PHYS 4392
Fall 2014
TE Coan
Due: 5 Sep '14 6:00 pm

Homework 1

1. Prove the vector identity

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$$

2. Now try to prove this monster, also found on the inside of Mr. Griffiths' text

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$$

3. This problem hopefully reinforces your knowledge of Stokes' theorem

$$\int_S (\nabla \times \mathbf{G}) \cdot d\mathbf{S} = \oint \mathbf{G} \cdot d\mathbf{l},$$

and Gauss' theorem (sometimes called the divergence theorem)

$$\int_V \nabla \cdot \mathbf{G} \, dV = \oint \mathbf{G} \cdot d\mathbf{S}.$$

a. Show that $\int_V (\nabla T) \, dV = \oint T \, d\mathbf{S}$. Hint: Let $\mathbf{G} = \mathbf{c}T$, where $\mathbf{c}$ is a constant vector, in the divergence theorem and use the product rules found on the inside cover of Griffiths. T is a scalar function.

b. Show that $\int_S \nabla T \times d\mathbf{S} = -\oint T \, d\mathbf{l}$. Hint: Let $\mathbf{G} = \mathbf{c}T$ and use Stokes' theorem.

4. This problem has nothing to do with vector calculus but will be useful later. It even requires no knowledge of PHYS 4392. Compute the number N of atoms in a cubic micron of solid aluminum. (One or two significant figures is fine.) This number is important because we will need to think carefully about exactly what electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields we are talking about when we discuss these fields inside matter.

5. We say a vector field $\mathbf{V}$ is *conservative* if it can be written as the gradient of some scalar function f. That is, if $\mathbf{V} = \nabla f$, $\mathbf{V}$ is conservative. Recalling two important theorems I mentioned in class and which those present claimed they knew, determine if the following vector fields are conservative.

a. $\mathbf{F_1} = -2y\mathbf{i} + (z - 2x)\mathbf{j} + (y + z)\mathbf{k}$.

b. $\mathbf{F_2} = y\mathbf{i} + 2x\mathbf{j}$.