# PHYS 3344 Exam 3

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**Printed Name** _____

## DIRECTIONS:

**1.** If I can't read it, I can't grade it.

**2.** **Show** your work to receive credit.

3. **BOX YOUR FINAL ANSWERS**

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**5.** Paginate all pages. Label the problem number clearly.

**6.** Staple your pages together, in order.

**7.** Good luck.

**Q1 15 pts.** A solid and uniform steel sphere of mass $m$ and radius $\rho$ rolls without slipping along the inside lower half of a hollow pipe of radous $R$. It rolls up and down the inner sides of this pipe. The moment of inertia $I$ of a solid sphere is $\frac{2}{5}mR^2$, where the symbols have the normal meaning.

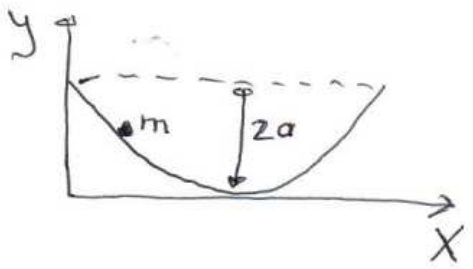
**a) 5 pts** Write down the lagrangian $L$ for this system. Box the answer.

**b) 5 pts** Write down the lagrangian equation(s) of motion for this system. Box the answer.

**c) 5 pts** Find the frequency $\omega$ of small oscillation. Box the answer.

**Q2 10 pts.** Consider a damped simple harmonic oscillator that has the property that after 4 cycles its amplitude has dropped to $1/e$ of its original value. If its *undamped* frequency is $\omega_0$, what is the ratio of the frequency $\omega$ of the damped oscillator to its undamped frequency? Box that answer.

**Q3 20 pts.** A particle of mass $m$ slides without friction on a cycloid, which is given by $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$. The angle $\theta$ is constrained to be $0 \leq \theta \leq 2\pi$. See the figure.




**a) 5 pts** Write down the lagrangian $L$ for this system. Box the answer.

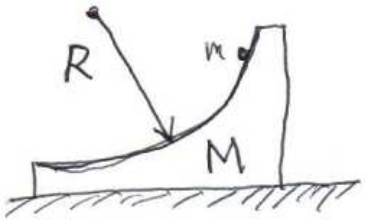
**b) 5 pts** Using the identity, $\cot(\theta/2) = \sin\theta/(1 - \cos\theta)$, show that the equation of motion for the small mass is given by

$$\ddot{\theta} + \frac{1}{2}\cot(\frac{\theta}{2})\dot{\theta}^2 - \frac{g}{2a}\cot(\frac{\theta}{2}) = 0$$

**c) 5 pts** Use the identity $u = \cos(\theta/2)$ to rewrite the answer in part (b) to find a simple differential equation for $u$. Box the answer

**d) 5 pts** Using the answer in part (c), what is the frequency of oscillation for the mass? Box the answer.

**Q4 15 pts.** A particle of mass $m$ slides down a smooth circular wedge of mass $M$ as shown in the figure. The wedge itself rests on a flat surface and can slide without friction horizontally. You can take the zero of potential energy to be the top of the wedge. **HINT:** Use the angular variable $\theta$, measured with respect to a horizontal line that just skims the top of the wedge, to help measure the position of $m$.




**a) 5 pts** Write down the lagrangian $L$ for this system. Box the answer.

**b) 5 pts** Write down the equation of motion for the wedge. Box the answer.

**c) 5 pts** Write down the equation of motion for the small mass $m$.