TEC ϕ

FOR A PROBABILITY DENSITY $f(x)$

w/ $\int_{-\infty}^{\infty} dx\, f(x) = 1$ AND MEAN μ

WE HAVE

$$\sigma^2 \equiv \int_{-\infty}^{\infty} dx\, f(x)\, (x-\mu)^2$$

$$= \int_{-\infty}^{\infty} dx\, f(x)\, x^2 - 2\mu \int_{-\infty}^{\infty} dx\, f(x)\, x + \mu^2 \int_{-\infty}^{\infty} dx\, f(x)$$

$$= \overline{x^2} - 2\mu^2 + \mu^2$$

$$\boxed{\sigma^2 = \overline{x^2} - \mu^2}$$

TRUE FOR ANY RANDOM PROBABILITY DENSITY (& PROBABILITY DISTRIBUTION)

e.g., BINOMIAL, GAUSSIAN, POISSON, ...

TAYLOR 4.6

(a) $\bar{x} = \dfrac{\Sigma x_i}{N}$ w/ $N = 20$

$$\boxed{\bar{x} = 10.7}$$

$$\sigma_x = \left[\frac{\sum_1 (x_i - \bar{x})^2}{N-1} \right]^{1/2}$$

$$\boxed{\sigma_x = 3.4}$$

(b) $\boxed{\sigma_x \simeq \sqrt{\bar{x}} = 3.3 \simeq 3.4}$

TAYLOR 4.7

(a) $\bar{x} = \sum_i x_i / N$ w/ $N = 5$

$$\boxed{\bar{x} = 22.2}$$

$$\sigma = \left[\frac{\sum (x_i - \bar{x})^2}{N-1}\right]^{1/2}$$

$$\boxed{\sigma = 4.3}$$

$$\boxed{\sigma \simeq \sqrt{\bar{x}} = \sqrt{22.2} = 4.7}$$

NOTE $4.3 \simeq 4.7$, RESULTS CONSISTENT

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T 4.7

(b) $y = \sqrt{\nu}$. SEEK ERROR IN y

$$\sigma_y^2 = \left(\frac{\partial f}{\partial \nu}\right)^2 \sigma_\nu^2$$ SEE CHAPS 1-3

$$= \left(\frac{1}{2}\right)^2 \frac{1}{\nu} \sigma_\nu^2$$

$$\sigma_y = \frac{1}{2} \frac{\sigma_\nu}{\sqrt{\nu}}$$, BUT $\sigma_\nu = \sqrt{\nu}$ (CHAP. 3)

SO,

$$\sigma_y = \frac{1}{2} = 0.5$$

NOTE, THIS IS THE UNCERTAINTY IN THE STD DEV., <u>NOT</u> THE STD DEV. ITSELF ??

STD DEVIATIONS COMPUTED IN pt (a) ARE CONISTENT w/ THIS RESULT.

TEC 1

a) $\text{BKG/MIN} = \dfrac{58 \pm \sqrt{58}}{5\ \text{MIN}}$

$\overline{\text{BKG}} = 11.6 \pm 1.5\ \text{MIN}^{-1}$

CORRECTED COUNTS = $\overline{\text{RAW}} - \overline{\text{BKG}}$

$\sigma^2(\text{CC}) = \sigma^2(\overline{\text{RAW}}) + \sigma^2(\overline{\text{BKG}})$

$\overline{\text{RAW}} = \dfrac{1232}{10\ \text{MIN}} \pm \dfrac{\sqrt{1232}}{10\ \text{MIN}}$

$= 123.2 \pm 3.5\ \text{MIN}^{-1}$

CORRECTED COUNTS $= (123.2 - 11.6)\ \text{MIN}^{-1}$

$= 111.6\ /\text{MIN}$

w/ ERROR $= \left[1.5^2 + 3.5^2\right]^{1/2}/\text{MIN} = 3.8/\text{MIN}$

$$\boxed{\text{CC} = 111.6 \pm 3.8\ \text{MIN}^{-1}}$$

NOTE: NO ERROR IN LENGTH OF TIME PERIOD DURING WHICH DATA IS COLLECTED.