PHYS 5383
Spring 2013
TE Coan
Due: 12 Apr '13 6:00 pm

Homework 4

1. Consider the spinor $\frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\end{pmatrix}$. What is the probability that a measurement of $(3S_x + 4S_y)/5$ will yield the value $-\hbar/2$? Hint: What are the possible eigenvalues of this operator?

2. Consider the spin state $|1/2\ -1/2\rangle$ built from 2 spins, $s_1 = 3/2$ and $s_2 = 1$. Express $|1/2\ -1/2\rangle$ in terms of $|s_1\, m_1\rangle$ and $|s_2\, m_2\rangle$.

3. Let's construct the operator matrices L_x, L_y, L_+ and L_- for the case of $l = 1$. This is done the same way we did for the case $s = 1/2$. The essential machinery is identical but some details change, like the entries in the various matrices that represent operators. Recall that

$$[L^2, L_z] = 0$$
$$\langle lm'|\, L_z\, |lm\rangle = \hbar\, m\, \delta_{m'm}$$
$$\langle lm'|\, L_\pm\, |lm\rangle = \hbar\, \sqrt{l(l+1) - m(m\pm 1)}\, \delta_{m',m\pm 1}$$

Because of the commutation relation, we know that L_z and L^2 will have the same eigenstates. We also know that since $l = 1$, $m \in \{1, 0, -1\}$. This tells us what the eigenvalues for the operator S_z must be for the case $l = 1\,\hbar$. From this you should be able to verify that the matrix operator S_z is

$$S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

when acting on the eigenspinors

$$|11\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad |10\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix} \quad |1\,-1\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

The various matrix elements can be calculated in a straightforward manner. For S_z, the matrix entries are

$$S_z = \begin{pmatrix} \langle 11|\, S_z\, |11\rangle & \langle 11|\, S_z\, |10\rangle & \langle 11|\, S_z\, |1\,{-1}\rangle \\ \langle 10|\, S_z\, |11\rangle & \langle 10|\, S_z\, |10\rangle & \langle 10|\, S_z\, |1\,{-1}\rangle \\ \langle 1\,{-1}|\, S_z\, |11\rangle & \langle 1\,{-1}|\, S_z\, |10\rangle & \langle 1\,{-1}|\, S_z\, |1\,{-1}\rangle \end{pmatrix}$$

For your own understanding, see if you can tell how each row and column entry are labeled by the various m values of the bras and kets. The same scheme was used for the case $S = \hbar/2$ and is used for spins greater than $1\,\hbar$.

a. Determine the matrix form of the operators S_+ and S_- for the case of spin $= 1\,\hbar$. Hint: There will only be 2 non-zero entries for each matrix. Remember to include the value $\hbar$ somewhere appropriately or you will lose points.

b. Find the matrix operators for S_x and S_y, again for the case of spin $= 1\,\hbar$.

4. We saw in lecture (and you also saw in the text) the operation of the exchange or swap operator $\hat{P}$. It had the property that it swapped or exchanged the position of two particles in an *arbitrary* wavefunction Ψ

$$P\Psi(\mathbf{r_1}, \mathbf{r_2}) = \Psi(\mathbf{r_2}, \mathbf{r_1}).$$

For the special cases of fully symmetric or fully antisymmetric wave functions Ψ_S and Ψ_A, respectively, we had the relations $P\Psi(\mathbf{r_1}, \mathbf{r_2})_S = +\Psi(\mathbf{r_2}, \mathbf{r_1})_S$ and $P\Psi(\mathbf{r_1}, \mathbf{r_2})_A = -\Psi(\mathbf{r_2}, \mathbf{r_1})_A$. Furthermore, we also convinced ourselves that if the potential $V(r_1, r_2) = V(r_2, r_1)$ then the Hamiltonian operator and the exchange operator commuted $[\hat{H}, \hat{P}] = 0$. Show now that if a state is fully symmetric or fully antisymmetric at some arbitrary time t, it *always* remains symmetric or antisymmetric. (This problem is a one or two liner!)