#1

STRATEGY: INTEGRATE USING DUMMY function $f(x)$

EXAMINE BEHAVIOR OF $\frac{d}{dx}(\text{SGN}(x))$.

$$\int_{-b}^{b} f(x) \frac{d}{dx} \text{SGN}(x)\, dx = \text{SGN}(x) f(x)\Big|_{-b}^{b} - \int_{-b}^{b} f'(x)\, \text{SGN}(x)\, dx$$

$$= f(b) - - f(-b) - \int_{-b}^{0} f'(x)\, \text{SGN}(x)\, dx - \int_{0}^{b} f'(x)\, \text{SGN}(x)\, dx$$

$\text{SGN}(-b) = -1$

$$= f(b) + f(-b) + \int_{-b}^{0} f'(x)\, dx - \int_{0}^{b} f'(x)\, dx$$

$$= f(b) + f(-b) + f(x)\Big|_{-b}^{0} - f(x)\Big|_{0}^{b}$$

$$= 2 f(0). \qquad (1)$$

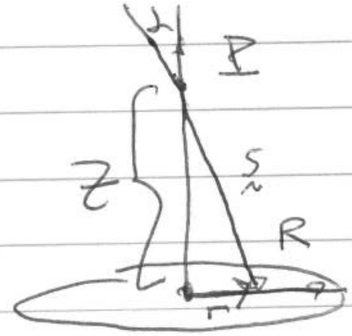
NOTE THAT THIS RESULT IS EQUIVALENT TO

$$\int_{-b}^{b} f(x) \cdot 2\,\delta(x)\, dx = 2 f(0). \qquad (2)$$

EQ (1) = EQ. (2)

$\therefore$ RESULT VERIFIED.

#2.



$$\underset{\sim}{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq\,\hat{s}}{s^2}$$

STRATEGY: ADD UP CONTRIBUTIONS FROM RINGS OF CHARGE ON DISK. USE SYMMETRY TO DEAL w/ DIRECTION

BY SYMMETRY, ONLY DIRECTION ALONG $\hat{z}$ MATTERS. ALSO, INCLUDE PROJECTION FACTOR OF $\cos\alpha$ TO ACCOUNT FOR PARTIAL CANCELLATION OF $\underset{\sim}{E}$ FROM SYMMETRY.

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\,dS}{s^2}\cos\alpha$$

w/ dS = CIRCULAR STRIP

$$= \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, r\,dr\,d\theta}{r^2+z^2}\cos\alpha$$

w/ r = RADIUS OF CIRCULAR STRIP

$$= \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, r\,dr\,d\theta\ \ z}{(r^2+z^2)^{3/2}}$$

w/ $\cos\alpha = z/s$

#2.

$$E = \frac{\sigma z}{4\pi\varepsilon_0}\int \frac{r\,dr\,d\theta}{(r^2+z^2)^{3/2}}$$

$$= \frac{2\pi\sigma z}{4\pi\varepsilon_0}\int_0^R \frac{r\,dr}{(r^2+z^2)^{3/2}}$$

$$= \frac{\sigma z}{2\varepsilon_0}\left[-(r^2+z^2)^{-1/2}\right]_0^R$$

$$= \frac{\sigma z}{2\varepsilon_0}\left[(r^2+z^2)^{-1/2}\right]\Big|_R^0$$

$$\underset{\sim}{E} = \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right]\hat{z}$$

$$\boxed{\underset{\sim}{E} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]\hat{z}}$$

(b) $$\lim_{R\to\infty} \underset{\sim}{E} = \frac{\sigma}{2\varepsilon_0}\left[1-0\right]\hat{z}$$

$$\boxed{\lim_{R\to\infty} \underset{\sim}{E} = \frac{\sigma}{2\varepsilon_0}\hat{z}}$$

I.e., SAME $\underset{\sim}{E}$ AS FOR INFINITE PLANE

2(C). FOR $z \gg R$, SAME AS $R/z \ll 1$.

