#1. $M_\odot = 2\times10^{30}$ kg

$R_\odot = 7\times10^5$ km

$$\bar{\rho} = M_\odot / \tfrac{4}{3}\pi R_\odot^3$$

$$\boxed{\begin{aligned}\bar{\rho} &= 1.4\times10^3\ \mathrm{kg/m^3} \\ \rho &= 1.4 = 1.4\ \mathrm{g/cm^3}\end{aligned}}$$

AVERAGE DENSITY ABOUT THE SAME AS PVC PIPE.

#2. $E = pc = \hbar k c$ RELATIVISTIC CASE

$$P_{deg} = -\partial E_{TOT}/\partial V$$

THE KEY IS TO FIND E_{TOT}.
FOLLOW EXAMPLE IN CLASS THAT USES SHOEBOX POTENTIAL. WE WILL USE A CUBICAL SHOEBOX

$$k = \left[\left(\frac{n_x\pi}{L}\right)^2 + \left(\frac{n_y\pi}{L}\right)^2 + \left(\frac{n_z\pi}{L}\right)^2\right]^{1/2}$$

$$E_F = \frac{\hbar\pi c}{L}\left(n_x^2 + n_y^2 + n_z^2\right)^{1/2} = \frac{\hbar\pi c\, n}{L} \qquad (1)$$

$n = \dfrac{L E_F}{\hbar\pi c}$. n A KIND OF RADIUS IN SPACE THAT LABELS STATES.

II. FIND # of e^-'s YOU CAN FIT IN THIS SPACE AS A FUNCTION OF E_F.

$$N_e = 2 \cdot \frac{1}{8} \cdot \frac{4}{3}\pi R^3 = \frac{\pi}{3} n^3 = \frac{\pi}{3} \frac{E_F^3 L^3}{\hbar^3 \pi^3 c^3}$$

SPIN

ONLY POS n_x, n_y, & n_z ALLOWED

$$\boxed{N_e = \frac{1}{3\pi^2} \frac{E_F^3 L^3}{\hbar^3 c^3}} \qquad (2)$$

$$\Rightarrow E_F^3 = 3\pi^2 \hbar^3 c^3 N_e / L^3$$

$$\boxed{E_F = \hbar c \left(3\pi^2 n_e\right)^{1/3}} \quad \text{w/ } n_e = e^- \text{ NUMBER DENSITY} \qquad (3)$$

COMPUTE E_{TOT} USING TECHNIQUE IN TEXT. FIND E_F FOR A SHELL IN n-SPACE, COUNT # OF e^-'s IN THE SHELL, MULTIPLY TOGETHER AND THEN SUM OVER ALL SHELLS.

$$E_{TOT} = \frac{2}{8} \frac{\hbar \pi c}{L} \int n \, d^3 n \quad \text{w/ } d^3 n = n^2 \, dn \sin\theta \, d\theta \, d\varphi$$

$$E_{TOT} = \frac{1}{4}\,\frac{\hbar \pi c}{L}\cdot 4\pi \int_0^R n^3\, dn$$

$$= \frac{\hbar \pi^2 c}{L}\, R^4/4 \qquad \text{w/ } R = \text{max } n \text{ VALUE}$$

FIND R^4: $E_F = \frac{\hbar \pi c\, n_{max}}{L}$

$$\Rightarrow R = n_{max} = \frac{L E_F}{\hbar \pi c}$$

$$R^3 = \left(\frac{L E_F}{\hbar \pi c}\right)^3 = \frac{1}{\pi^3}\left(\frac{L E_F}{\hbar c}\right)^3 = \frac{3\pi^2}{\pi^3} N_e$$

USING (1)

$$\boxed{R^4 = (R^3)^{4/3} = \left(\frac{3 N_e}{\pi}\right)^{4/3}} \qquad (5)$$

HENCE

$$E_{TOT} = \frac{\hbar \pi^2 c}{4L}\left(\frac{3 N_e}{\pi}\right)^{4/3}$$

$$\boxed{E_{TOT} = \frac{\hbar \pi^2 c}{4} V^{-1/3}\left(\frac{3 N_e}{\pi}\right)^{4/3}} \qquad (6)$$

$$= \frac{\hbar \pi^2 c}{4}\,\frac{L^3}{L^4}\left[\frac{3 N_e}{\pi}\right]^{4/3}$$

$$\boxed{E_{TOT} = \frac{\hbar \pi^2 c}{4} V\left(\frac{3 n_e}{\pi}\right)^{4/3} \quad \text{w/ } n_e = e^- \text{ DENSITY.}} \qquad (7)$$

~~FINALLY~~ NOTE THAT $E_{TOT} \propto N_e^{4/3}$ FOR RELATIVISTIC CASE, EQ. (6) WHILE FOR NON-REL CASE, $E_{TOT} \propto N_e^{5/3}$. HENCE WE EXPECT DEGENERACY PRESSURE TO BE LESS FOR "COLD" RELATIVISTIC e^-'s THAN FOR NON-RELATIVISTIC e^-'s.

$$P_{DEG} = -\partial E_{TOT}/\partial V$$

$$\boxed{P_{DEG} = +\frac{\hbar \pi^2 c}{12}\left(\frac{3 n_e}{\pi}\right)^{4/3}}$$

RELATIVISTIC CASE

#3. FIND R_{EQ} AT WHICH

$$P_{DEG} = P_{GRAVITY}$$

USE NR $P_{DEG.}$ FROM TEXT.

FIND $P_{GRAVITY}$

$$P_{GR} = -\partial U_G/\partial V$$ w/ U_G = GRAV POT. E.

