16 April 2015

Physics 1307 Examination 3

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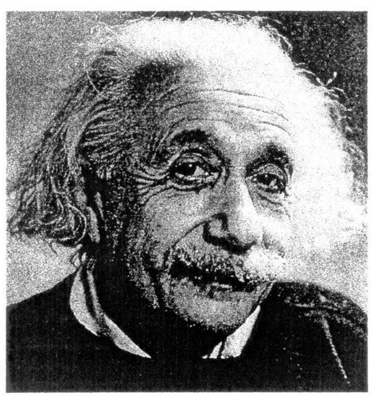
Spring 2015

Please PRINT your name so that we can read it.

Name: Solutions

Yes, BOX your final answers.
SHOW work for maximum credit.



Al says ....Be Smokin' Smart!

Q1 10 pts Albertina and Evelyn have ropes attached to them by which they hang from the ceiling. They are face to face with each other and push off against each other. Albertina has a mass of 80 kg and Evelyn has a mass of 56 kg. Following the push, Albertina swings upward to a height of 0.65 m above her starting point. How high h above her starting point does Evelyn rise?

let m_a = ~~alegelina~~ Albertina's mass

h_a = Albertina's height

m_e = Evelyn's mass

h_e = Evelyn's height

$|\vec{V}_a|$ = albertina's velocity

$|\vec{V}_e|$ = evelyn's velocity

use conservation of energy to find albertina's initial velocity:

$$\frac{1}{2} m_a v_a^2 = m_a g h_a$$

$$v_a = \sqrt{2 g h_a}$$

conservation of momentum: $|\vec{p}_i| = |\vec{p}_f|$

total momentum just before push off

total mamentum just after

$$0 = m_a |\vec{v}_a| - m_e |\vec{v}_e|$$

$$m_a v_a = m_e v_e \longrightarrow v_e = \frac{m_a v_a}{m_e} = \frac{m_a \sqrt{2 g h_a}}{m_e}$$

conservation of energy to find Evelyn's height:

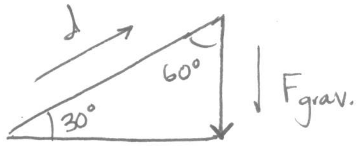
$$\frac{1}{2} m_e v_e^2 = m_e g h_e$$

$$h_e = \frac{v_e^2}{2g} = \frac{m_a^2}{m_e^2}(2 g h_a) \cdot \frac{1}{2g} = \frac{m_a^2}{m_e^2} \cdot h_a = \left(\frac{80\,kg}{56\,kg}\right)^2 \cdot .65\,m$$

$$\boxed{h_e = 1.33\,m}$$

Q2 10 pts A runaway truck lane heads uphill at a angle of 30° to the horizontal. If a truck of mass $m = 16,000$ kg goes out of control with a speed $v = 110$ km/hr, what distance s *along* the ramp does the truck go? Box that answer.

convert velocity to m/s: $V = 110\,\frac{km}{hr} \cdot \frac{1\,hr}{3600\,s} \cdot \frac{1000\,m}{km} = 30.5\,m/s$



note, 60°, not 30°

$$W_{grav} = \vec{F} \cdot \vec{d} = Fd\cos(60°)$$

$W_{grav.} = \Delta KE$ (note: work done by gravity is negative)

$$-Fd\cos(60°) = \cancel{\tfrac{1}{2}mv_f^2}^{\,0} - \tfrac{1}{2}mv_i^2$$

$$Fd\cos(60°) = \tfrac{1}{2}mv_i^2$$

F=mg

$$d = \tfrac{1}{2}mv_i^2 \cdot \frac{1}{F\cos(60°)} = \tfrac{1}{2}\cancel{m}v_i^2 \cdot \frac{1}{\cancel{m}g\cos(60°)} = \frac{v_i^2}{2g\cos(60°)}$$

$$d = \frac{(30.5\,m/s)^2}{2(9.8\,m/s^2)\cos(60°)} \approx 95.3\,m$$

$$\boxed{d \approx 95\,m}$$

Q3 10 pts Suppose the moon had an orbit around earth such that it never seemed to move in the night sky. That is, the moon appeared fixed in the night sky so that its orbit around earth took very close to 30 days to complete. If the mass of earth $M_E = 6.0 \times 10^{24}$ kg and Newton's gravitational constant $G = 6.67 \times 10^{-11}$N-m^2/kg^2, what would be the radius R of the moon's orbit about earth?

convert period from days to seconds:

$$T = 30 \text{ days} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{3600 \text{ s}}{1 \text{ hr}} = 2.59 \times 10^6 \text{ s}$$

radius of orbit related to period by:

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left[\frac{(6.67\times10^{-11}\,\text{Nm}^2/\text{kg}^2)(6\times10^{24}\,\text{kg})(2.59\times10^6\,\text{s})^2}{4\pi^2}\right]^{1/3}$$

$$\boxed{r = 4 \times 10^8 \text{ m}}$$

Q4 10 pts A friend of yours has an "ambitious" scheme to launch iron ore mined on the moon to a set of manufacturing satellites in orbit about the moon. This crazy scheme calls for a large spring to launch iron ore-filled bins of mass $m = 1000\,\text{kg}$. How large must the spring constant k of these springs be? The escape speed of the moon is $v = 2.4\,\text{km/s}$. Recall that the "escape" speed is the speed required by an object to escape or overcome the gravitational force of a planet or moon that the object is launched from. Be sure to box your answer.

Note: missing information, assume spring is compressed by 100 m from equilibrium.

conservation of energy: $E_{final} = E_{initial}$

$$\cancel{\tfrac{1}{2}} m v_{esc.}^2 = \cancel{\tfrac{1}{2}} k x^2$$

$$k = \frac{m v_{esc.}^2}{x^2} = \frac{(1000\text{ kg})(2400\text{ m/s})}{(100\text{ m})^2}$$

$$\boxed{k = 5.76 \times 10^5\ \text{N/m}}$$

Q5 10 pts A drunk balloonist is floating along the countryside at a height of 400 meters. She accidentally drops a cask of wine overboard (she's drunk, remember). Fortunately, for her, the cask falls in a layer of hay with no significant damage to the cask or its contents. Assume that the speed of the cask at impact was 40 m/s (terminal speed) and that the mass of the cask was 80 kg. Further assume the maximum force the cask can withstand without damage is 1.0×10^5 N.

a) 5 pts What minimum depth d of hay would stop the cask safely? Are you going to box your answer? Yes, you are.

minimum depth, d_{min} results in maximum force, F_{max}

work done by hay to stop cask: $W_{hay} = -F_{max} d_{min}$

$$W_{hay} = \Delta KE$$

$$-F_{max} d_{min} = \cancel{\tfrac{1}{2} m v_f^2} - \tfrac{1}{2} m v_i^2$$

$$d_{min} = \frac{\tfrac{1}{2} m v_i^2}{F_{max}} = \frac{(80\,kg)(40\,m/s)^2}{2(1 \times 10^5\,N)}$$

$$\boxed{d_{min} = 0.64\,m}$$

b) 5 pts What is the magnitude of the impulse J on the cask from the hay?

$$|\vec{J}| = |\Delta\vec{p}| = |m\Delta\vec{v}| = |(80\text{ kg})(-40\text{ m/s})|$$

$$\boxed{|\vec{J}| = 3200\text{ Ns}}$$