Welcome Back to
Physics 1308

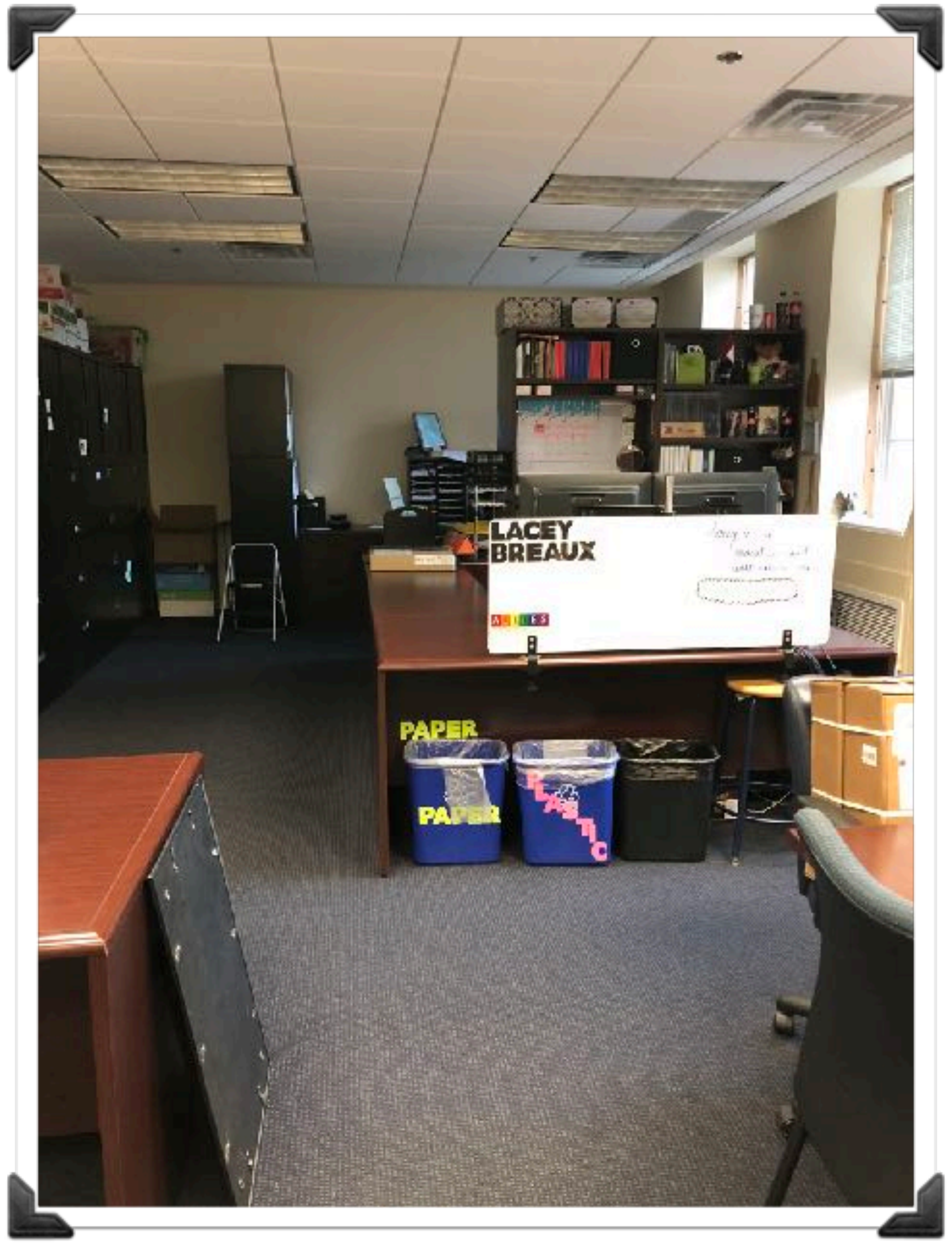
Gauss' Law

Johann Carl Friedrich Gauss
1777 - 1855

Announcements

- **Assignments for Tuesday, September 11th:**
  - Reading: Chapter 24.1 - 24.3
  - Watch Video: https://youtu.be/hloHDhI8BdM — Lecture 6 - The Electric Field and Energy Concepts
- **Homework 3 Assigned - due before class on Tuesday, September 11th.**
- Dr. Cooley's office hours are canceled for tomorrow - Friday, September 7th. Please email her if you need any assistance.

