PHYS 3344
Fall 2019
TE Coan
Due: 18 Oct '19 6:00 pm

Homework 6

1. The frequency of a damped harmonic oscillator is one-half the frequency of the same oscillator with no damping. Find the ratio R of the maxima of successive oscillations. Box that answer.

2. Derive the expression for the total energy $E(t)$ of an underdamped oscillator as a function of time. Box your answer. You may find the trigonometric identities useful: $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ and $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$.

3. Plot the *velocity* resonance curve for a driven, damped harmonic oscillator with $Q = 6$.

a) Do this by first finding the expression for the amplitude v_0 of the velocity function and finding the resonance frequency ω_v for v_0. Box both answers.

b) Show that the full width $\Delta\omega$ of the curve between the points corresponding to $\dot{x}_{\max}/\sqrt{2}$ is approximately equal to $\omega_0/6$. Box that answer.

c) Now make the plot as a function of ω. The plot **must** be machine made or no credit. You'll need to pick some numerical value for ω_0. You could make it equal 5 or 10 or ...

4. If you drive along a "washboard" road (it looks like a road with a sequence of regularly spaced speed bumps), the bumps will cause the wheels to oscillate on the springs. Find the speed v at which your call resonates by using the following information.

a) When 4 80-kg passengers climb into the car, the car's body sinks by 2 centimeters. Estimate the spring constant k of each of the 4 springs. Box the answer.

b) If the axle assembly (two wheels and whatever else is oscillating) has a total mass of 50 kg, what is the natural frequency f of the oscillating mass on the two springs? Box.

c) If the bumps on the road are 80 cm apart, at what approximate speed v would cause these oscillations to go into resonance. Box your answer.

5. Suppose you drive an *undamped* simple harmonic oscillator with natural frequency ω_0 with an external sinusoidal driving force also oscillating at frequency ω_0 and with force per unit mass of f_0. If it helps you visualize things, you can assume this oscillating system is a mass m attached to a massless spring of spring constant k.

a) Show that the amplitude $x(t)$ of the oscillator from its equilibrium position is given by

$$x(t) = A\cos\omega_0 t + B\sin\omega_0 t + \frac{f_o t}{2\omega_0}\sin\omega_0 t.$$

Note: You must *derive* this equation from first principles. Show your work and box your final answer.

b) Plot the 3rd term above. Machine made plot only, no scribbles allowed. Pick some convenient value for ω_0 and f_0 for the plot. How about 1? Also, make the time t positive, $t \geq 0$. Note that the purely oscillatory piece of the solution does not damp away at large t. Your plot should have t along the x-axis and $x(t)$ along the y-axis.