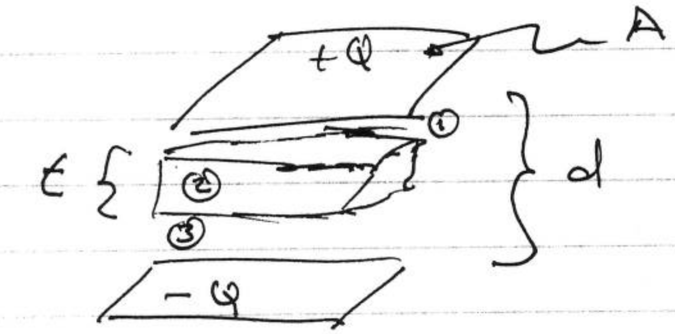
Q1.



$C = Q/V$ FIND V FROM $\underset{\sim}{E}$ IN 3 REGIONS

NOTE: $\oint \underset{\sim}{D} \cdot d\underset{\sim}{s}' = Q_{f_{enc}}$

$D^{\perp}_{ABOVE} - D^{\perp}_{BELOW} = \sigma_f \Rightarrow D_{\perp}$ CONTINUOUS ACROSS DIELECTRIC

$D_1 = \varepsilon_0 E_1 = Q_F/A = \sigma_f$

$E_1 = \sigma_F/\varepsilon_0$. REGION 1

$D_1 = D_2$ (NO FREE CHARGE, REGION 2)

$D_1 = \varepsilon E_2 = \sigma_f$

$E_2 = \sigma_F/\varepsilon$

$E_3 = \sigma_f/\varepsilon_0$ (LIKE REGION 1)

NOTE $\underset{\sim}{E} \parallel \underset{\sim}{D}$

Q1 $V = -\oint \underset{\sim}{E} \cdot d\underset{\sim}{l}$

$$= \frac{\sigma_f}{\varepsilon_0}(d-t) + \sigma_f/\varepsilon \, t$$

$$= \frac{Q}{A}\left[\frac{d-t}{\varepsilon_0} + t/\varepsilon\right]$$

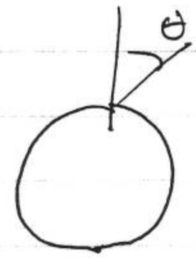
SINCE $C = Q/V$

$$\boxed{C = \frac{A}{\left[\frac{d-t}{\varepsilon_0} + t/\varepsilon\right]}}$$

NOTE THAT IN LIMIT OF $t \to 0$, WE RECOVER STD RESULT FOR A PARALLEL PLATE CAP.

Q2:

$$\underset{\sim}{P} = \varepsilon_0 \chi_e \underset{\sim}{E}$$



$$\sigma_P = \underset{\sim}{P} \cdot \hat{n}$$

$$= \varepsilon_0 \chi_e \underset{\sim}{E} \cdot \hat{n}$$

$\uparrow E_0 \quad \uparrow E_0$

$$* \quad \sigma_p = \frac{3 \varepsilon_0 \chi_e E_0 \cos\theta}{\varepsilon_r + 2}$$

Now, $\varepsilon = \varepsilon_0 (1 + \chi_e)$

$$\varepsilon / \varepsilon_0 = \varepsilon_r = 1 + \chi_e$$

$$\Rightarrow \chi_e = \varepsilon_r - 1$$

COLLECTING TERMS,

$$\boxed{\sigma_p = \frac{3 \varepsilon_0 (\varepsilon_r - 1)}{\varepsilon_r + 2} E_0 \cos\theta}$$

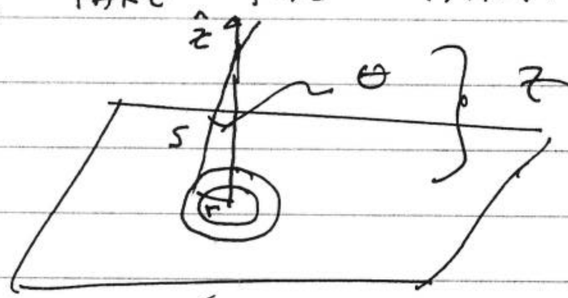
Q3. EDGE VIEW: $+ \quad + \quad + \quad + \quad \ldots$

