PHYS 5382
Fall 2016
TE Coan
Due: 22 Nov '16 6:00 pm

Homework 7

0. Box your **entire** answer for each problem or lose points.

1a. Let the Hamiltonian for a spin-$\frac{1}{2}$ particle be given by

$$\hat{H} = \omega_o \hat{S}_x$$

.

If at time $t = 0$, $|\Psi(0)\rangle = \left|\frac{3}{2}, \frac{3}{2}\right\rangle$, determine the probability that the particle is in the state $\left|\frac{3}{2}, -\frac{3}{2}\right\rangle$ at time t. Note these kets are written in the z-basis. The eigenstates for the operator $\hat{S}_x$, written in the z-basis, are given by

$$\left|\frac{3}{2}, \frac{3}{2}\right\rangle_x \longrightarrow \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ \sqrt{3} \\ \sqrt{3} \\ 1 \end{pmatrix} \qquad \left|\frac{3}{2}, \frac{1}{2}\right\rangle_x \longrightarrow \frac{1}{2\sqrt{2}} \begin{pmatrix} \sqrt{3} \\ 1 \\ -1 \\ -\sqrt{3} \end{pmatrix}$$

$$\left|\frac{3}{2}, -\frac{1}{2}\right\rangle_x \longrightarrow \frac{1}{2\sqrt{2}} \begin{pmatrix} \sqrt{3} \\ -1 \\ -1 \\ -\sqrt{3} \end{pmatrix} \qquad \left|\frac{3}{2}, -\frac{3}{2}\right\rangle_x \longrightarrow \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ -\sqrt{3} \\ \sqrt{3} \\ -1 \end{pmatrix}$$

(1b). Evaluate this probability when $t = \pi/\omega_o$ and explain your result.

2. The lifetime of a hydrogen atom in the $2p$ state to decay to the $1s$ ground state is 1.6×10^{-9} s. Estimate the uncertainty ΔE in energy of this excited state. What is the corresponding "linewidth" $\Delta\lambda$ in nanometers? Box your answer.

3a. A bit of a review and a test if you followed the ammonia molecule discussion and read the relevant pages in Townsend! Suppose, suppose the matrix representation of the Hamiltonian for a three-state system with basis states $|1\rangle, |2\rangle$ and $|3\rangle$ is given by

$$\begin{pmatrix} E_0 & 0 & A \\ 0 & E_1 & 0 \\ A & 0 & E_0 \end{pmatrix}$$

If the state of the system at time $t = 0$ is $|\Psi(0)\rangle = |2\rangle$, what is $|\Psi(t)\rangle$?

3b. If the state of the system at time $t = 0$ is $|\Psi(0)\rangle = |3\rangle$, what is $|\Psi(t)\rangle$?

4. A beam of spin-$\frac{1}{2}$ particles in the $|+\mathbf{z}\rangle$ state enters a uniform magnetic field B_0 in the x-z plane oriented at an angle θ with respect to the z axis. At a time T later, the particles enter a SG**y** device. What is the probability $P(S_y = \hbar/2)$ the particles will be found with $S_y = \hbar/2$? Check your results by evaluating the special cases $\theta = 0$ and $\theta = \pi/2$. **Hint:** You will first need to express the energy eigenstates of the Hamiltonian in terms of the z-basis states $|+\mathbf{z}\rangle$ and $|-\mathbf{z}\rangle$. These energy eigenstates are just the spin eigenstates (given below) of a spin-$\frac{1}{2}$ particle along a direction $\hat{\mathbf{n}}$ that is parallel to the **B**-field. See the figure from HW1, problem 2.

$$|+\mathbf{n}\rangle = \cos\frac{\theta}{2}\,|+\mathbf{z}\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|-\mathbf{z}\rangle$$

$$|\text{-}\mathbf{n}\rangle = \sin\frac{\theta}{2}\,|+\mathbf{z}\rangle - e^{i\phi}\cos\frac{\theta}{2}\,|-\mathbf{z}\rangle$$

You should now be able to determine $|\Psi(t)\rangle$ and to form any inner product you want.