PHYS 3344
Fall 2019
TE Coan
Due: 8 Nov '17 6:00 pm

Homework 8

1. A particle of mass m rests on a smooth square plate. The plate is tilted to an inclination angle θ at a steady rate α ($\theta = 0$ at $t = 0$) so that it pivots about one edge of the plate. This causes the particle to slide down the plate toward the downhill edge of the plate. Determine the equation of motion $r(t)$ of the particle, where $r(t)$ is the position of the particle from the downhill edge. **HINT:** At some point you will arrive at an inhomogeneous differential equation. Keep your wits about you and remember that in such circumstances the general solution is the sum of the homogeneous solution $r_h(t)$ plus the particular solution $r_p(t)$. For the particular solution, why not try $r_p(t) = C_1 \sin \alpha t + C_2 \cos \alpha t$, where C_1 and C_2 are some constants? Take it from there. Do box your answer.

2. Consider a simple pendulum with string length l and bob of mass m attached to the ceiling of an elevator that is accelerating upward with magnitude a. Using the Lagrangian technique, determine the angular frequency ω for small oscillations of the pendulum. Box that answer.

3. A smooth wire is bent in the form of a helix, with cylindrical polar coordinates $\rho = R$ and $z = \lambda\phi$, where R and λ are constants and the z axis is vertically upward (and gravity vertically downward). A bead of mass m is threaded onto the wire. Find the equation for the bead's azimuthal motion and its vertical acceleration $\ddot{z}$. Box the answer.

4. Here is a twist on a simple problem that makes the problem interesting. Consider the one-dimensional motion along the x-axis of a cart of mass m attached by a spring of spring constant k to a fixed wall. The cart will oscillate back and forth with a well known angular frequency ω you learned about in PHYS 1303. However, when you derived this result you assumed the spring was massless. Well, it ain't and we want to see what effect this has on the oscillation frequency ω of our cart if the spring has mass M.

a) Consider the spring uniform as well as its stretching. Show that its kinetic energy is $\frac{1}{6}M\dot{x}^2$ and write down the Lagrangian L for the cart-spring system. Box your answer.

b) Now write down the Lagrange equation for this system and show that the cart still executes SHM but now with an angular frequency $\omega = \sqrt{k/(m + M/3)}$. That is, accounting for the spring mass is the same as the massless case but with the cart mass increased by $M/3$. Boxes will come in handy.