USE TECHNIQUE FROM PHYS 3344:
COMPUTE GRAV PE FROM SPHERE w/ MASS SHELL WRAPPED AROUND.

$$dU_G = -G\,\frac{\left(\frac{4}{3}\pi \rho r^3\right)\cdot 4\pi\rho r^2 dr}{r}$$

$$= -\frac{(4\pi)^2 G\rho^2 r^4 dr}{3}$$

w/ ρ = MASS DENSITY
ASSUME CONST.

$$U_G = -\frac{(4\pi)^2 G\rho^2}{3}\int_0^R r^4\,dr = -\frac{(4\pi)^2}{15} G\rho^2 R^5$$

BUT $\frac{4}{3}\pi\rho R^3 = M = N m_n$ w/ m_n = NUCLEON MASS.

#3.

$$U_G = -\frac{3}{5}\left(\frac{4\pi}{3}\right)^{1/3} G(Nm_n)^2 V^{-1/3}$$

$$\Rightarrow P_G = -\partial U_G/\partial V = -\frac{1}{5}\left(\frac{4\pi}{3}\right)^{1/3} G(Nm_n)^2 V^{-4/3}$$

$-$ SIGN MEANS P_G INWARD.

WHEN DO 2 PRESSURES BALANCE?

$$\frac{1}{5}\left(\frac{4\pi}{3}\right)^{1/3} G(Nm_n)^2 V^{-4/3} = \frac{\hbar^2}{15 m_e}\pi^3\left(\frac{3Ne}{\pi}\right)^{5/3} V^{-5/3} \quad (1)$$

NOW $R_{eq} = \left(\frac{3}{4\pi}\right)^{1/3} V^{1/3}$

SO FIND $V^{1/3}$ FROM EQ (1)

$$V^{1/3} = \frac{\hbar^2 \pi^3}{G m_e m_n^2} \cdot \frac{1}{3} \, 2^{5/3} \left(\frac{3}{4\pi}\right)^{7/3} N^{-1/3}$$

$$= \frac{\hbar^2}{G m_e m_n^2} \, \frac{2}{3} \, 2^{2/3} \cdot \pi \left(\frac{9}{16}\right)\left(\frac{3}{4\pi}\right)^{1/3} N^{-1/3}$$

$$= \frac{\hbar^2}{G m_e m_n^2} \cdot \frac{3\pi}{8} \, 2^{2/3} \, \frac{3^{1/3}}{2^{2/3} \cdot \pi^{1/3}} N^{-1/3}$$

$$= \frac{\hbar^2}{G m_e m_n^2} \, \frac{3\pi}{8} \, \frac{3^{1/3}}{\pi^{1/3}} N^{-1/3}$$

#3.

So, $R_{EQ} = \left[\frac{\hbar^2}{G m_e m_n^2}\right] N^{-1/3} \left(\frac{3}{4\pi}\right)^{1/3} \frac{3\pi}{8} \left(\frac{3}{\pi}\right)^{1/3}$

$$= \left[\quad\right] N^{-1/3} \left[\frac{9 \cdot 3^3 \pi^3}{4\pi^2}\right]^{1/3} \frac{1}{8}$$

$$R_{EQ} = \frac{\hbar^2}{G m_e m_n^2} \cdot N^{-1/3} \left[\frac{9\pi}{4}\right]^{1/3} \frac{3}{8}$$

$$\boxed{R_{EQ} = \frac{3}{8} \left[\frac{9\pi}{4}\right]^{1/3} \frac{\hbar^2}{G m_e m_n^2} N^{-1/3}}$$

$$\boxed{\begin{aligned} R_{EQ} &\simeq 1.76 \times 10^7 \text{ m} \\ R_{EQ} &\cong 1.8 \times 10^4 \text{ km} \end{aligned}}$$

w $N = 10^{57}$
$m = 0.9\, m_\odot$

#4 (a) FROM PROB. #3.

$$R_{eq} = \tfrac{3}{8}\left(\frac{9\pi}{4}\right)^{1/3} \frac{\hbar^2}{G m_n^3} N^{-1/3}$$

$$= \frac{m_e}{m_N}\left(\frac{1.8}{0.9}\right)^{-1/3} R_{EQ}(\text{WHITE DWARF})$$

$$\simeq \frac{1}{1800} R_{EQ}(\text{WHITE DWARF})$$

$$\simeq \frac{1.76 \times 10^7 \text{ m}}{1800}$$

$$\boxed{R_{EQ} \simeq 9.8 \times 10^3 \text{ m} \simeq 10 \text{ km}}$$

(b) $$\bar{\rho} = M/V = \frac{1.8 \times 2\times 10^{30} \text{ kg.}}{\frac{4}{3}\pi\,(10^4)^3 \text{ m}^3}$$

$$\boxed{\begin{aligned}\bar{\rho} &\simeq 8.6 \times 10^{17} \text{ kg/m}^3 \text{ !}\\ &\simeq 8.6 \times 10^{14} \text{ g/cm}^3 \text{ !}\end{aligned}}$$

$$\rho_{Pb} \simeq 11.35 \text{ g/cm}^3$$

$$\boxed{\therefore\ \rho_{NS}/\rho_{COCONUT} \simeq 10^{14}}$$

#5. SEE PROB #2, EQ (7):

$$\boxed{E_F = \hbar c \left(3\pi^2 n_e\right)^{1/3}}$$ MASSLESS FERMIONS

w/ n_e = # DENSITY OF MASSLESS FERMIONS.