Homework/Quiz Return




KEEP
CALM
AND
ITS
QUIZ TIME

Review Question 1

An electron traveling horizontally to the right enters a region where a uniform electric field is directed downward. What is the direction of the force exerted on the electron once it has entered the field?

A) upward
B) downward
C) to the right
D) to the left
E) The force is zero newtons.

Key Concepts

Electric Flux

$$\Phi = \int \vec{E} \cdot d\vec{A} \quad \text{(total flux).}$$

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad \text{(net flux).}$$

An inward piercing field is negative flux. An outward piercing field is positive flux. A skimming field is zero flux.

Gauss' Law

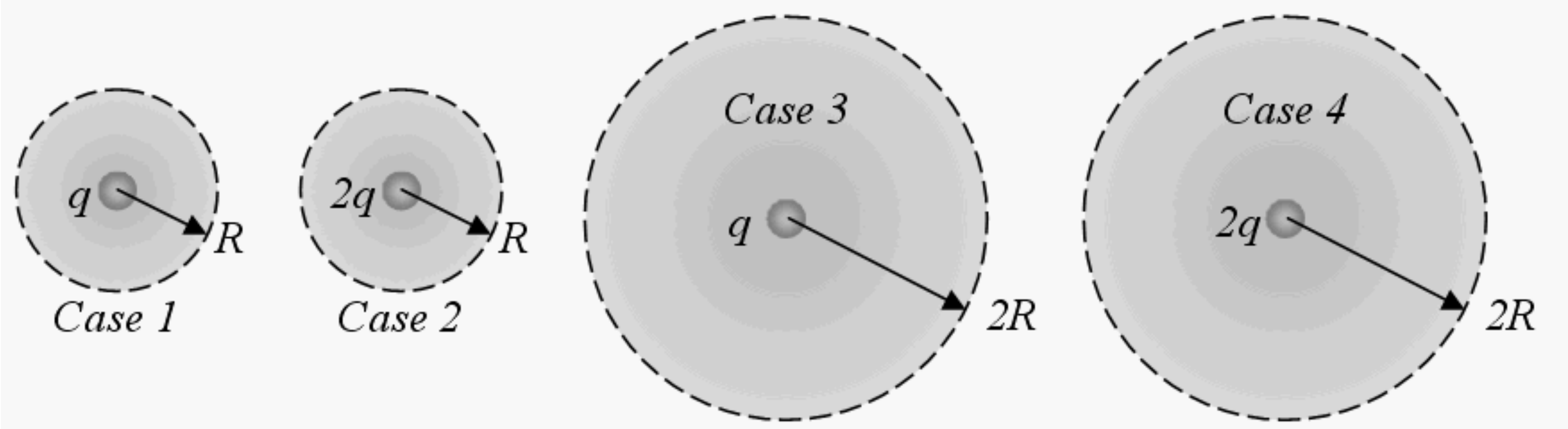
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \quad \text{(Gauss' law).}$$

Isolated Conductors

If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

Question 1

Consider the five situations shown. Each one contains either a charge q or a charge 2q. A Gaussian surface surrounds the charged particle in each case. Considering the electric flux through each of the Gaussian surfaces, which of the following comparative statements is correct?



