Q1 (a)

FIND U_E & U_B

$$U_E = \frac{1}{2}\varepsilon_0 \int E^2 \, dV'$$

$$= \frac{1}{2}\varepsilon_0 \left(\frac{e}{4\pi\varepsilon_0}\right)^2 \int_R^\infty \frac{4\pi \; r^2 dr}{r^4}$$

$$= \frac{1}{2} \cdot \frac{1}{4\pi} \frac{e^2}{\varepsilon_0 R} \left(-1/r\right)\Big|_R^\infty$$

$$\boxed{U_E = \frac{1}{8\pi} \frac{e^2}{\varepsilon_0 R}}$$

$$U_B = \frac{1}{2\mu_0} \int_{\text{ALL SPACE}} B^2 \, dV$$

$$\underset{\sim}{B}(r<R) = \frac{2}{3}\mu_0 \sigma \omega R \hat{z} \qquad \text{EX 5.11}$$

$\sigma = e/4\pi R^2$

$$\underset{\sim}{B}(r<R) = \cancel{\frac{2}{3}\mu_0 \frac{e}{}} \; \frac{\mu_0 e \omega}{6\pi R} \hat{z}$$

$$B^2(r<R) = \frac{\mu_0^2 e^2 \omega^2}{36\pi^2 R^2}$$

$$U_B^{IN} = \frac{1}{2\mu_0} \frac{\mu_0^2 e^2 \omega^2}{36\pi^2 R^2} \; \frac{4}{3}\pi R^3$$

$$\boxed{U_B^{IN} = \frac{1}{54\pi} \mu_0 e^2 \omega^2 R}$$

Q1 (e) FOR $r \geq R$, $\underset{\sim}{B}$ IS LIKE DIPOLE

$$\underset{\sim}{B} = \frac{\mu_0}{4\pi r^3}\left[3(\underset{\sim}{m} \cdot \hat{r})\hat{r} - \underset{\sim}{m}\right]$$

NEED TO FIND MAGNETIC MOMENT $\underset{\sim}{m}$.

From EXAMPLE 6.1

$$\underset{\sim}{m} = \underset{\sim}{M} \cdot \frac{4}{3}\pi R^3$$

$$\underset{\sim}{K} = \sigma \omega R \sin\theta \ \hat{\varphi}$$

$$\underset{\sim}{K} = \underset{\sim}{M} \times \hat{n} = M \sin\theta \ \hat{\varphi}$$

$$\Rightarrow M = \sigma \omega R$$

$$= \frac{e}{4\pi R^2} \omega R$$

$$\boxed{\underset{\sim}{M} = \frac{e\omega}{4\pi R} \hat{z}}$$

$$\underset{\sim}{m} = \frac{e\omega R^2}{3} \hat{z}$$

Q1(b) CONT.

$$B^2 = \left(\frac{\mu_0}{4\pi r^3}\right)^2 \left[9(\underline{m}\cdot\hat{r})^2 + m^2 - 2\cdot 3(\underline{m}\cdot\hat{r})\hat{r}\cdot\underline{m}\right]$$

$$= (\quad)^2 \left[9m^2\cos^2\theta + m^2 - 6m^2\cos^2\theta\right]$$

$$= (\quad)^2 \left[3m^2\cos^2\theta + m^2\right]$$

$$= (\quad)^2 m^2 \left[3\cos^2\theta + 1\right]$$

$$B^2 = \left(\frac{\mu_0}{4\pi r^3}\right)^2 \cdot \frac{e^2\omega^2R^4}{9}\left[3\cos^2\theta + 1\right]$$

$$= C_0\left[3\cos^2\theta + 1\right]/r^6 \qquad w/\ C_0 = \left(\frac{\mu_0}{4\pi}\right)^2 \frac{e^2\omega^2R^4}{9}$$

$$U_B^{OUT} = \frac{1}{2\mu_0} C_0 \int_R^\infty \frac{1}{r^6}\left[3\cos^2\theta + 1\right] r^2 dr \sin\theta d\theta d\phi$$

$$= \frac{2\pi}{2\mu_0} C_0 \int_{\theta=0}^{\pi}\int_{r=R}^{\infty} \frac{3\cos^2\theta\sin\theta\, d\theta\ r^2 dr}{r^6}$$

