#1

$$B_z = \frac{\mu_0 I a^2}{2} \frac{1}{\left[a^2 + \left(z + \frac{l}{2}\right)^2\right]^{3/2}}$$

$$+ \frac{\mu_0 I a^2}{2} \frac{a^2}{\left[a^2 + \left(z - l/2\right)^2\right]^{3/2}}$$

$$= \frac{\mu_0 I a^2}{2} \left[\frac{1}{\left[a^2 + \left(z + \frac{\alpha a}{2}\right)^2\right]^{3/2}} + \frac{1}{\left[a^2 + \left(z - \frac{\alpha a}{2}\right)^2\right]^{3/2}} \right]$$

~~$B_z(0)$~~ SET DERIVATIVE(S) OF B_z TO ZERO:

$$B_z'(0) = \frac{\mu_0 I a^2}{2} \left[-\frac{3}{2}\left(a^2 + \left(z + \frac{\alpha a}{2}\right)^2\right)^{-5/2} \cdot 2\left(z + \frac{\alpha a}{2}\right) - \frac{3}{2}\left(a^2 + \left(z - \frac{\alpha a}{2}\right)^2\right)^{-5/2} 2\left(z - \frac{\alpha a}{2}\right) \right]\Bigg|_{z=0}$$

$$0 = \frac{\alpha a/2}{\left[a^2 + \left(\frac{\alpha a}{2}\right)^2\right]^{5/2}} - \frac{\alpha a/2}{\left[a^2 + \left(\frac{\alpha a}{2}\right)^2\right]^{5/2}}$$

ANY α WORKS SO NEED 2^{ND} DERIVATIVE:

CON'T.

#1.

$$B_z''(0) = 0 = \Big\{ -\tfrac{5}{2}\left[D_+^2\right]^{-7/2} 2\left(z + \tfrac{\alpha a}{2}\right)^2 + \left(D_+^2\right]^{-5/2}$$

$$-\tfrac{5}{2}\left[D_-^2\right]^{-7/2} 2\left(z - \tfrac{\alpha a}{2}\right)^2 + \left[D_-^2\right]^{-5/2} \Big\}\Big|_{z=0}$$

$$\text{w/ } D_\pm^2 = a^2 + \left(z \pm \tfrac{\alpha a}{2}\right)^2$$

$$0 = -\tfrac{5}{2}\left[D^2\right]^{-7/2} \cdot 2\left(\tfrac{\alpha a}{2}\right)^2 + \left(D^2\right)^{-7/2} D^2$$

$$-\tfrac{5}{2}\left[D^2\right]^{-7/2} \cdot 2\left(\tfrac{\alpha a}{2}\right)^2 + \left(D^2\right)^{-7/2} D^2$$

$$\text{w/ } D^2 = a^2 + \left(\tfrac{\alpha a}{2}\right)^2$$

$$= -\tfrac{5}{2} D^{-7} \cdot 2\left(\tfrac{\alpha a}{2}\right)^2 + D^{-5} - \tfrac{5}{2} D^{-7} \cdot 2\left(\tfrac{\alpha a}{2}\right)^2 + D^{-5}$$

$$0 = -5D^{-7}\left(\tfrac{\alpha a}{2}\right)^2 \cdot 2 + 2D^{-5}$$

$$0 = -5D^{-2}\left(\tfrac{\alpha a}{2}\right)^2 + 1$$

$$\left(\tfrac{\alpha a}{2}\right)^2 = D^2/5$$

$$\frac{\alpha^2 a^2}{4} = \frac{a^2 + \frac{\alpha^2 a^2}{4}}{5}$$

$$\alpha^2/4 = \left(1 + \alpha^2/4\right)/5$$

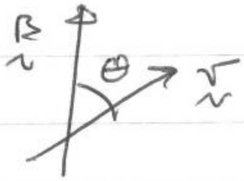
$$\alpha = 1, \quad \boxed{\therefore L = a}$$ SEPARATE COILS BY THEIR RADIUS.

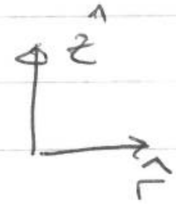
#1 (b)

$$B_z(0) = \frac{2\mu_0 I}{2} \frac{a^2}{\left(a^2 + \left(\frac{a}{2}\right)^2\right)^{3/2}}$$

$$\boxed{B_z(0) = \frac{\mu_0 I}{a}\left(4/5\right)^{3/2}}$$

#2





$\underset{\sim}{F} = Q \underset{\sim}{v} \times \underset{\sim}{B}$

$F_{\parallel} = 0$ PARALLEL TO $\underset{\sim}{B}$

$F_{\perp} = q v B \sin\theta$ PERPENDICULAR TO B

PARTICLE MOVES IN SPIRAL

TTL z- DISTANCE $= L = \overline{v_z t} = v(\cos\theta) \cdot t$

w/ t = TRAVEL TIME

TTL R- DISTANCE $= v_r t$

$= v \sin\theta \cdot L / v \cos\theta = L \tan\theta$

TTL DISTANCE $= \left[z^2 + r^2\right]^{1/2}$

$= \left[L^2 + L^2 \tan^2\theta\right]^{1/2}$

$\boxed{\text{DISTANCE} = L / \cos\theta}$

THIS ASSUMES SPIRAL RADIUS IS LESS THAN SOLENOID RADIUS.

#3. MAX Q OCCURS WHEN WE HAVE BREAKDOWN IN AIR.

$E_b = 3\times10^6$ V/m $= 30$ kV/cm

(ACTUALLY DEPENDS ON AIR CONDITIONS.)

E-FIELD @ SURFACE OF SPHERE $= V/R$

w/ V = SPHERE VOLTAGE

R = SPHERE RADIUS $= 5'' = 12.5$ cm

$V @ E_b = 30$ kV/cm $\times 12.5$ cm

$= 375$ kV (STD VALUE FOR LECTURE VDG'S.)

$Q = CV$

$C = 4\pi\varepsilon_0 R$ FOR SPHERE

$C = 4\pi \times 8.85\times10^{-12} \times 0.125$ F

$= 13.9$ pF

$Q = (13.9\times10^{-12})(375\times10^{3})$ C

$$\boxed{Q \triangleq 5.2\times10^{-6}\ \text{C}}$$