PHYS 4392
Fall 2014
TE Coan
Due: 2 Dec '14 in lecture

Homework 12

1. Consider the following naive model of an electron: a charged spherical shell with charge e, radius R and spinning with angular velocity ω.

a Find the total energy contained in the electromagnetic fields. Do not be afraid to use results from earlier homework or text example problems. Do identify where your previous results came from.

b Find the total angular momentum contained in the electromagnetic field.

2. Here is a problem that involves superconductivity. A superconductor is a material that has zero electrical resistance. Its conductivity σ is infinite, so $\mathbf{E} = 0$. (Recall $\mathbf{J} = \sigma\mathbf{E}$.) Any net charge resides on its surface. Superconductors also have the fascinating property that $\mathbf{B} = 0$ inside them. Superconductivity is a distinctly quantum mechanical effect. However, Maxwell's equations still allow us to make sensible statements about the $\mathbf{E}$ and $\mathbf{B}$ fields in them.

a. Show that the magnetic field in a superconductor is constant. That is, that $\partial\mathbf{B}/\partial t = 0$.

b. Show that the magnetic flux through a perfectly conducting loop is constant. You can answer this in 1-2 lines. This effect is often called the Meissner effect. Try not to google for the answer.

c. Show that the current in a superconductor is confined to its surface. That is $\mathbf{J} = 0$. Hint: Stare at Maxwell's equations. Actually, you only have to stare at one of them. You can use the ones for vacuum in this case.

d. Superconductors are only superconducting below a certain temperature, called the critical temperature T_c, which varies from material to material. Consider a sphere of radius a initially above its particular T_c. You hold it in a region of constant magnetic field $B_0\hat{\mathbf{z}}$ and cool it below its T_c. Calculate the induced surface current $\mathbf{K}$ as a function of the polar angle θ. You can go ahead and use a result from a text example problem, but be sure to identify which one it is!