$$+ \frac{4\pi}{2\mu_0} C_0 \int_R^\infty \frac{1}{r^4} dr$$

$$= \frac{2\pi}{2\mu_0} C_0 \left(-\frac{r^{-3}}{3}\right)\Bigg|_R^\infty \int_{\theta=0}^{\pi} 3\cos^2\theta\sin\theta\, d\theta$$

$$+ \frac{4\pi}{2\mu_0} C_0 \left(\frac{-r^{-3}}{3} \right) \Big|_R^{\infty}$$

$$U_B^{OUT} = \frac{\pi}{\mu_0} C_0 \left(\frac{1}{3R^3} \right) \left(-\cos^3\theta \Big|_0^{\pi} \right)$$

$$+ \frac{2\pi}{\mu_0} C_0 \left(\frac{1}{3R^3} \right)$$

$$U_B^{OUT} = \frac{4\pi}{3\mu_0} \frac{C_0}{R^3} = \frac{4\pi}{3\mu_0 R^3} \left(\frac{\mu_0}{4\pi} \right)^2 \frac{e^2 \omega^2 R^4}{9}$$

$$\boxed{U_B^{OUT} = \frac{\mu_0}{27 \cdot 4\pi} e^2 \omega^2 R}$$

$$U_B^{TTL} = U_B^{IN} + U_B^{OUT}$$

$$\boxed{U_B^{TTL} = \frac{1}{36\pi} \mu_0 e^2 \omega^2 R}$$

$$U_{TTL} = U_E + U_B$$

$$\boxed{U_{TTL} = \frac{1}{8\pi} \frac{e^2}{\varepsilon_0 R} + \frac{1}{36\pi} \mu_0 e^2 \omega^2 R}$$

Q1 (b) $\underset{\sim}{\ell} = \varepsilon_0 [\underset{\sim}{r} \times (\underset{\sim}{E} \times \underset{\sim}{B})]$

$$\underset{\sim}{L} = \int_{\text{ALL SPACE}} \underset{\sim}{\ell} \, dV$$

$\underset{\sim}{E} (r < R) = 0$

$$\underset{\sim}{E} (r \geq R) = \frac{e \, \hat{r}}{4\pi \varepsilon_0 r^2}$$

$$\underset{\sim}{B} (r \geq R) = \frac{\mu_0 m}{4\pi r^3} \left[2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta} \right]$$

$\underset{\sim}{B}(r<R)$ IRRELEVANT SINCE $\underset{\sim}{E}=0$ THERE

ALSO, $m = eωR^2/3$ FROM COMPARING EXAMPLE 5.11 EXPRESSION FOR $\underset{\sim}{B}$ AND SECTION 5.4.3 FOR $\underset{\sim}{B}_{\text{DIPOLE}}$.

NOW,

$$\underset{\sim}{E} \times \underset{\sim}{B} = \frac{e}{4\pi \varepsilon_0 r^2} \frac{\mu_0 m}{4\pi r^3} \hat{r} \times \left[2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta} \right]$$

w/ $m = e\omega R^2/3$

$$= \frac{\mu_0 e^2 \omega R^2 \sin\theta}{3 \varepsilon_0 (4\pi)^2 r^5} \, \hat{r} \times \hat{\theta}$$

NOW,

$$\hat{r} \times \hat{\theta} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \end{vmatrix}$$

$$= \hat{x}\left[-\sin^2\theta \sin\varphi - \cos^2\theta \sin\varphi\right]$$
$$-\hat{y}\left[-\sin^2\theta\cos\varphi - \cos^2\theta\cos\varphi\right]$$
$$+\hat{z}\left[\sin\theta\cos\theta\sin\varphi\cos\varphi - \sin\theta\cos\theta\sin\varphi\cos\varphi\right]$$
$$= -\hat{x}\sin\varphi + \hat{y}\cos\varphi$$

$$\boxed{\hat{r} \times \hat{\theta} = \hat{\varphi}}$$

SO, $\underset{\sim}{l} = \varepsilon_0 \underset{\sim}{r} \times (\underset{\sim}{E} \times \underset{\sim}{B}) = \dfrac{\mu_0 e^2 \omega R^2 \sin\theta}{3(4\pi)^2 r^4}\, \hat{r} \times \hat{\varphi}$

Since WE NEED TO INTEGRATE ~~THIS~~ $\underset{\sim}{l}$, WE NEED TO GET ITS DIRECTION.

