PHYS 4392
Fall 2014
TE Coan
Due: 24 Oct '14 6:00 pm

Homework 8

1. Consider a dielectric sandwich made from a parallel plate capacitor of plate width w and length l, and a partially inserted dielectric slab between the plates. See figure 4.30 in Griffiths. Show that the capacitance C of this arrangement is

$$C = \frac{\epsilon_0 w}{d}(\epsilon_r l - \chi_e x)\,,$$

where $\epsilon_r = 1 + \chi_e$. Hint: Recall an important integral theorem to get started.

2. Consider now a multiplate capacitor of six parallel plates, as shown below. Alternating plates are connected to opposite terminals of a battery. Calculate the capacitance C of this set of the plates if each plate has an A, separation d from its neighbor by a material of permittivity ϵ. Do not be afraid to use a result from PHYS 1304 and even PHYS 1106.

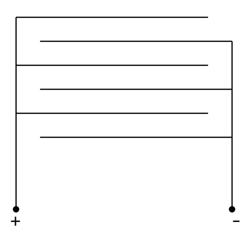

Figure 1: Multiplate capacitor.

3. Let's take a whack at solving another capacitor problem. Suppose, suppose, you have a pair of coaxial tubes of radius a and b. The tubes are upright and you dunk them in oil (a dielectric of permittivity ϵ and mass density ρ). See the figure below for a side view of the geometry. A voltage V is applied between the tubes. (It actually doesn't matter which tube is more negative than the other.) How high h does the oil rise above the surface of the oil? Before having a nervous breakdown, recall from your reading that the electrical force F on a dielectric in a capacitor is given by $F = \frac{1}{2}V^2\,dC/dx$, even if the potential difference V applied to the capacitor is fixed. (We only did the case of constant charge in lecture.) Furthermore, it will help to think of the capacitance of the air filled portion and the oil filled portion as capacitors in parallel. Take it from there.

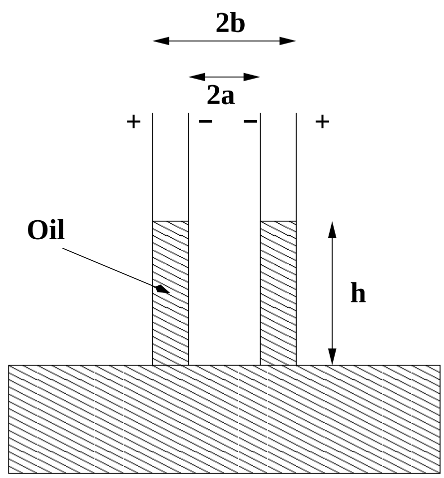

Figure 2: Side view of the coaxial cylinders dunked in oil and with a voltage difference V applied to the tubes.

4. The boundary conditions for $\mathbf{D}$ at the interface between two dielectrics imply that the electric field $\mathbf{E}$ will bend at the interface. See if you can show

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2},$$

where the angles are defined with respect to the normal at the interface. See the figure below. As usual, ϵ labels the permittivity of the relevant dielectric.

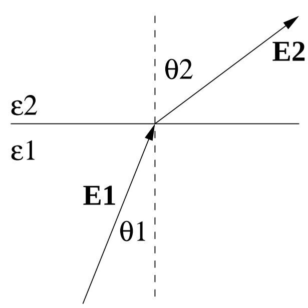

Figure 3: Bending of an electric field $\mathbf{E}$ at the interface between two dielectrics.