#1

Q IS CONSTANT ON PLATES. MOVE JUST 1 PLATE AND ASK HOW MUCH WORK W REQ'D. COMPUTE FORCE ON ONE PLATE DUE TO THE OTHER

$$\sigma = Q/A = C_1 V_0/A = \varepsilon_0 V_0/d$$

$$W = \sigma A E \cdot (d'-d) = \frac{\sigma^2 A}{2\varepsilon_0}(d'-d)$$

$$= \frac{\varepsilon_0^2 V_0^2}{d^2} \cdot \frac{A}{2\varepsilon_0}(d'-d)$$

$$\boxed{W = \frac{1}{2} A \varepsilon_0 V_0^2 \left(\frac{d'-d}{d^2}\right)}$$

OTHER WAYS POSSIBLE ∵

$$\boxed{W = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A}(d'-d)}$$

#2.

USING HINT: $dw = (dq)V$

w/ dq = CHARGE IN SHELL OF THICKNESS dr

$$dw = \rho \cdot 4\pi r^2 dr \cdot \frac{q}{4\pi\varepsilon_0 r}$$ w/ q = CHARGE ALREADY THERE.

$$dW = \rho \cdot 4\pi r^2 dr \cdot \frac{\left(Q r^2/R^3\right)}{4\pi\varepsilon_0}$$ & $\rho = Q/\frac{4\pi}{3}R^3$

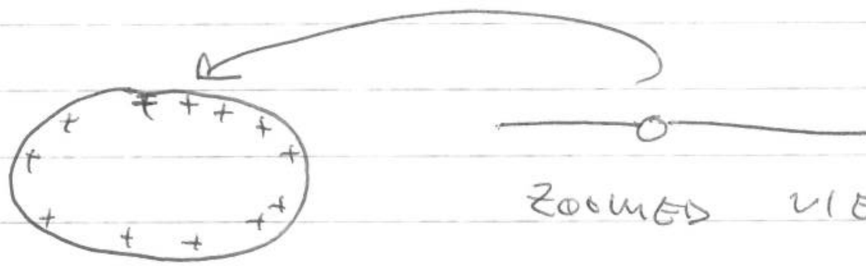
$$dW = \frac{\rho \cdot 4\pi}{4\pi\varepsilon_0} \cdot \frac{Q}{R^3} r^4 dr$$

$$W = \frac{\rho Q}{\varepsilon_0 R^3} \cdot \int_0^R r^4 dr$$

$$= \frac{\rho Q}{\varepsilon_0} R^2/5 = \frac{3Q^2}{4\pi\varepsilon_0} \cdot \frac{1}{5R}$$

$$\boxed{W = \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 R}}$$

#3.



E @ SURFACE DUE TO CHARGE IN A CIRCULAR PATCH **PLUS** E FROM OTHER CHARGES.

AT SURFACE $E_{PATCH} = \frac{\sigma}{2\varepsilon_0}$

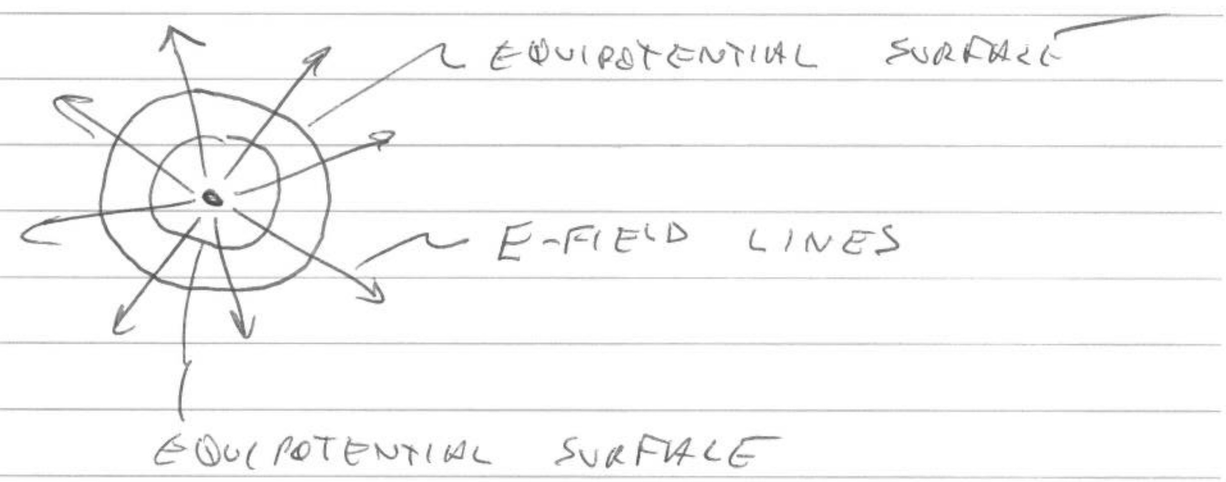
JUST THINK OF E FROM A PLANE.