a) $\Phi_2 = \Phi_4 > \Phi_1 = \Phi_3$

b) $\Phi_1 = \Phi_3 > \Phi_2 = \Phi_4$

c) $\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3$

d) $\Phi_3 = \Phi_4 > \Phi_2 = \Phi_1$

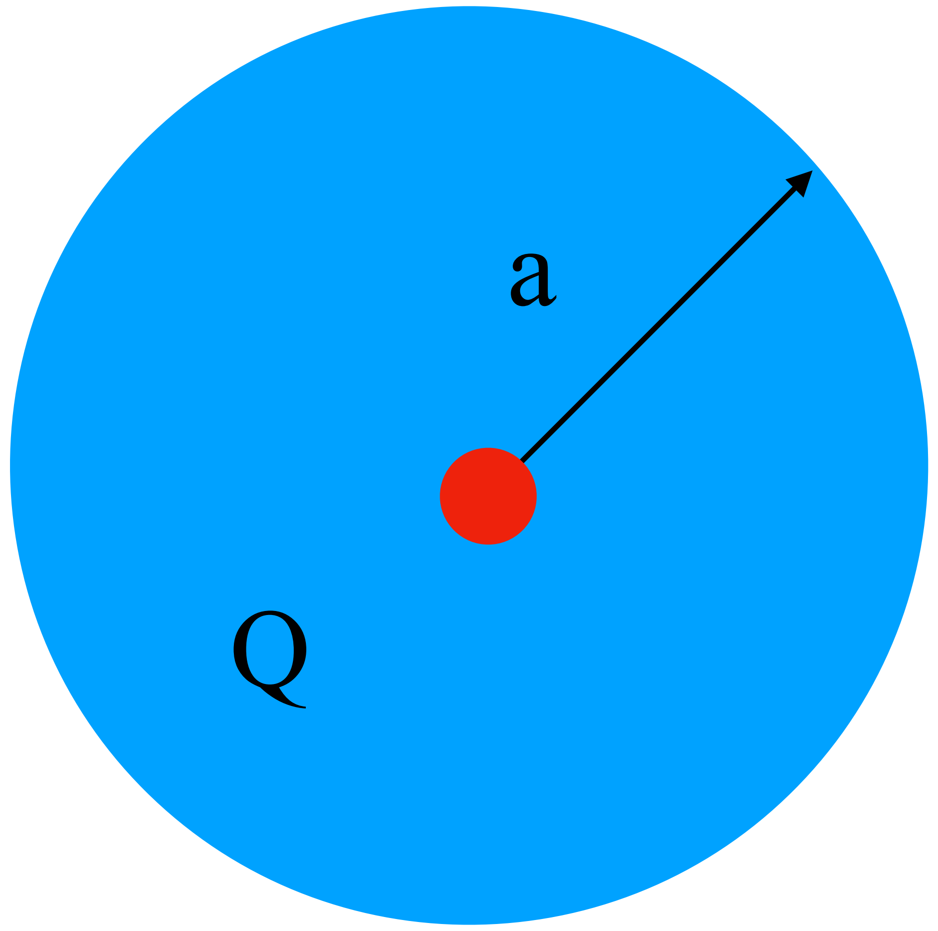
e) $\Phi_4 > \Phi_3 > \Phi_2 > \Phi_1$

Question 2

A large sheet of electrically insulating material has a uniform charge density . Let’s compare the electric field produced by the insulating sheet with that produced by a thin metal (electrically conducting) slab with /2 charge density distributed on one large surface of the slab and /2 distributed over the surface on the opposite side. How does the electric field at a distance d from each surface compare?

A) The electric field near the insulating sheet is four times that near the conducting slab.
B) The electric field near the insulating sheet is twice that near the conducting slab.
C) The electric field near the insulating sheet is the same as that near the conducting slab.
D) The electric field near the insulating sheet is one half that near the conducting slab.
E) The electric field near the insulating sheet is one fourth that near the conducting slab.

