PHYS 3344
Fall 2019
TE Coan
Due: 4 Oct '19 6:00 pm

Homework 4

1. Given $\mathbf{F}_1 = 2xz\mathbf{i} + y\mathbf{j} + x^2\mathbf{k}$, and $\mathbf{F}_2 = y\mathbf{i} - x\mathbf{j}$, which $\mathbf{F}$, if either, is conservative? Show your work and box your answer.

2. Show that

$$\mathbf{F} = y^2 z \sinh(2xz)\mathbf{i} + y\cosh(2xz)\mathbf{j} + y^2 x \sinh(2xz)\mathbf{k}$$

is conservative and find a scalar potential U such that $\mathbf{F} = -\nabla U$. Box that answer.

3. A uniform rope has a total mass $m = 0.4\,\text{kg}$ and a total length $L = 4$ meters. Suppose a length $s = 0.6\,\text{m}$ of the rope is hanging vertically down off a table. How much work W is required to put all the rope on the table? Do not forget proper units! No units, no credit.

4. Suppose you have a mass m on the end of a spring of force constant k and constrained to move along the horizontal x axis. If you place the origin at the spring's equilibrium position, the potential energy is $\frac{1}{2}kx^2$. At time $t = 0$ the mass is sitting at the origin and given a sudden kick to the right so that it moves out to a maximum displacement of $x_{\text{max}} = A$ and then continues to oscillate about the origin.

a) Write down the equation for the conservation of energy **and** solve it to give the mass's velocity $\dot{x}$ in terms of the position x and the total energy E.

b) Show that $E = \frac{1}{2}kA^2$, and use this to eliminate E from your expression for $\dot{x}$.

c) Use the result, discussed in the text (4.58) and the videos, $t = \int dx'/\dot{x}(x')$, to find the time for the mass to move from the origin to a position x.

d) Find x as a function of t and show that the mass executes simple harmonic motion with period $2\pi\sqrt{m/k}$.

5. Consider the bead shown in Fig 4.13 of your text threaded onto a curved rigid wire. The bead's position is measured by its distance s measured along the wire from its origin.

a) Show that the bead's speed v is just $v = \dot{s}$. **Hint:** Write $\mathbf{v}$ in terms of its components, dx/dt,etc., and find its magnitude using Pythagoras' theorem.

b) Show that $m\ddot{s} = F_{\text{tang}}$, the tangential component of the net force on the bead. **Hint:** One way to do this to take the time derivative of the expression $v^2 = \mathbf{v} \cdot \mathbf{v}$. The left side should lead to $\ddot{s}$ and the right side to F_{tang}.

c) One force on the bead is the normal force **N** of the wire that constrains the bead to follow the wire. If all the other forces (e.g., gravity) on the bead are conservative, then their resultant can be derived from a potential energy U. Show that $F_{\text{tang}} = -dU/ds$. This shows that one-dimensional systems of this type can be treated just like linear systems, with x replaced by s and F_x by F_{tang}.