#1 NOTE THAT OPERATOR IS OF FORM

$$\hat{S}\cdot\hat{n} = \frac{3}{5}S_x + \frac{4}{5}S_y$$ w/ $\hat{n}$ POINTING

IN SOME DIRECTION. ROTATE YOUR COORDINATE SYSTEM UNTIL YOUR Z-AXIS IS ALLIGNED w/ $\hat{n}$ DIRECTION. THEN

$|\langle\uparrow|SPIN\rangle|^2$ = PROBABILITY OF MEASURING SPIN UP

$$\left|(1\ 0)\frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\end{pmatrix}\right|^2 = 4/5$$

$|\langle\downarrow|SPIN\rangle|^2$ = PROB OF MEASURING SPIN DOWN

$$\left|(0\ 1)\frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\end{pmatrix}\right|^2 = 1/5$$

$$\boxed{\text{PROB OF MEASURING } -\hbar/2 = 20\%}$$

#2 $\boxed{\left|\tfrac{1}{2}\,-\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{6}}\left|\tfrac{3}{2}\,\tfrac{1}{2}\right\rangle|1\,-1\rangle - \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2}\,-\tfrac{1}{2}\right\rangle|1\,0\rangle + \sqrt{\tfrac{1}{2}}\left|\tfrac{3}{2}\,-\tfrac{3}{2}\right\rangle|1\,1\rangle}$

#3

SUBSTITUTE THE $\hat{S}_+$ OPERATOR FOR $\hat{S}_z$ IN THE EXAMPLE IN THE PROBLEM.

NOTE $S_+|11\rangle = 0$

$S_+|10\rangle = \sqrt{2}\,\hbar\,|11\rangle$

$S_+|1-1\rangle = \sqrt{2}\,\hbar\,|10\rangle$

$$\Rightarrow \boxed{\hat{S}_+ = \hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}}$$

SIMILARLY,

$S_-|11\rangle = \sqrt{2}\,\hbar\,|1\,0\rangle$

$S_-|1\,0\rangle = \sqrt{2}\,\hbar\,|1-1\rangle$

$S_-|1-1\rangle = 0$

OR $S_- = S_+^\dagger$

$$\text{BOTH} \Rightarrow \boxed{S_- = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}}$$

#3 CONT.

$$S_+ = S_x + i S_y$$

$$S_- = S_x - i S_y$$

$$\Rightarrow S_x = \frac{1}{2}(S_+ + S_-)$$

$$S_y = \frac{1}{2i}(S_+ - S_-)$$

FROM ABOVE,

$$\boxed{S_x = \hbar \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 0 \end{pmatrix}}$$

$$\boxed{S_y = \hbar \begin{pmatrix} 0 & -i/\sqrt{2} & 0 \\ i/\sqrt{2} & 0 & -i/\sqrt{2} \\ 0 & i/\sqrt{2} & 0 \end{pmatrix}}$$

4 JUST SHOW $\frac{d}{dt}\langle p \rangle = 0$

RECALL FOR AN ARBITRARY OBSERVABLE
WITH CORRESPONDING OPERATOR $\hat{A}$

$$\frac{d}{dt}\langle \hat{A} \rangle = \left\langle \frac{\partial}{\partial t} A \right\rangle + \frac{\langle [\hat{A}, \hat{H}] \rangle}{i\hbar}$$

FOR US, $\hat{P} = \hat{A}$.

$$d/dt \langle \hat{p} \rangle = 0 + \frac{\langle [P, H] \rangle}{i\hbar} = 0$$

SINCE $[P, H] = 0$

$\therefore \langle \hat{p} \rangle$ IS CONSTANT IN TIME

FOR FULLY ASYMMETRIC/SYMMETRIC STATES