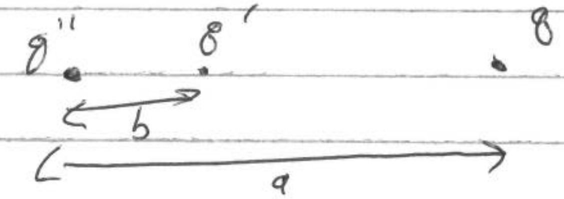
#1 STRATEGY:

① FIND q' IMAGE CHARGE TO MAKE SPHERE POTENTIAL $V = 0$ VOLTS
FOLLOW EXAMPLE IN TEXT & CLASS.

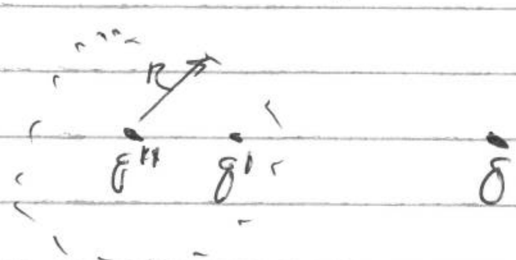
② THEN FIND 2ND IMAGE CHARGE $q'' = -q'$ AND PLACE AT SPHERE CENTER TO RAISE SPHERE POTENTIAL TO V_0.

STEP 1:

$$q' = -q\, R/a \quad \text{PLACED AT } b = R^2/a$$



R = SPHERE RADIUS.



STEP 2. $q'' + q' = 0$ SINCE SPHERE IS NEUTRAL.

$$q'' = -q' = +q R/a$$

NOTE THAT q'' HAS <u>SAME</u> SIGN AS q.

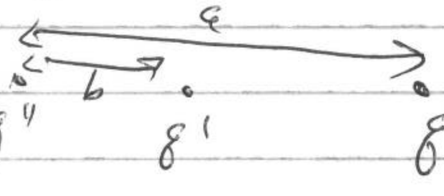
~~ALSO~~ ALSO NOTE THAT SPHERE IS AT POTENTIAL V'' WHEN q'' IS INSERTED:

$$V_0 = V'' = \frac{q''}{4\pi\varepsilon_0 R}$$

$$V_0 = \frac{+q}{4\pi\varepsilon_0 a}$$

THIS SHOULD NOT BE SURPRISING SINCE AVERAGE VALUE OF POTENTIAL OVER A SPHERICAL SURFACE CONTAINING NO REAL CHARGE IS SAME V AT CENTER.

TO CONTINUE, WE THROW AWAY SPHERICAL SURFACE AND COMPUTE FORCE ON q FROM IMAGE CHARGES q' & q'':



$$F_q = \frac{-q q'}{4\pi\varepsilon_0} \frac{1}{(a-b)^2} + \frac{q q''}{4\pi\varepsilon_0 a^2}$$

$$= \frac{q q'}{4\pi\varepsilon_0} \left[-\frac{1}{(a-b)^2} + \frac{1}{a^2} \right]$$

$$= \frac{q^2 R/a}{4\pi\varepsilon_0} \left[\frac{-a^2 + a^2 - 2ab + b^2}{a^2[a^2 - 2ab + b^2]} \right]$$

$$F_q = \frac{q^2}{4\pi\varepsilon_0} R/a \left[\frac{-2R^2 + (R^2/a)^2}{a^2\left(a^2 - 2a\frac{R^2}{a} + (R^2/a)^2\right)} \right]$$

$$= \frac{q^2}{4\pi\varepsilon_0} \frac{R^3}{a^3} \left[\frac{-2 + (R/a)^2}{a^2 - 2R^2 + R^4/a^2} \right]$$

$$F_q = \frac{q^2}{4\pi\varepsilon_0} \left(R/a\right)^3 \left[\frac{-2a^2 + R^2}{[a^2 - R^2]^2} \right]$$

$$= \frac{-q^2}{4\pi\varepsilon_0} \left(\frac{R}{a}\right)^3 \left[\frac{2a^2 - R^2}{[a^2 - R^2]^2} \right]$$

SINCE $a > R$, $F_q < 0$, WHICH MEANS IT IS ATTRACTIVE. MAGNITUDE OF ATTRACTIVE FORCE $|F_q|$:

$$\boxed{|F_q| = \frac{q^2}{4\pi\varepsilon_0} \left(\frac{R}{a}\right)^3 \left[\frac{2a^2 - R^2}{[a^2 - R^2]^2} \right]}$$

AGAIN, F_q IS ATTRACTIVE.

$$\boxed{|F_q| = \frac{q^2}{4\pi\varepsilon_0} \left(\frac{R}{r_0}\right)^3 \left[\frac{2r_0^2 - R^2}{[r_0^2 - R^2]^2} \right]}$$