NOTE: THIS IS NOT THE SAME AS $z \to \infty$!

$$\underset{\sim}{E} = \frac{\sigma}{2\varepsilon_0}\left[1 - z \cdot \frac{1}{z}\left(1 + R^2/z^2\right)^{-1/2}\right]\hat{z}$$

$$\simeq \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1 - \frac{1}{2}R^2/z^2\right)\right]\hat{z}$$

$$\simeq \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{2}R^2/z^2\right)\hat{z}$$

$$\simeq \frac{\sigma\pi R^2}{4\pi\varepsilon_0 z^2}\hat{z}$$

$$\boxed{\underset{\sim}{E} \simeq \frac{Q}{4\pi\varepsilon_0 z^2}\hat{z}}$$

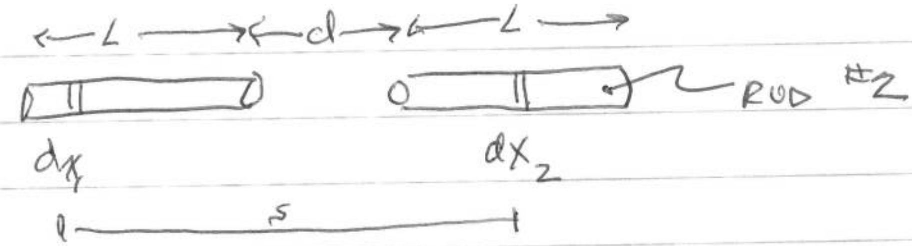
DISK LOOKS LIKE A PT. CHARGE Q A DISTANCE z AWAY.

#3. BY SIMPLE MULTIPLICATION.

$$\boxed{T \simeq \pi \times 10^{7} \text{ sec}}$$

#4. $\boxed{e = 1.60 \times 10^{-19} \text{ C}}$

#5



STRATEGY: DIVIDE EACH ROD INTO THIN SLICES. CALCULATE THE FORCE ON ONE SLICE IN ONE ROD FROM ALL THE OTHER SLICES IN THE OTHER ROD. THEN, SUM OVER ALL SLICES IN THE ORIGINAL ROD

$$d^2F_1 = \frac{dq_1\,dq_2}{4\pi\varepsilon_0 s^2} = \frac{\lambda\,dx_1\,\lambda\,dx_2}{4\pi\varepsilon_0\left((L-x_1)+d+x_2\right)^2}$$

$$= \frac{\lambda^2 dx_1 dx_2}{4\pi\varepsilon_0 (z_0+x_2)^2} \quad w/\ z_0 = L - x_1 + d$$

$$dF_1 = \frac{\lambda^2}{4\pi\varepsilon_0}\int_{x_2=0}^{L}\frac{dx_2}{(z_0+x_2)^2} = \frac{\lambda^2 dx_1}{4\pi\varepsilon_0}\int_{z_0}^{z_0+L}\frac{dt}{t^2}$$

$$= \frac{\lambda^2 dx_1}{4\pi\varepsilon_0}\left(-1/t\right)\Big|_{z_0}^{z_0+L}$$

$$= \frac{\lambda^2 dx_1}{4\pi\varepsilon_0}\left[\frac{1}{z_0} - \frac{1}{z_0+L}\right]$$

$$= \frac{\lambda^2 dx_1}{4\pi\varepsilon_0}\left[\frac{L}{z_0(z_0+L)}\right]$$

$$dF_1 = \frac{\lambda^2 L}{4\pi\varepsilon_0} \frac{dx_1}{(L-x_1+d)(2L-x_1+d)}$$

$$= \frac{\lambda^2 L}{4\pi\varepsilon_0} \frac{-d\eta}{\eta(L+\eta)} \quad w/\ \eta \equiv L-x_1+d$$

$$F_1 = \frac{-\lambda^2 L}{4\pi\varepsilon_0} \int_{L+d}^{d} \frac{d\eta}{\eta(L+\eta)}$$

$$= +\frac{\lambda^2 L}{4\pi\varepsilon_0} \left. \frac{\ln\left(\frac{L+\eta}{\eta}\right)}{L} \right|_{L+d}^{d}$$

$$= \frac{\lambda^2}{4\pi\varepsilon_0}\left[\ln\left(\frac{L+d}{d}\right) - \ln\left(\frac{2L+d}{L+d}\right)\right]$$

$$\boxed{F_1 = \frac{\lambda^2}{4\pi\varepsilon_0} \ln\left[\frac{(L+d)^2}{d(2L+d)}\right]}$$

F_1 = TTL FORCE ON ROD 1 BY #2.

