PHYS 3344
Fall 2019
TE Coan
Due: 22 Nov '19 6:00 pm

Homework 10

1. A bucket of water is set spinning about its symmetry axis with an angular velocity of magnitude Ω. What is the shape of the water surface after it has reached equilibrium? **HINT:** The surface of the water is an equipotential under the combined effects of gravity and the centrifugal force. The shape is one you are familiar with and using cylindrical polar coordinates will make your life easier. You will do what with your answer? Yes, use our favorite shape.

2. Determine how much greater Δg the gravitational acceleration g is at the pole than at the equator. Assume Earth is spherical. If the actual measured difference is $\Delta g = 52\,\text{mm/s}$, explain the difference between your calculations and the measurement. Use the values $\Omega = 7.3 \times 10^{-5}\,\text{rad/s}$ and $R = 6370\,\text{km}$ for your calculations.

3. Approximate Lake Superior as a circle of radius 162 km at a latitude of 47°. Assume the water is at rest with respect to Earth and find the depth h that the center of the lake is suppressed with respect to the shore due to the centrifugal force. Box the answer.

4. Suppose you are on a perfectly spherical planet and measure the free-fall acceleration to be $g = g_0$ at the North Pole. (This planet is not necessarily Earth!) At the equator, you measure $g = \lambda g_0$ (with $0 \leq \lambda \leq 1$). Find $g(\theta)$, the free-fall acceleration at colatitude θ as a function of θ. Be sure to box your answer.

5. Suppose some particle of mass m is confined to move, without friction, in a vertical plane, with axes x horizontal and y vertically up. The plane is forced to rotate with angular velocity of magnitude Ω about the y axis. Find the equation of motion for x and y, solve them, and describe the possible motions. This is not meant to be a lagrangian problem.