Instructor Problem: Spherically Symmetric Charge Distribution



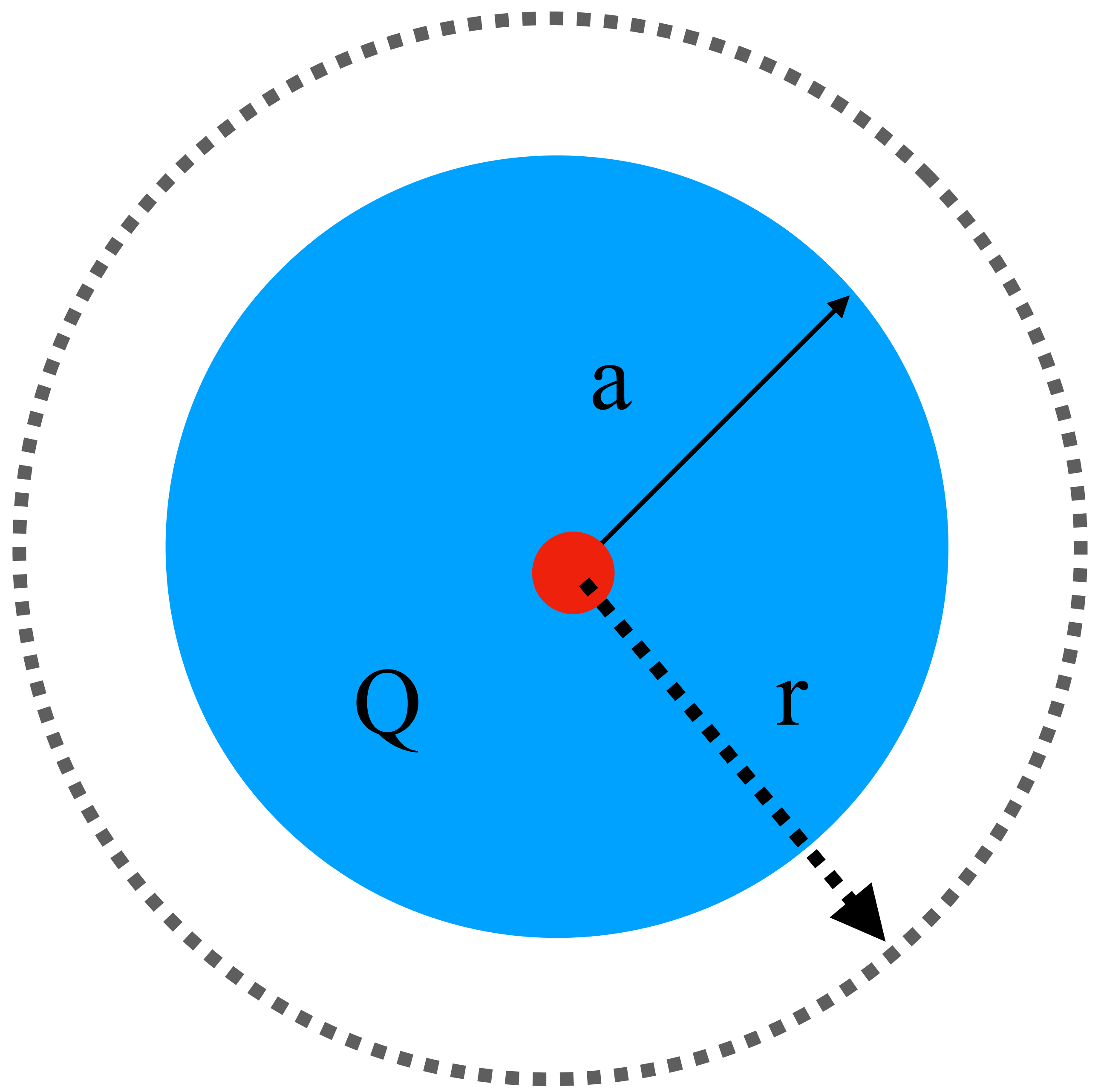
An insulating sphere of radius a has an uniform volume charge density ρ and carries a total positive charge Q.

A) Calculate the magnitude of the electric field at a point outside the sphere.
B) Find the electric field at a point inside the sphere.

A) First we should choose our Gaussian surface and sketch the problem.

Next — use Gauss' Law to find the E-field.

$$\Phi = \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$



The E-field is normal to the surface everywhere. Thus ,

$$\epsilon_0 \oint E dA = q_{enc}$$

By symmetry E has the same value everywhere on the surface.
Thus, we can take it outside the integral.

$$\epsilon_0 E \oint dA = q_{enc}$$

Next we need to calculate the total area of the sphere and the total charge enclosed.

$$\epsilon_0 E(4\pi r^2) = Q \quad \longrightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0 r^2} \quad \text{for r > a}}$$

Note that — this is exactly Coulomb's Law!