REPULSIVE ALONG THEIR AXIS

$F_1 = F_2$ BY NEWTON'S 3RD LAW

#6. STRATEGY: USE GAUSS' LAW & SUPERPOSITION. FOR A POINT P IN THE OVERLAP REGION, SUM $\tilde{E}_+$ (FIELD FROM $+\rho$ SPHERE) AND $\tilde{E}_-$ (FIELD FROM $-\rho$ SPHERE).

FOR $+\rho$ SPHERE, GAUSS' LAW GIVES

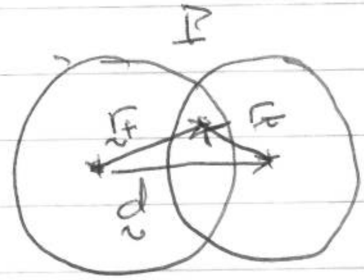
$$E_+ \cdot 4\pi r_+^2 = Q_{enc}/\varepsilon_0$$
$$= \tfrac{4}{3}\pi r_+^3 \rho/\varepsilon_0$$

$$E_+ = \frac{\rho r_+}{3\varepsilon_0}$$; r_+ = RADIUS VECTOR MAGNITUDE FROM + SPHERE CENTER.

$$\tilde{E}_+ = \frac{\rho \tilde{r}_+}{3\varepsilon_0}$$

FOR, $-$SPHERE,

$$\tilde{E}_- = \frac{-\rho \tilde{r}_-}{3\varepsilon_0}$$ w/ $\tilde{r}_-$ = RADIUS VECTOR TO P FROM $-$ SPHERE CENTER



$$\tilde{r}_+ - \tilde{r}_- = \tilde{d}$$

$$\tilde{E} = \tilde{E}_+ + \tilde{E}_- = \frac{\rho}{3\varepsilon_0}(\tilde{r}_+ - \tilde{r}_-)$$

$$\boxed{\tilde{E} = \frac{\rho \tilde{d}}{3\varepsilon_0}}$$

#7 SOLVE BY SUPERPOSITION !

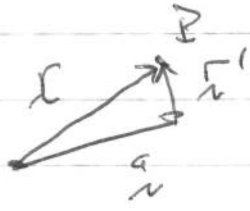
A HOLLOW CAVITY IS EQUIVALENT TO 2 IDENTICAL SPHERES OF OPPOSITE CHARGE DENSITY SUPERIMPOSED ON EACH OTHER.

LET $\underset{\sim}{r}$ = RADIUS VECTOR FROM CENTER OF BIG SPHERE TO $\underline{P}$

$\underset{\sim}{r}'$ = RADIUS FROM CENTER OF HOLLOW SPHERICAL CAVITY TO $\underline{P}$.

$\underset{\sim}{a}$ = RADIUS VECTOR FROM BIG SPHERE CENTER TO HOLLOW CAVITY.

$$\underset{\sim}{r} = \underset{\sim}{r}' + \underset{\sim}{a}$$



NOW, FIRST FILL HOLLOW CAVITY w/ CHARGE DENSITY ρ.

USE GAUSS' LAW TO FIND $\underset{\sim}{E}$ IN THIS CIRCUMSTANCE.

$$E_\rho \cdot 4\pi r^2 = Q_{enc}/\varepsilon_0$$

$$= \frac{4}{3}\pi r^3 \rho/\varepsilon_0$$

$$\underset{\sim}{E}_\rho = \frac{\rho r}{3\varepsilon_0}\hat{r} \qquad (1)$$

FOR CAVITY FILLED w/ $-\rho$, AGAIN USE GAUSS' LAW, AND WE GET

$$\underset{\sim}{E}_{-\rho} = \frac{-\rho r' \hat{r}'}{3\varepsilon_0}$$

$$\underset{\sim}{E} = \underset{\sim}{E}_\rho + \underset{\sim}{E}_{-\rho} = \frac{\rho \underset{\sim}{r}}{3\varepsilon_0} - \frac{\rho \underset{\sim}{r}'}{3\varepsilon_0}$$

$$= \frac{\rho}{3\varepsilon_0}(\underset{\sim}{r} - \underset{\sim}{r}')$$

$$\boxed{\underset{\sim}{E} = \frac{\rho}{3\varepsilon_0}\underset{\sim}{a} \quad \text{(FROM ABOVE)} \qquad |\underset{\sim}{E}| = \frac{\rho a}{3\varepsilon_0}}$$