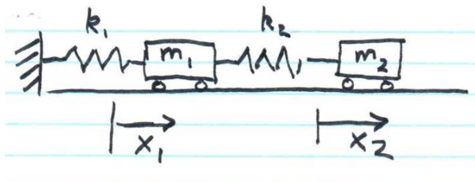
PHYS 3344
Fall 2019
TE Coan
Due: 3 Dec '19 6:00 pm

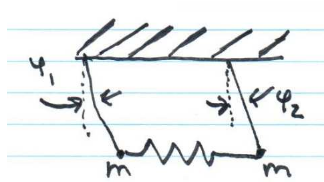
Homework 11

1a.) Find the normal frequencies, ω_1 and ω_2, for the two carts shown in the figure below. The variables x_1 and x_2 represent the displacements of the respective carts from their equilibrium positions. Call ω_1 the highest of the normal frequencies and assume $m_1 = m_2 = m$, $k_1 = k_2 = k$.



b.) Find and describe the motion for each of the normal modes. **Hint:** You may find it useful to set the spring constant $k_3 = 0$ in the text's Fig. 11.1 and modify the associated matrices **M** and **K**.

2.) Consider the two coupled pendulums shown in the figure below. Each of the pendulums has a length L and the spring constant is k. The pendulums' position can be specified by the angles ϕ_1 and ϕ_2. The relaxed length of the spring is such that the equilibrium position of the pendulums is at $\phi_1 = \phi_2 = 0$ with the two pendulums vertical.

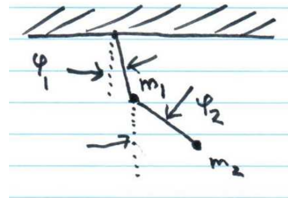


a.) Find the lagrangian L of this system. You can assume the angular deflections are "small" at all times so that the extension of the spring is well approximated by $L(\phi_2 - \phi_1)$. Box your answer.

b.) Write down the lagrangian equations of motion. Box.

c.) Find and describe the normal modes of this system. Clearly label the frequencies and describe the normal modes. Call the highest oscillation frequency ω_1. BBBBox.

3. Back to the double pendulum we explored in class and video, and shown in the figure below. Assume that $\phi_1, \phi_2, \dot{\phi}_1$ and $\dot{\phi}_2$ remain small throughout their motion. Let the masses and lengths be such that $m_1 = 8m_2 = 8m$, and $L_1 = L_2 = L$.



a.) Find the normal frequencies. Label the highest frequency ω_1. Box your answers.

b.) Find the actual motion $[\phi_1(t), \phi_2(t)]$ if the pendulum is released from rest with $\phi_1(t) = 0$ and $\phi_2 = \alpha$. I just need the relative sizes of the $\phi(t)$ amplitudes and matched with their corresponding normal frequencies. I do not need the phase angles. **Note:** This situation described here is different from the chaotic lobby double pendulum because in the lobby we do not make the small angle approximation and we are not able to release the pendulum with the initial conditions we have here. Hence,the motion is very different. Remember to box your answer(s).