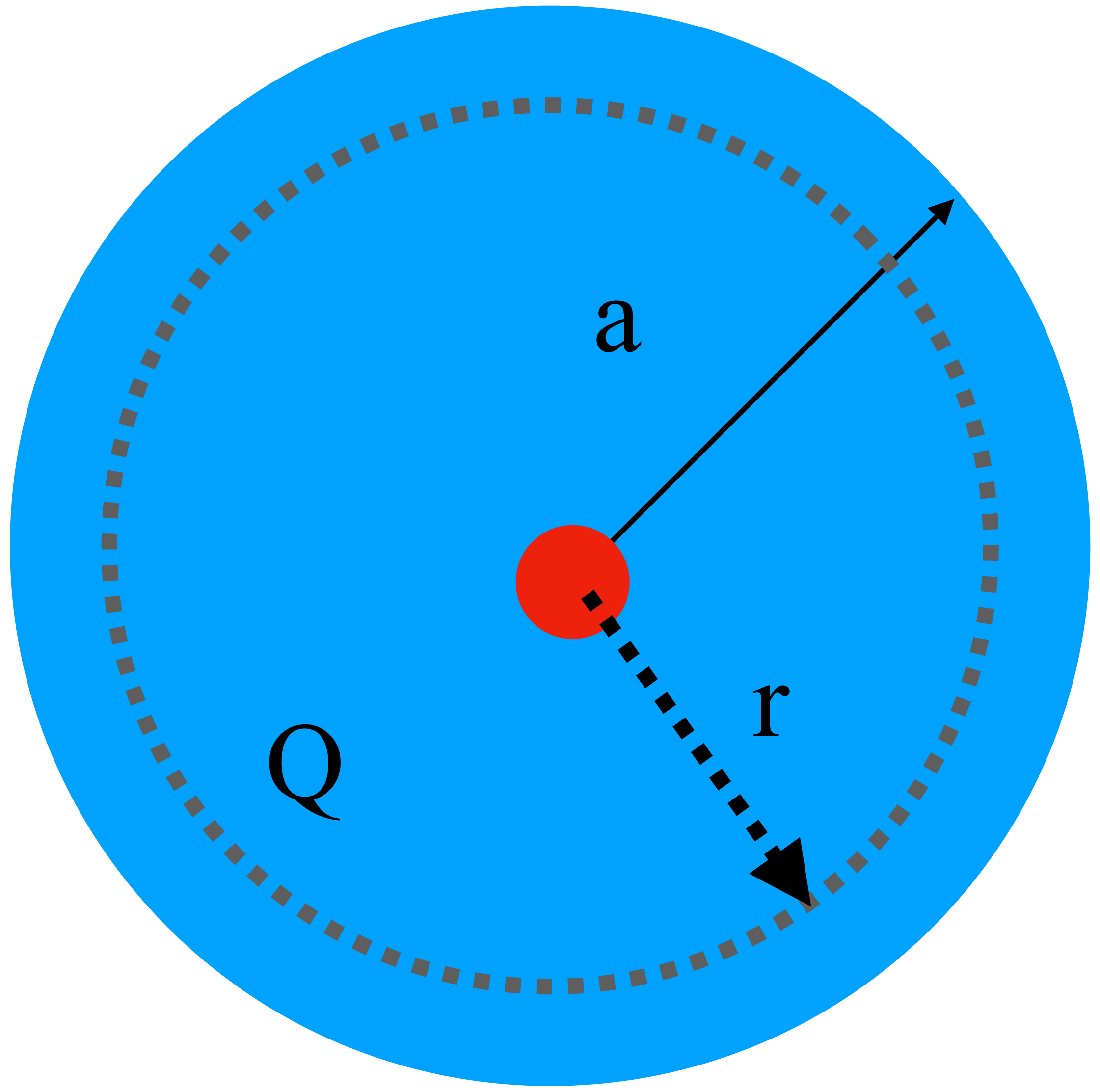
B) First we should choose our Gaussian surface and sketch the problem.

Again we will use Gauss' Law.

$$\Phi = \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

In this case, we need to calculate $q_{enc.}$ We will take advantage of the definition of volume density.

$$q_{enc} = \rho V' = \rho(\frac{4}{3}\pi r^3)$$



The E-field is still normal to the surface everywhere and it is still constant.

$$\epsilon_0 \oint E dA = \frac{4}{3}\rho\pi r^3$$

$$\epsilon_0 E \oint dA = \frac{4}{3}\rho\pi r^3$$

$$\epsilon_0 E(4\pi r^2) = \frac{4}{3}\rho\pi r^3$$

$$E = \frac{\rho r}{3\epsilon_0}$$

Recall definition of ρ

$$\rho = \frac{Q}{V} = \frac{Q}{4/3\pi a^3}$$

Substitute

$$E = \frac{3Q}{4\pi a^3}\frac{r}{3\epsilon_0} \longrightarrow \boxed{E = \frac{Qr}{4\pi\epsilon_0 a^3} \text{ for } r < a}$$

