PHYS 5382
Fall 2016
TE Coan
Due: 28 Oct '28 6:00 pm

Homework 5

0. Box your **entire** answer for each problem or lose points.

1. A spin-1 particle is in the state

$$|\Psi\rangle \underset{\text{S}_z\text{basis}}{\longrightarrow} \frac{1}{\sqrt{14}} \begin{pmatrix} i \\ 2 \\ 3i \end{pmatrix}.$$

What is the probability that a measurement of S_x yields the value $\hbar$ for this state? (Did you do the reading...)

2. Arsenic atoms in the ground state are spin-$\frac{3}{2}$ particles. Suppose an arsenic beam is shot through an SG**x** device, one with its inhomogeneous magnetic field oriented along the **x**-direction. Atoms with $S_x = \frac{1}{2}\hbar$ then enter an SG**z** device. Determine the fraction of the atoms that exit the SG**z** device with $S_z = \frac{3}{2}\hbar, S_z = \frac{1}{2}\hbar, S_z = -\frac{1}{2}\hbar$ and $S_z = -\frac{3}{2}\hbar$. The eigenstates of the operator $\hat{S}_x$ in the S_z basis are:

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle_x \longrightarrow \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ \sqrt{3} \\ \sqrt{3} \\ 1 \end{pmatrix} \qquad \left|\frac{3}{2},\frac{1}{2}\right\rangle_x \longrightarrow \frac{1}{2\sqrt{2}} \begin{pmatrix} \sqrt{3} \\ 1 \\ -1 \\ -\sqrt{3} \end{pmatrix}$$

$$\left|\frac{3}{2},-\frac{1}{2}\right\rangle_x \longrightarrow \frac{1}{2\sqrt{2}} \begin{pmatrix} \sqrt{3} \\ -1 \\ -1 \\ -\sqrt{3} \end{pmatrix} \qquad \left|\frac{3}{2},-\frac{3}{2}\right\rangle_x \longrightarrow \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ -\sqrt{3} \\ \sqrt{3} \\ -1 \end{pmatrix}$$

3. A spin-$\frac{3}{2}$ particle is in the state

$$|\Psi\rangle \underset{\text{S}_z\text{basis}}{\longrightarrow} N \begin{pmatrix} i \\ 2 \\ 3 \\ 4i \end{pmatrix}.$$

(a) Determine N so that $|\Psi\rangle$ is properly normalized.

(b) What is $\langle S_x \rangle$ for this state? Recall that the text has the matrix representation for the operator $\hat{S}_x$ in one of the examples in chapter 3.

(c) What is the probability $P(S_x = +\frac{\hbar}{2})$ that a measurement of S_x yields the value $\hbar/2$ for this state?

4a. What is the matrix representation for $\hat{S}_y$ for a spin-$\frac{3}{2}$ particle?

(b) What is the normalized eigenstate of $\hat{S}_y$ with eigenvalue $\frac{3}{2}\hbar$ for the same spin-$\frac{3}{2}$ particle?