#1. EXPRESS / EXPAND TRITIUM W. F. IN TERMS OF EIGENSTATES OF He^3 $(Z=2)$:

$$\psi(\text{TRITIUM}) = \sum_{n=1}^{\infty} \psi_n (He^3)$$

PROBABILITY OF REMAINING IN GROUND STATE <u>AFTER</u> TRANSITION IS

$$\text{PROB} = \left| \langle \psi(\text{TRITIUM GND STATE}) | \psi(He^3 \text{ G.S.}) \rangle \right|^2$$

$$\psi_{100}(\text{TRITIUM}) = \frac{2}{\sqrt{4\pi}} \left(\frac{1}{a}\right)^{3/2} e^{-r/a}$$

$$\psi_{100}(\text{TRITIUM}) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$$

$$\text{w/ } a = \frac{4\pi\varepsilon_0 \hbar^2}{m e^2}$$

$$\psi_{100}(He^3) = \sqrt{\frac{8}{\pi a^3}}\, e^{-2r/a} \qquad (\text{2 PROTONS})$$

NOTE THAT FOR $Z \neq 1$, $a = \dfrac{4\pi\varepsilon_0 \hbar^2}{m Z e^2}$

$$\text{SO, } P = \left[\frac{1}{\sqrt{\pi a^3}} \int_0^{\infty} e^{-r/a} \cdot \sqrt{\frac{8}{\pi a^3}}\, e^{-2r/a}\, r^2\, dr\, d\Omega \right]^2$$

#1.

$$P = \left[\frac{\sqrt{8}}{\pi a^3} \cdot 4\pi \int_0^\infty e^{-3r/a} r^2 \, dr \right]^2$$

$$= \left[\frac{4\sqrt{8}}{a^3} \cdot \int_0^\infty e^{-y} y^2 \, dy \cdot \frac{a^3}{27} \right]^2$$

w/ $y \equiv 3r/a$

NOW, $\int_0^\infty e^{-y} y^2 \, dy = 2$ VIA WOLFRAM INTEGRATOR

HENCE,

$$P = \left[\frac{4\sqrt{8} \cdot 2}{27} \right]^2$$

$$\boxed{P = 0.70}$$

#2. $\int_0^a u^2\,dr = 1$ w/ $u = A\sin kr + B\cos kr$

$= A\sin kr$

since $u(a) = 0$.

$\int_0^a A^2 \sin^2 kr\,dr = 1$ w/ $k = \sqrt{2mE}/\hbar$

$\int_0^a A^2 \frac{(1-\cos kr)}{2}\,dr = 1$

$A^2\left[\frac{r}{2} - \frac{\sin kr}{kr}\right]\Big|_0^a = 1$

$A^2\,a/2 = 1$ since $\sin ka = 0$

$$\boxed{A = \sqrt{2/a}}$$

#3. FOR H-ATOM

$$E_n = -\left[\frac{m_e}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\right]\cdot\frac{1}{n^2}$$

$$= -13.6\ eV/n^2 \qquad w/\ m_e = e^-\ \text{MASS}$$

MORE GENERALLY

$$E_n = -\frac{\mu}{m_e}\left[\frac{m_e}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\right]\cdot\frac{1}{n^2}$$

$$E_n = -\frac{\mu}{m_e}\,\frac{13.6\ eV}{n^2} \qquad \mu = \text{REDUCED MASS}$$

FOR IONIZATION, $n = 1 \rightarrow \infty$

AND $\mu = \dfrac{m_\mu m_p}{m_\mu + m_p}$

$$= \frac{206\ m_e}{\frac{m_\mu}{m_p} + 1}$$

$\Rightarrow \mu \simeq 182\ m_e$

So,

$$\boxed{E_{IONIZATION} \simeq 182 \times 13.6\ eV \simeq 2.5\ keV}$$

$\mu^- p$ "ATOM" VERY TIGHTLY BOUND!

#4. $E_{e^+e^-} = -\left[\frac{\mu}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\right]\cdot 1/n^2$

$= -\frac{m_e/2}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\Big]\cdot 1/n^2$

$E_{e^+e^-} = -\frac{1}{2}\ 13.6\ eV/n^2$

BALMER SERIES $n_f = 2$

LOWEST E TRANSITIONS: $3\to2,\ 4\to2,\ 5\to2.$

$E_\gamma(3\to2) = \frac{13.6\,eV}{2}\left(\frac{1}{4} - 1/9\right)$

$\boxed{E_\gamma(3\to2) = 0.94\ eV}$

$\lambda(nm) = \frac{1240\ eV\text{-}nm}{E_\gamma(eV)}$ FROM LECTURE

$\boxed{\lambda(3\to2) = 1313\ nm \quad \underline{\text{NOT}}\ \text{VISIBLE!}}$

NOTE:

$\sim 390\ nm \leq$ VISIBLE LIGHT $\leq \sim 700\ nm$

$E_\gamma(4\to2) = \frac{13.6}{2}\left(1/4 - \frac{1}{16}\right) = 1.28\,eV$

$\lambda = \frac{1240\ eV\text{-}nm}{1.28\ eV}$

$\lambda = 970\ nm$ $\underline{\text{NOT}}$ VISIBLE!

#4. $E_\gamma(5 \to 2) = \frac{13.6}{2}\, eV \left(\frac{1}{4} - \frac{1}{25} \right)$

$$\boxed{\begin{array}{l} E_\gamma(5 \to 2) = 1.43\ eV \\ \lambda(5 \to 2) = 868\ nm \quad \underline{\text{NOT}}\ \text{VISIBLE!} \end{array}}$$