Q1

EARTH'S CHARGE

TREAT EARTH AS EQUIPOTENTIAL.
CHARGE RESIDES ON SURFACE.

$E = Q/4\pi\epsilon_0 R^2$ w/ R = EARTH'S RADIUS = 6400 km.

$Q = 4\pi\epsilon_0 R^2 \cdot E$

$$\boxed{Q \simeq 4.6 \times 10^5 \text{ Coulombs}}$$

Q2.

(a) $\underset{\sim}{M} = \chi_m \underset{\sim}{H}$

$\oint \underset{\sim}{H} \cdot d\underset{\sim}{l} = I_f = I_0$

$H = I_0/2\pi r \quad r > r_0$

$$\boxed{\underset{\sim}{M}(\underset{\sim}{r}) = \chi_m \, I_0/2\pi r \; \hat{\varphi} \qquad r > r_0}$$

(b) $\underset{\sim}{J}_b = \underset{\sim}{\nabla} \times \underset{\sim}{M}$ USE SPHERICAL COORDS.

$= \frac{1}{r}\left[\frac{\partial}{\partial r}(r M_\varphi)\right]\hat{\theta}, \quad M_\theta = M_r = 0$

$\boxed{\underset{\sim}{J}_b = 0}$ NO VOLUME CURRENT IN H_2O

(c) AT WIRE SURFACE $\underset{\sim}{K}_b = \underset{\sim}{M}(r = r_0) \times \hat{n}$

$= \frac{\chi_m I_0}{2\pi r_0} \hat{\varphi} \times (-\hat{r})$ H_2O NORMAL POINTS TOWARD WIRE

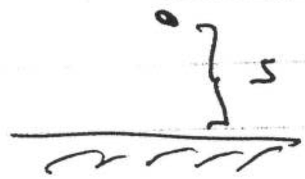
$= \frac{\chi_m I_0}{2\pi r_0} \hat{z}$ (PARALLEL TO I_0)

$I_b = K_b \cdot 2\pi r_0 = \chi_m I_0$

$$\boxed{I_T = (1 + \chi_m) I_0}$$

Q3

— <u>FALLING</u> <u>CHARGE</u>



$s(t=0) = s_0$

USE IMAGES !

$$F = \frac{1}{4\pi\varepsilon_0} \, q^2/(2s)^2$$

$$m\ddot{s} = \frac{-q^2}{16\pi\varepsilon_0}\frac{1}{s^2}$$ FORCE IS ATTRACTIVE

$$m\ddot{s} = m\frac{dv}{ds}\cdot ds/dt \Rightarrow v\, dv/ds = -C_0/s^2$$

$\cancel{w/\, C_0 = q^2/16\pi m\varepsilon_0}$

$$v\,dv/ds = -C_0/s^2 \qquad w/\; C_0 = q^2/16\pi m\,\varepsilon_0$$

$$\int v\,dv = \int -C_0\, ds/s^2$$

$$v^2/2 = C_0/s \quad + \text{const}$$

$$= C_0/s - C_0/s_0 \qquad v(t=0)=0$$

$$ds/dt = -\left[2C_0\left(\frac{1}{s} - 1/s_0\right)\right]^{1/2}$$

↑ v IS NEGATIVE, TOWARD PLANE

Q3

$$-\int_0^T dt = \int_{s_0}^{0} - \left[\frac{s_0 s}{2c_0 (s_0 - s)}\right]^{1/2} ds$$

$$= \int_{s_0}^{0} -\sqrt{\frac{cs}{s_0 - s}}\, ds \qquad w/\ c \equiv \frac{s_0}{2c_0}$$

$$T = \sqrt{c}\left(\sqrt{s_0 s - s^2} - s_0 \tan^{-1}\sqrt{\frac{s}{s_0 - s}}\right)\Bigg|_{s_0}^{0}$$

