#1

$$(\underset{\sim}{A} \times \underset{\sim}{B}) \cdot (\underset{\sim}{C} \times \underset{\sim}{D}) \overset{?}{=} (\underset{\sim}{A} \cdot \underset{\sim}{C})(\underset{\sim}{B} \cdot \underset{\sim}{D}) - (\underset{\sim}{A} \cdot \underset{\sim}{D})(\underset{\sim}{B} \cdot \underset{\sim}{C})$$

EXPAND LHS OF ABOVE EQUATION

$$LHS = \varepsilon_{ijk} A_j B_k \cdot \varepsilon_{irs} C_r D_s$$

$$= \varepsilon_{ijk} \varepsilon_{irs} A_j B_k C_r D_s$$

$$= \varepsilon_{jki} \varepsilon_{rsi} A_j B_k C_r D_s = (\delta_{jr}\delta_{ks} - \delta_{js}\delta_{kr}) A_j B_k C_r D_s$$

$$= \delta_{jr} A_j C_r \delta_{ks} B_k D_s - \delta_{js} A_j D_s \delta_{kr} B_k C_r$$

$$= A_r C_r B_s D_s - A_s D_s B_r C_r$$

$$= (\underset{\sim}{A} \cdot \underset{\sim}{C})(\underset{\sim}{B} \cdot \underset{\sim}{D}) - (\underset{\sim}{A} \cdot \underset{\sim}{D})(\underset{\sim}{B} \cdot \underset{\sim}{C})$$

$$\boxed{\therefore \; (\underset{\sim}{A} \times \underset{\sim}{B}) \cdot \underset{\sim}{C} \times \underset{\sim}{D} = (\underset{\sim}{A} \cdot \underset{\sim}{C})(\underset{\sim}{B} \cdot \underset{\sim}{D}) - (\underset{\sim}{A} \cdot \underset{\sim}{D})(\underset{\sim}{B} \cdot \underset{\sim}{C})}$$

#2

$$\underset{\sim}{\nabla} \times (\underset{\sim}{A} \times \underset{\sim}{B}) \overset{?}{=} \underset{\sim}{A}(\underset{\sim}{\nabla} \cdot \underset{\sim}{B}) - \underset{\sim}{B}(\underset{\sim}{\nabla} \cdot \underset{\sim}{A}) + (\underset{\sim}{B} \cdot \underset{\sim}{\nabla})\underset{\sim}{A} - (\underset{\sim}{A} \cdot \underset{\sim}{\nabla}) \cdot \underset{\sim}{B}$$

$$= \varepsilon_{ijk} \frac{\partial}{\partial x_j} \varepsilon_{krs} A_r B_s$$

$$= \varepsilon_{ijk} \varepsilon_{krs} \frac{\partial}{\partial x_j} A_r B_s$$

$$= (\delta_{ir}\delta_{js} - \delta_{is}\delta_{jr}) \left[B_s \frac{\partial}{\partial x_j} A_r + A_r \frac{\partial}{\partial x_j} B_s \right]$$

$$= \delta_{ir} \left(\frac{\partial}{\partial x_j} A_r \right) \delta_{js} B_s - \delta_{is} B_s \delta_{jr} A_r + \delta_{ir} A_r \delta_{js} \frac{\partial}{\partial x_j} B_s$$

$$- \delta_{is} \left(\frac{\partial}{\partial x_j} B_s \right) \delta_{jr} A_r$$

$$= B_s \frac{\partial}{\partial x_s} A_i - B_i \frac{\partial}{\partial x_s} A_s + A_i \frac{\partial}{\partial x_i} B_s - \left(\frac{\partial}{\partial x_j} B_i \right) A_j$$

$$= (\underset{\sim}{B} \cdot \underset{\sim}{\nabla})\underset{\sim}{A} - \underset{\sim}{B} \cdot (\underset{\sim}{\nabla} \cdot \underset{\sim}{A}) + \underset{\sim}{A}(\underset{\sim}{\nabla} \cdot \underset{\sim}{B}) - (\underset{\sim}{A} \cdot \underset{\sim}{\nabla})\underset{\sim}{B}$$

REARRANGING

$$\boxed{\underset{\sim}{\nabla} \times (\underset{\sim}{A} \times \underset{\sim}{B}) = \underset{\sim}{A}(\underset{\sim}{\nabla} \cdot \underset{\sim}{B}) - \underset{\sim}{B}(\underset{\sim}{\nabla} \cdot \underset{\sim}{A}) + (\underset{\sim}{B} \cdot \underset{\sim}{\nabla})\underset{\sim}{A} - (\underset{\sim}{A} \cdot \underset{\sim}{\nabla})\underset{\sim}{B}}$$

3(c) $\int \nabla T \, dV \stackrel{?}{=} \int T \, dS$

START w/ THE DIVERGENCE THEOREM

$$\int \nabla \cdot G \, dV = \oint G \cdot dS$$

AND USE THE HINT $G = c\,T$ w/ c = CONST VECTOR AND T IS A SCALAR FUNCTION.

$$\int \nabla \cdot (c\,T) \, dV = \oint c\,T \cdot dS$$

$$c \cdot \int \nabla T \, dV = c \cdot \oint T \, dS$$

UNDOING THE DOT PRODUCT w/ c

$$\boxed{\int \nabla T \, dV = \oint T \, dS}$$

NOTE THAT $\nabla \cdot c = 0$.

