a\cos\omega t + b\sin\theta$$
$$y = a\sin\omega t - b\cos\theta$$

$$\dot{x} = -a\omega\sin\omega t + b\dot{\theta}\cos\theta$$
$$\dot{y} = a\omega\cos\omega t + b\dot{\theta}\sin\theta$$

θ = GENERALIZED COORDINATE

t = INDEPENDENT VARIABLE

$$T = \frac{m}{2}(\dot{x}^2 + \dot{y}^2)$$
$$U = mgy$$

$$L = \frac{m}{2}(-a\omega \sin\omega t + b\dot\theta\cos\theta)^2$$

$$+\frac{m}{2}(a\omega\cos\omega t + b\dot\theta\sin\theta)^2$$

$$- mg(a\sin\omega t - b\cos\theta)$$

$$= \frac{m}{2}\left[a^2\omega^2\sin^2\omega t - 2a\omega b\dot\theta\sin\omega t\cos\theta + b^2\dot\theta^2\cos^2\theta\right.$$

$$\left. + a^2\omega^2\cos^2\omega t + 2a\omega b\dot\theta\cos\omega t\sin\theta + b^2\dot\theta^2\sin^2\theta\right]$$

$$- mg(a\sin\omega t - b\cos\theta)$$

$$\boxed{L = \frac{m}{2}\left[a^2\omega^2 + b^2\dot\theta^2 + 2a\omega b\dot\theta\sin(\theta-\omega t)\right] - mg(a\sin\omega t - b\cos\theta)}$$

RECALL, $\sin(A-B) = \sin A\cos B - \cos A\sin B$

NOW, WE NEED $\frac{d}{dt}\frac{\partial L}{\partial\dot\theta}$ & $\frac{\partial L}{\partial\theta}$

$$\frac{\partial L}{\partial\dot\theta} = m\left[b^2\dot\theta + a\omega b\sin(\theta-\omega t)\right]$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot\theta} = m\left[b^2\ddot\theta + a\omega b(\dot\theta-\omega)\cos(\theta-\omega t)\right]$$

~~$\frac{\partial L}{\partial\theta} = m\left[a\omega b\cos(\theta-\omega t) - gb\sin\theta\right]$~~

$$\frac{\partial L}{\partial \theta} = m\left[a\omega b\dot{\theta}\cos(\theta-\omega t) - gb\sin\theta \right]$$

FOR THE EQN OF MOTION

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{\partial L}{\partial \theta}$$

$$mb^2\ddot{\theta} + ma\omega b(\dot{\theta}-\omega)\cos(\theta-\omega t)$$

$$= ma\omega b\dot{\theta}\cos(\theta-\omega t) - mgb\sin\theta$$

$$mb^2\ddot{\theta} - ma b\omega^2\cos(\theta-\omega t) = -mgb\sin\theta$$

$$\boxed{\ddot{\theta} = \frac{a\omega^2}{b}\cos(\theta-\omega t) - \frac{g}{b}\cdot\sin\theta}$$

SOLVE NUMERICALLY, E.G. BY MMA

NOTE THAT FOR $\omega = 0$ WE RECOVER RESULT FOR SIMPLE PENDULUM