$$= 0 - \left(-\sqrt{c}\, s_0 \cdot \pi/2\right)$$

$$T = \sqrt{\frac{s_0}{2c_0}}\, s_0 \cdot \pi/2$$

$$= s_0^{3/2}\left(\pi/2\right)\left[\frac{8\pi m \varepsilon_0}{g^2}\right]^{1/2}$$

$$\boxed{T = \left[\frac{2\pi^3 m \varepsilon_0 s_0^3}{g^2}\right]^{1/2}}$$

Q4. ENERGY IN EARTH'S B-FIELD

$$E = \frac{1}{2\mu_0}\int B^2\, dV$$

$$B^2 = \left[\frac{\mu_0}{4\pi} \frac{1}{r^3}\right]^2 \left[9(\underset{\sim}{m}\cdot\hat{r})^2 + m^2 - 6(\underset{\sim}{m}\cdot\hat{r})(\underset{\sim}{m}\cdot\hat{r})\right]$$

$$= [\quad]^2 \left[3(\underset{\sim}{m}\cdot\hat{r})^2 + m^2\right]$$

$$= [\quad]^2 \left[3m^2\cos^2\theta + m^2\right]$$

$$dV = r^2 dr\, d\varphi\, d(-\cos\theta)$$

COMBINING

$$E = -\frac{\mu_0 m^2}{2\cdot 16\pi^2}\int_{R_E}^{\infty}\frac{dr}{r^4}\int_0^{2\pi} d\varphi \int_1^{-1}(3\cos^2\theta + 1)\, d(\cos\theta)$$

$$= -\frac{\mu_0 m^2}{2\cdot 16\pi^2}\left[-\frac{1}{3r^3}\right]_{R_E}^{\infty} \cdot 2\pi \left[\cos^3\theta + \cos\theta\right]\Big|_1^{-1}$$

$$= \frac{-\mu_0 m^2}{16\pi}\left(\frac{1}{3R_E^3}\right)\left[\cos^3\theta + \cos\theta\right]\Big|_1^{-1}$$

$$\boxed{E = +\frac{\mu_0 m^2}{12\pi R_E^3}}$$

$\mu_0 = 4\pi\times10^{-7}$ T·m/A

$m \simeq 10^{23}$ A·m^2

$R_E \simeq 6400$ km

$$\boxed{E = 1.27\times10^{18} \text{ Joules}}$$

Q5 MAGNETIC HORN

(a) $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I_{enc}$

$B \cdot 2\pi r = \mu_0 \dfrac{\pi r^2 I}{\pi R^2}$

$$\boxed{\underset{\sim}{B} = \frac{\mu_0 I r}{2\pi R^2} \hat{\phi}}$$

i.e., $\underset{\sim}{B}$ IS AZIMUTHAL

(b) $\underset{\sim}{F} = q \underset{\sim}{v} \times \underset{\sim}{B} = -qvB\hat{r}$ FORCE TOWARD CYLINDER AXIS.

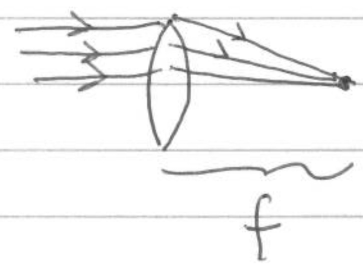
$dP_r/dt = qvB$ NEWTON'S 2^{ND} LAW!

$P_r = \int qvB \, dt$

$= qvB(L/v) = qBL$

$$\boxed{P_r = \frac{q\mu_0 I r}{2\pi R^2} \quad \propto r, \text{ RADIAL MOMENTUM}}$$

(c) THINK OF A LENS FROM PHYS 1304 OR 1106.



BY SIMILAR TRIANGLES

$$r/f = P_r/P \qquad \text{SINCE } P_r \propto r$$

$$f = P \cdot r/P_r = P \cdot r \Big/ \frac{q \mu_0 I L r}{2\pi R^2}$$

$$\boxed{f = \frac{2\pi R^2 P}{q \mu_0 I L}}$$

INDEPEDENT OF r