PHYS 5383
Spring 2013
TE Coan
Due: 22 Feb '13 6:00 pm

Homework 2

1. The operator $\hat{A}$ has only non-degenerate eigenvectors $\{\phi_n\}$ and eigenvalues $\{a_n\}$. What are the eigenvectors and eigenvalues of the inverse operator $\hat{A}^{-1}$?

2. Recall from the faraway land of differential equations that the linear independence of two functions $s(x)$ and $v(x)$ may be specified by their Wronskian:

$$W(s, v) = \begin{vmatrix} s & v \\ s' & v' \end{vmatrix}.$$

If s and v are the solutions to a linear, second order differential equation and $W(s, v) \neq 0$ over some interval, then s and v are linearly independent over that interval. Using this criterion, establish that the two solutions to the radial Schrödinger in one dimension with an infinite spherical well and with $l = 0$ are independent over the size of the well. What value for W do you find in this case?

3. Here is a bit of review. At $t = 0$ it is known that of 1000 neutrons in a one-dimensional box of width 10^{-7} m, 100 have energy $4E_1$ and 900 have energy $225E_1$.

a. Construct a wavefunction with these properties. (Coefficients of the eigenfunctions may be complex.)

b. What is the density $\rho(x)$ of neutrons per unit length?

c. How many neutrons are in the left hand side of the "box"?

4. Consider the situation where it is equally likely that an electron has momentum $\pm\mathbf{p}_0$. Measurement at a given instant of time finds the value $+\mathbf{p}_0$. A pesky student concludes that the electron must have had this value of momentum prior to the measurement. Is the student correct? Explain. **Briefly.**

5. As threatened in lecture, prove that the coefficient A for the function $u(r) = A \sin kr + B \cos kr$ that satisfies the radial equation for the infinite spherical well with $l = 0$ is $A = \sqrt{2/a}$, where a is the radius of the well.