PHYS 3344
Fall 2019
TE Coan
Due: 11 Oct '19 6:00 pm

Homework 5

1. Our text makes the (correct) statement that a force $\mathbf{F}(\mathbf{r})$ that is central and spherically symmetric is conservative. Show that, in spherical polar coordinates, such a force has the property $\nabla \times \mathbf{F} = 0$.

2. See if you can prove the so-called **virial theorem**. This is a statement that relates the average kinetic energy $\langle T \rangle$ of a stable system to the average potential energy $\langle U \rangle$ of the system. It applies when the force between two particles of the system has a corresponding potential energy U of the form $U = kr^n$, where r is the separation of the particles and n is some real number. So, suppose a mass m moves in a circular orbit about the origin in the field of an attractive central force with potential energy $U = kr^n$. Show that the average kinetic energy T of the particle is $\langle T \rangle = n\langle U \rangle/2$. You did something like this in PHYS 1303 but the virial theorem was never mentioned.

3. Suppose the potential energy (PE) U of a mass m a distance r from the origin in an effective one-dimensional universe is given by

$$U(r) = U_0 \left(\frac{r}{R} + \lambda^2 \frac{R}{r} \right),$$

for $0 < r < \infty$, and with U_0, R and λ all positive constants.

a) Find the equilibrium position r_0. Box the answer.

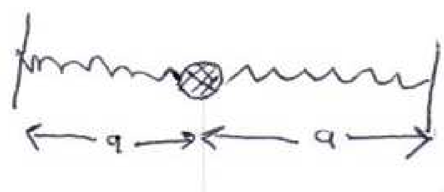
b) If you call x the distance from the equilibrium position, show that for small x, the PE has the form $U = \text{const} + \frac{1}{2}kx^2$. What is the angular frequency ω_0 of small oscillations? Box your answers.

4. In class we haven't yet talked about oscillators in more than one dimension. However, what we learned carries over to such oscillators. So-called **isotropic** oscillators have the same spring constant in each of the dimensions they oscillate in. So-called **anisotropic** oscillators can have different spring constants, and hence oscillation frequencies, for each of their dimensions. Consider a two-dimensional anisotropic oscillator. Its motion is described by Eq. (5.23) in Taylor.

a) Show that if the ratio of frequencies is rational (i.e., $\omega_x/\omega_y = p/q$ where p and q are integers) then the motion is periodic. What is the period T? Box that answer.

b) Show that if the ratio of frequencies is irrational, then the motion never repeats itself.

5. This problem is a bit challenging. Consider a mass m resting on a horizontal, frictionless table as shown in the figure below. The table lies in the $x - y$ plane and both identical springs have spring constant k and unstretched length l_0. At equilibrium the mass rests at the origin and the distances a are not necessarily equal to l_0 (i.e., they may already be stretched or compressed).



a) If the mass moves to a position (x, y), with x and y "small" compared to a, show that the potential energy U has the form $U = \frac{1}{2}(k_x x^2 + k_y y^2)$, typical of an anisotropic oscillator, where the two spring constants refer to spring constants in the x and y directions. You may find the Taylor series expansion $(1 + \epsilon)^{1/2} \simeq 1 + \frac{1}{2}\epsilon - \frac{1}{8}\epsilon^2 + \ldots$ useful. Box your answer.

b) Show that if $a < l_0$ the equilibrium position is unstable and explain why. Box that answer.