$- \quad - \quad - \quad - \quad \ldots$

JUST LIKE 2 INFINITE SHEETS OF UNIFORM BUT OPPOSITE CHARGE-
BY SUPERPOSITION TTL $\tilde{E}$-FIELD ABOVE OR BELOW SHEET IS ZERO!

$$\boxed{\tilde{E} = 0 \;!}$$

FOR V, TAKE THE HINT.



$$V = \frac{1}{4\pi\epsilon_0} \frac{\tilde{p}' \cdot \hat{s}}{s^2} \qquad \tilde{p}' : \text{DIPOLE MOMENT}$$

~~[illegible]~~ IN PROBLEM $\tilde{p}$ = DIPOLE moment / UNIT-AREA

$$V(z) = \frac{1}{4\pi\epsilon_0} \int_{\theta=0}^{\pi/2} \int_{r=0}^{\infty} \frac{p\, dS' \cos\theta}{s^2}$$

$$= \frac{1}{4\pi\epsilon_0} \int_0^{\pi/2} \int_0^{\infty} \frac{p\, r\, dr\, d\varphi \cos\theta}{s^2}$$

Q3. ~~V(z)=~~ BUT $r = s \sin\theta$

$$dr = s \cos\theta \, d\theta$$

$$V(z) = \frac{1}{4\pi\varepsilon_0} \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{2\pi} p \frac{s^2 \sin\theta \cos\theta \, d\theta}{s^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \int_0^{\pi/2} \int_0^{2\pi} \sin\theta \cos\theta \, d\theta \, d\varphi$$

$$= \frac{p}{2\varepsilon_0} \left. \frac{\sin^2\theta}{2} \right|_0^{\pi/2}$$

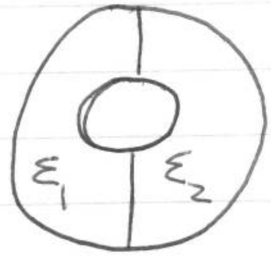
$$\boxed{V(z) = \frac{p}{4\varepsilon_0}}$$

w/ p = DIPOLE MOMENT / UNIT AREA

$$\boxed{V(z) = \text{const} \quad \text{SO} \quad \underset{\sim}{E} = -\underset{\sim}{\nabla} V = 0, \text{ AS ABOVE!}}$$

(2) ⇒ $E \cos\theta - \varepsilon_0 A . \cos\theta = \varepsilon ($ [illegible] $+$ [illegible] $\cos\theta)$

Q4



BC'S AT INTERFACE:

$$E_1^{\parallel} = E_2^{\parallel}$$

$$D_1^{\perp} = D_2^{\perp}$$

$\mathbf{E}$ ~~HAS~~ IS ASSUMED TO BE SPHERICALLY SYMMETRIC:

$$\mathbf{E}_1 = \mathbf{E}_2 = A\hat{r}/r^2$$

SATISFIES ABOVE BC

GAUSS' LAW FOR $\mathbf{D}$: $\oint \mathbf{D}_2 \cdot d\mathbf{S}' = Q$

$$2\pi D_1 r^2 + 2\pi D_2 r^2 = Q$$

$$2\pi \varepsilon_1 E_1 r^2 + 2\pi \varepsilon_2 E_2 = Q$$

$$2\pi (\varepsilon_1 + \varepsilon_2) A = Q \qquad (E_1 = E_2 = A/r^2)$$

$$\Rightarrow A = Q / 2\pi(\varepsilon_1 + \varepsilon_2)$$

Q4. CONT.

HENCE,

$$\boxed{\underset{\sim}{E}_1 = \underset{\sim}{E}_2 = \frac{Q\,\hat{r}}{2\pi(\varepsilon_1+\varepsilon_2)\,r^2}}$$

$$\boxed{\begin{aligned} \underset{\sim}{D}_1 &= \frac{\varepsilon_1 Q\,\hat{r}}{2\pi(\varepsilon_1+\varepsilon_2)\,r^2} \\ \underset{\sim}{D}_2 &= \frac{\varepsilon_2 Q\,\hat{r}}{2\pi(\varepsilon_1+\varepsilon_2)\,r^2} \end{aligned}}$$

Q5.

PASSENGER DOOR NOT VISIBLE.
BUS MOVING TO LEFT?