BUT E_{TTL} OUTSIDE CONDUCTOR IS $E_{TTL} = \frac{\sigma}{\varepsilon_0}$. IF YOU REMOVE LOCAL PATCH BY DRILLING A HOLE, THEN E IS DUE TO OTHER CHARGES

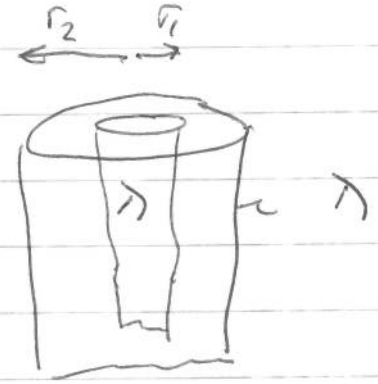
SO, $E_{HOLE} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = \sigma/2\varepsilon_0$

$$\boxed{E_{HOLE} = \frac{\sigma}{2\varepsilon_0}}$$

#4.

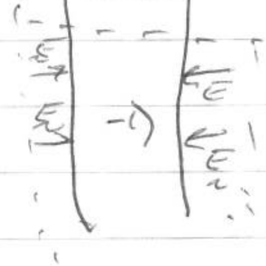

EQUIPOTENTIAL SURFACE
E-FIELD LINES
EQUIPOTENTIAL SURFACE

A5



a) EXPRESS V AS FUNCTION OF E_b, r_1 & r_2.
FROM SYMMETRY, FIND E & THEN V.

GAUSS' LAW AROUND INNER CYLINDER



$$\int \underset{\sim}{E}\cdot d\underset{\sim}{S} = Q_{enc}/\varepsilon_0$$

$$-E\, 2\pi r l = -\lambda l/\varepsilon_0$$

$\underset{\sim}{E}$ POINTS INWARD

$$\Rightarrow E = \lambda/2\pi\varepsilon_0 r$$

$$V_1 - V_2 = -\int_{r_2}^{r_1} \underset{\sim}{E}\cdot d\underset{\sim}{l} \qquad \text{w/ } V(r=r_1) = V_1 = 0$$

$$-V_2 = \frac{-\lambda}{2\pi\varepsilon_0}\int_{r_2}^{r_1} dr/r = \frac{+\lambda}{2\pi\varepsilon_0}\ln r_1/r_2$$

$$V_2 = E_b \frac{2\pi\varepsilon_0 r_1}{2\pi\varepsilon_0}\ln r_2/r_1$$

$V_2 = E_b r_1 \ln r_2/r_1$ (POTENTIAL DIFFERENCE BTWN CYLINDERS.)

NOTE THAT E IS MAX @ r_1 SINCE CHARGE PER UNIT AREA IS LARGEST THERE & $E \sim \sigma$ AT SURFACE OF CONDUCTOR.

FIND r_1 NOW: THAT MAXIMIZES V_2

$$\partial V/\partial r_1 = 0 = E_b \left[r_1 \left(-r_2/r_1^2\right) / \left(r_2/r_1\right) + \ln r_2/r_1 \right]$$

$$0 = -1 + \ln r_2/r_1$$

$$1 = \ln r_2/r_1$$

$$r_2/r_1 = e$$

(a) $\boxed{r_1 = r_2/e}$ YES, $r_1 < r_2$.

$$\Rightarrow V_{max} = E_b \frac{r_2}{e} \ln e$$

$$\boxed{V_{max} = E_b r_2/e}$$

#5

(b) WHAT r_1 MAXIMIZES $\frac{1}{2}CV^2$?