3(b) SHOW $\int \nabla \underset{\sim}{T} \times d\underset{\sim}{S} = -\oint T d\underset{\sim}{\ell}$

FROM STOKES THEOREM

$$\int (\underset{\sim}{\nabla} \times \underset{\sim}{G}) \cdot d\underset{\sim}{S} = \oint \underset{\sim}{G} \cdot d\underset{\sim}{\ell} \qquad (1)$$

$$\int (\nabla \times \underset{\sim}{c} T) \cdot d\underset{\sim}{S} = \int (T \underset{\sim}{\nabla} \times \underset{\sim}{c} - \underset{\sim}{c} \times \nabla \underset{\sim}{T}) \cdot d\underset{\sim}{S}$$

$$= -\int (\underset{\sim}{c} \times \underset{\sim}{\nabla} T) \cdot d\underset{\sim}{S} \qquad (2)$$

THE RHS OF (1) $= \oint \underset{\sim}{G} \cdot d\underset{\sim}{\ell} = \oint \underset{\sim}{c} T \cdot d\underset{\sim}{\ell}$

$$= \underset{\sim}{c} \cdot \oint T d\underset{\sim}{\ell} \qquad (3)$$

USING A VECTOR ID FROM GRIFFITHS, (2) CAN BE REWRITTEN

$$-\int (\underset{\sim}{c} \times \underset{\sim}{\nabla} T) \cdot d\underset{\sim}{S} = -\int \underset{\sim}{c} \cdot (\underset{\sim}{\nabla} T \times d\underset{\sim}{S})$$

$$= -\underset{\sim}{c} \cdot \int \underset{\sim}{\nabla} T \times d\underset{\sim}{S} \qquad (4)$$

USING (3) & (4)

$$-\underset{\sim}{c} \cdot \int \nabla \underset{\sim}{T} \times d\underset{\sim}{S} = \underset{\sim}{c} \cdot \oint T d\underset{\sim}{\ell}$$

$$\boxed{\int \nabla \underset{\sim}{T} \times d\underset{\sim}{S} = -\oint T d\underset{\sim}{\ell}}$$

4)

$$N = \frac{\text{MASS IN CUBE}}{\text{MOLAR MASS}} \cdot N_0 \ ; \ N_0 = \text{AVOGADRO'S NUMBER.}$$

EASIEST PERHAPS IF WE USE grams AND CENTIMETERS.

$$N \cong \frac{2.7 \ g/cm^3}{27 \ g} \times (10^{-4} \ cm)^3 \times 6.0 \times 10^{23}$$

$$N \cong \frac{1}{10} \times 10^{-12} \times 6 \times 10^{23}$$

$$\boxed{N \simeq 6 \times 10^{10}}$$

5) $\nabla \times \nabla f = 0 \quad \forall f$ w/ f = SCALAR FUNCTION

BUT IF $\underset{\sim}{G} = \nabla \underset{\sim}{f}$ FOR SOME SCALAR FUNCTION, WE SAY $\underset{\sim}{G}$ IS "CONSERVATIVE". SO,

IF $\nabla \times \underset{\sim}{G} = 0$, $\underset{\sim}{G}$ IS CONSERVATIVE.

EXAMINE IF $\underset{\sim}{\nabla} \times \underset{\sim}{F_1} = 0$:

$$\underset{\sim}{\nabla} \times \underset{\sim}{F_1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -2y & z-2x & y+z \end{vmatrix}$$

$$= \hat{i}(1-1) - \hat{j}(0) + \hat{k}(-2 - -2)$$

$\underset{\sim}{\nabla} \times \underset{\sim}{F_1} = 0 \Rightarrow$ $\boxed{\underset{\sim}{F_1} \text{ IS CONSERVATIVE}}$

SIMPLE CALCULATION YIELDS $\underset{\sim}{\nabla} \times \underset{\sim}{F_2} = \hat{k} \neq 0$

$\boxed{\underset{\sim}{F_2} \text{ IS } \underline{\text{NOT}} \text{ CONSERVATIVE}}$