PHYS 5382
Fall 2016
TE Coan
Due: 6 Dec 6:00 pm

Homework 8

0. Box your **entire** answer for each problem or lose points.

1a. Recall that positronium is a bound state of an electron and a positron, both of which are spin-$\frac{1}{2}$ particles. Suppose this bound system is in an external magnetic field in the z-direction so that its spin Hamiltonian is

$$\hat{H} = \frac{2A}{\hbar^2}\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2 + \omega_0(\hat{S}_{1z} - \hat{S}_{2z}).$$

Calculate the energy eigenvalues of this Hamiltonian. Box that answer.

2. More with positronium. Suppose that at $t = 0$ the electron and the positron are found to have a total spin angular momentum of zero at time $t = 0$. Suppose further this "atom" is in a uniform, static magnetic field B_0 in the z-direction.

2a. Ignoring the interaction between the electron and positron for simplicity, show that the spin Hamiltonian of positronium can be written as

$$\hat{H} = \omega_0(\hat{S}_{1z} - \hat{S}_{2z}),$$

where $\hat{\mathbf{S}}_1$ is the spin operator of the electron and $\hat{\mathbf{S}}_2$ is the spin operator of the positron, and ω_0 is a constant we have seen before.

2b. What is the spin state of the system at some later time t? Show that the state oscillates between spin-0 and spin-1. Determine the period T of this oscillation. Box your various answers.

2c. Measurements of S_{1x} and S_{2x} are made at time t. Calculate the probability $P(\frac{\hbar}{2}, \frac{\hbar}{2})$ that *both* of these measurements yield the value $\hbar/2$.

3. Consider the case where we have *three* spin-$\frac{1}{2}$ particles. The maximum spin angular momentum of this system is $\frac{3}{2}\hbar$ and the maximum z component of the angular momentum is also $\frac{3}{2}\hbar$. Denote this state by the standard notation $\left|\frac{3}{2}, \frac{3}{2}\right\rangle$. This state occurs when all three spins are each in the state $|+\mathbf{z}\rangle$, so that $\left|\frac{3}{2}, \frac{3}{2}\right\rangle = |+\mathbf{z}, +\mathbf{z}, +\mathbf{z}\rangle$. Find all 4 states with $s = \frac{3}{2}$ and write them in a way that denotes their individual spin states, similar to what was done for the case of two spin-$\frac{1}{2}$ particles. **Hint:** Why not build three particle raising and lowering operators and let them do the work for you? Eq. (5.36) in Townsend can serve as a guide.

4. We discussed the triangle rule in lecture when discussing the possible total angular momenta of a set of particles. Suppose you have two particles, each with spin $s = 1$. What are all the possible *total* spin states for the composite system? For *each* of those total spin states, what are the possible m values? Be systematic in the presentation of your answer so I can understand it. Box it.