Compare our results:

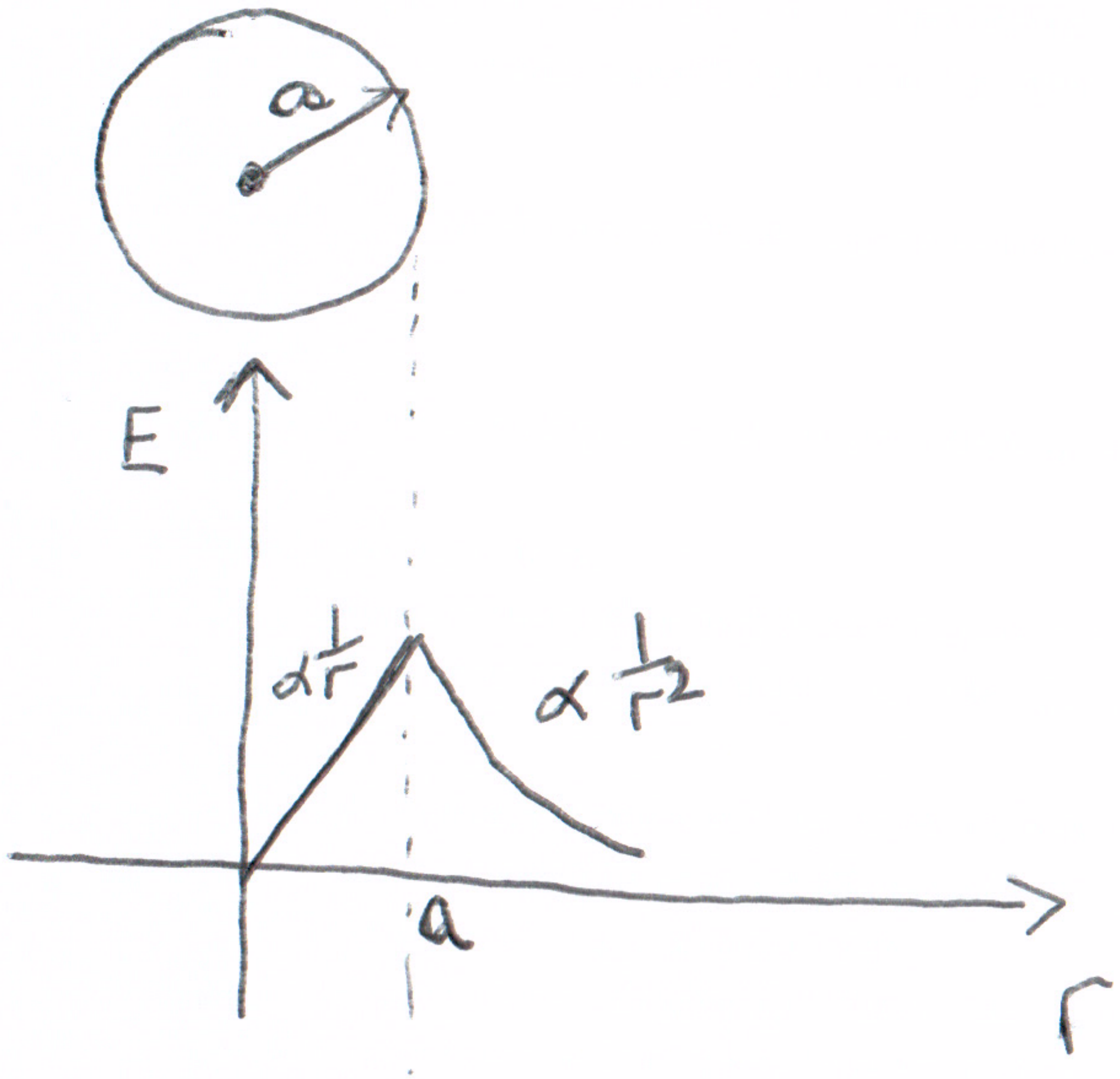
$$\longrightarrow \boxed{E = \frac{Q}{4\pi\epsilon r^2} \quad \text{for r > a}}$$

$$\longrightarrow \boxed{E = \frac{Qr}{4\pi\epsilon a^3} \quad \text{for r < a}}$$

What happens if r = a?