WE NEED $\hat{r} \times \hat{\varphi}$

②

AWAY WE GO:

$$\hat{r} \times \hat{\varphi} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ -\sin\varphi & \cos\varphi & 0 \end{vmatrix}$$

$$= -\hat{x}\cos\theta\cos\varphi \quad - \hat{y}\sin\varphi\cos\theta$$

$$+ \hat{z}\sin\theta$$

$$\hat{r} \times \hat{\varphi} = -\hat{\theta}$$

BECAUSE OF $\sin\theta$ TERM IN MAGNITUDE OF $\frac{l}{2}$, INTEGRATING OVER ~~θ~~ THETA YIELDS $\int_{\theta=0}^{\pi} \cos\theta \sin^2\theta \, d\theta = 0$ FOR $\hat{x}$ INTEGRATION

LIKEWISE, FOR $\hat{y}$ INTEGRATION OVER θ

$$\int \sin^2\theta \cos\theta \, d\theta = 0$$

ONLY $\hat{z}$ TERM WILL SURVIVE.

FOR $\hat{z}$ INTEGRATION

$$\underset{\sim}{L} = l\hat{z}$$

$$= -\frac{\mu_0 e^2 \omega R^2}{3(4\pi)^2} \int_R^{\infty} \frac{r^2 dr}{r^4} \int_0^{2\pi} d\varphi \int_0^{\pi} \sin^3\theta d\theta \, \hat{z}$$

$$= \frac{-\mu_0 e^2 \omega R^2}{3(4\pi)^2} \left(-\frac{1}{R} \Big|_R^{\infty} \right) \cdot 2\pi \cdot \frac{4}{3} \hat{z}$$

$$= -\frac{\mu_0 e^2 \omega R^2}{3(4\pi)^2} \frac{2\pi}{R} \cdot 4/3$$

$$\boxed{\underset{\sim}{L} = -\frac{\mu_0 e^2 \omega R}{18\pi} \hat{z}}$$

Q2 (a) $\nabla \times \underset{\sim}{E} = -\partial \underset{\sim}{B}/\partial t$

$\underset{\sim}{E} = 0 \Rightarrow \partial \underset{\sim}{B}/\partial t = 0$

(b) $\mathcal{E} = -d\Phi/dt$

$\oint \underset{\sim}{E} \cdot d\underset{\sim}{l} = -d\Phi/dt$

SINCE $\underset{\sim}{E} = 0$, $d\Phi/dt = 0 \Rightarrow \Phi = \text{const.}$

(C) $\nabla \times \underset{\sim}{B} = \mu_0 \underset{\sim}{J} + \varepsilon_0 \mu_0 \partial \underset{\sim}{E}/\partial t$

$\underset{\sim}{B} = 0 \Rightarrow \nabla \times \underset{\sim}{B} = 0$

$\underset{\sim}{E} = 0 \Rightarrow \partial \underset{\sim}{E}/\partial t = 0$

SUBSTITUTING INTO EQN $\Rightarrow \underset{\sim}{J} = 0$

(d) WE KNOW $\underset{\sim}{B} = 0$ & $\underset{\sim}{J} = 0$, CURRENT FLOWS ONLY AT SURFACE, SURFACE CURRENT MUST FLOW IN SUCH A WAY THAT THE B-FIELD $\underset{\sim}{B}_K$ IT PRODUCES EXACTLY CANCELS THE STATIC FIELD $\underset{\sim}{B_0}$

$\underset{\sim}{B}_K + \underset{\sim}{B}_0 = 0$

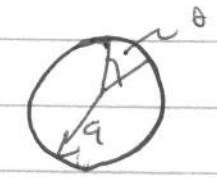
Q2

FROM EXAMPLE 5.11 WE KNOW THAT A SPHERICAL SHELL WITH CONST CHARGE DENSITY σ SPINNING w/ ANGULAR SPEED ω PRODUCES A STEADY B-FIELD INSIDE SHELL.

FROM EX 5.11 $\quad \mathbf{B} = \frac{2}{3}\mu_0 \sigma \omega a \hat{z}$

w/ a = SHELL RADIUS

RECAST IN TERMS OF $\mathbf{K}$



FIRST NOTE THAT WE NEED

$$\sigma \omega a = -\frac{3}{2} B_0 / \mu_0$$

NOW, $\mathbf{K} = \sigma \mathbf{v} = \sigma \omega a \sin\theta \, \hat{\varphi}$

$$\Rightarrow \boxed{\mathbf{K} = -\frac{3}{2}\frac{B_0}{\mu_0}\sin\theta \, \hat{\varphi}}$$