$$W = \frac{1}{2}CV^2$$

FIND C: $C = Q/V$

FROM ABOVE $Q = \lambda l$

$$V = E_b r_1 \ln r_2/r_1$$
$$= (\lambda/2\pi\varepsilon_0) \ln r_2/r_1$$

$$C = \frac{2\pi\varepsilon_0 l}{\ln(r_2/r_1)}$$

$$C/l = 2\pi\varepsilon_0 / \ln(r_2/r_1) \equiv C'$$

Joules PER UNIT LENGTH STORED W'

$$W' = \frac{1}{2}C'V^2$$
$$= \frac{1}{2}\, 2\pi\varepsilon_0/\ln(r_2/r_1) \cdot E_b^2 r_1^2 \left(\ln\left(r_2/r_1\right)\right)^2$$

$$W' = \pi E_b^2 r_1^2 \ln r_2/r_1$$

$$\partial W'/\partial r_1 = \pi E_b^2 \left[2r_1 \ln r_2/r_1 + r_1^2\left(-\frac{r_2}{r_1^2} \cdot r_1/r_2\right)\right] = 0$$

$$2r_1 \ln r_2/r_1 - r_1 = 0$$
$$2 \ln r_2/r_1 = 1$$
$$r_2/r_1 = e^{1/2}$$
$$\boxed{r_1 = r_2/\sqrt{e}}$$

e = Euler's const.

#5 (b).

$$V(r_1 = r_2/\sqrt{e}) = E_b \, r_2/\sqrt{e} \, \ln \frac{r_2}{r_2/\sqrt{e}}$$

$$V_{max} = E_b \, r_2/\sqrt{e} \cdot \ln e^{1/2}$$

$$\boxed{V_{max} = \frac{E_b \, r_2}{2\sqrt{e}}}$$

#5 (c) $E_b = 3 \times 10^{6}$ V/m

$r_2 = 1$ cm $= 10^{-2}$ m

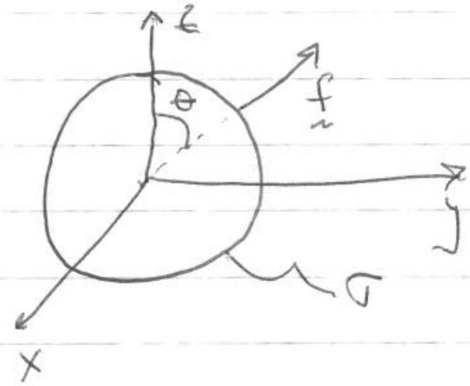
PT a: $V_{max} = E_b \, r_2/e$

$$\boxed{V_{max} = 1.10 \times 10^{4} \, V}$$

PT b: $V_{max} = E_b \, r_2/2\sqrt{e}$

$$\boxed{V_{max} = 9.1 \times 10^{3} \, V}$$

#6.



$$\underset{\sim}{f} = \frac{\sigma^2}{2\varepsilon_0}\,\hat{\underset{\sim}{n}}$$

<u>NET</u> FORCE ON 1 HEMISPHERE ALONG $\hat{z}$:

$$F_N = \int \underset{\sim}{f} \cdot \hat{z}\, dS$$

F_N DUE TO REPULSION FROM ALL OTHER CHARGES.

$$F_N = \frac{\sigma^2}{2\varepsilon_0}\int \cos\theta\, dS = \frac{\sigma^2}{2\varepsilon_0}\int \cos\theta \sin\theta\, d\theta R^2 d\varphi$$

$$= \frac{\sigma^2 R^2}{2\varepsilon_0}\int_0^{\pi/2} \sin\theta\cos\theta\, d\theta \int_0^{2\pi} d\varphi$$

NOTE θ INTEGRATION IS FOR 1 HEMISPHERE

$$F_N = \frac{2\pi\sigma^2 R^2}{2\varepsilon_0}\left.\frac{\sin^2\theta}{2}\right|_0^{\pi/2}$$

$$= \frac{2\pi}{4\varepsilon_0}\left(\frac{Q}{4\pi R^2}\right)^2 \cdot R^2$$

$$\boxed{F_N = \frac{1}{8}\left(\frac{Q^2}{4\pi\varepsilon_0 R^2}\right)}$$