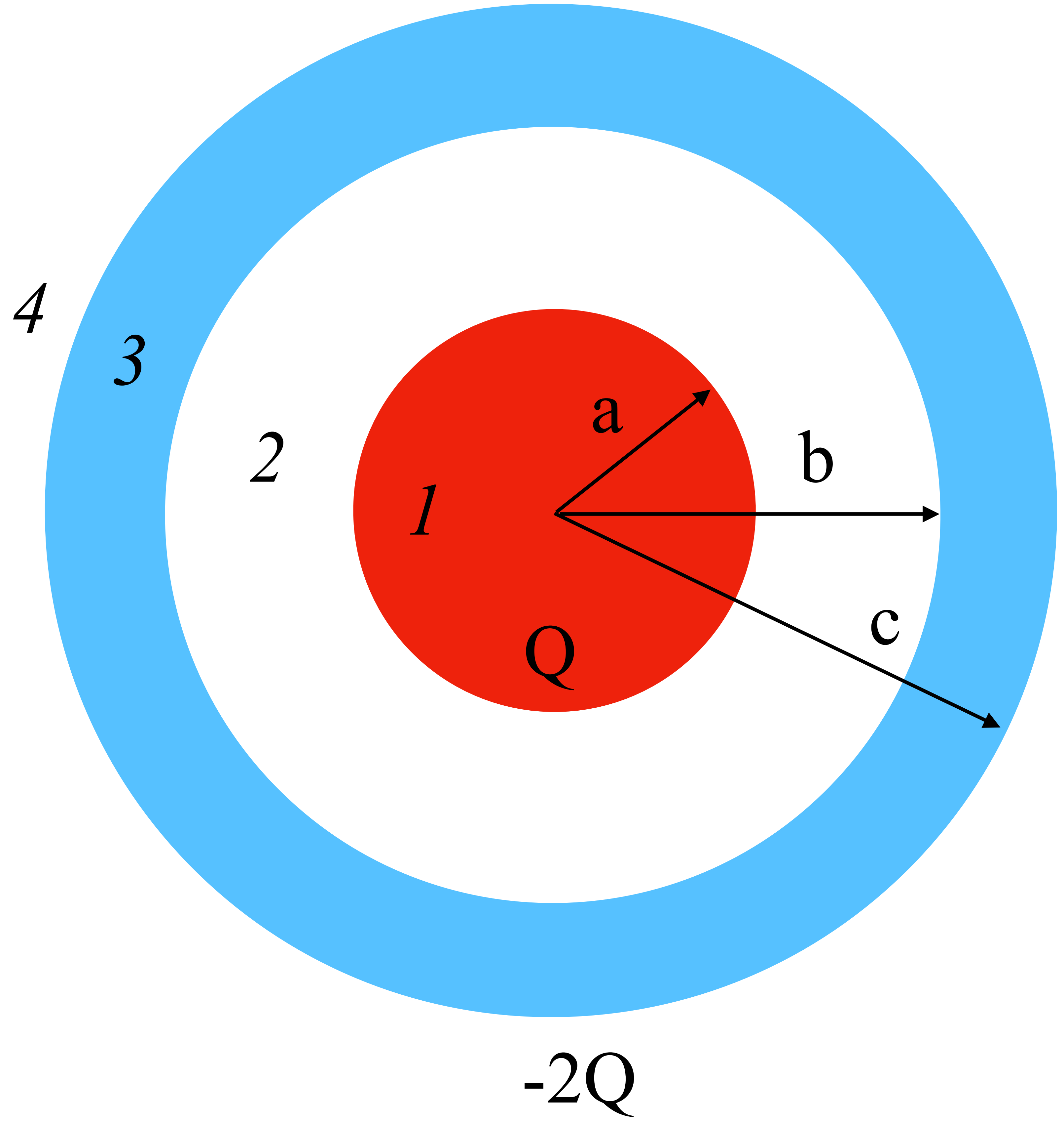
$$E = \lim_{r \to a} \left(\frac{Q}{4\pi\epsilon a^2} \right)$$

$$E = \lim_{r \to a} \left(\frac{Qa}{4\pi\epsilon a^3} \right) = \lim_{r \to a} \left(\frac{Q}{4\pi\epsilon a^2} \right)$$



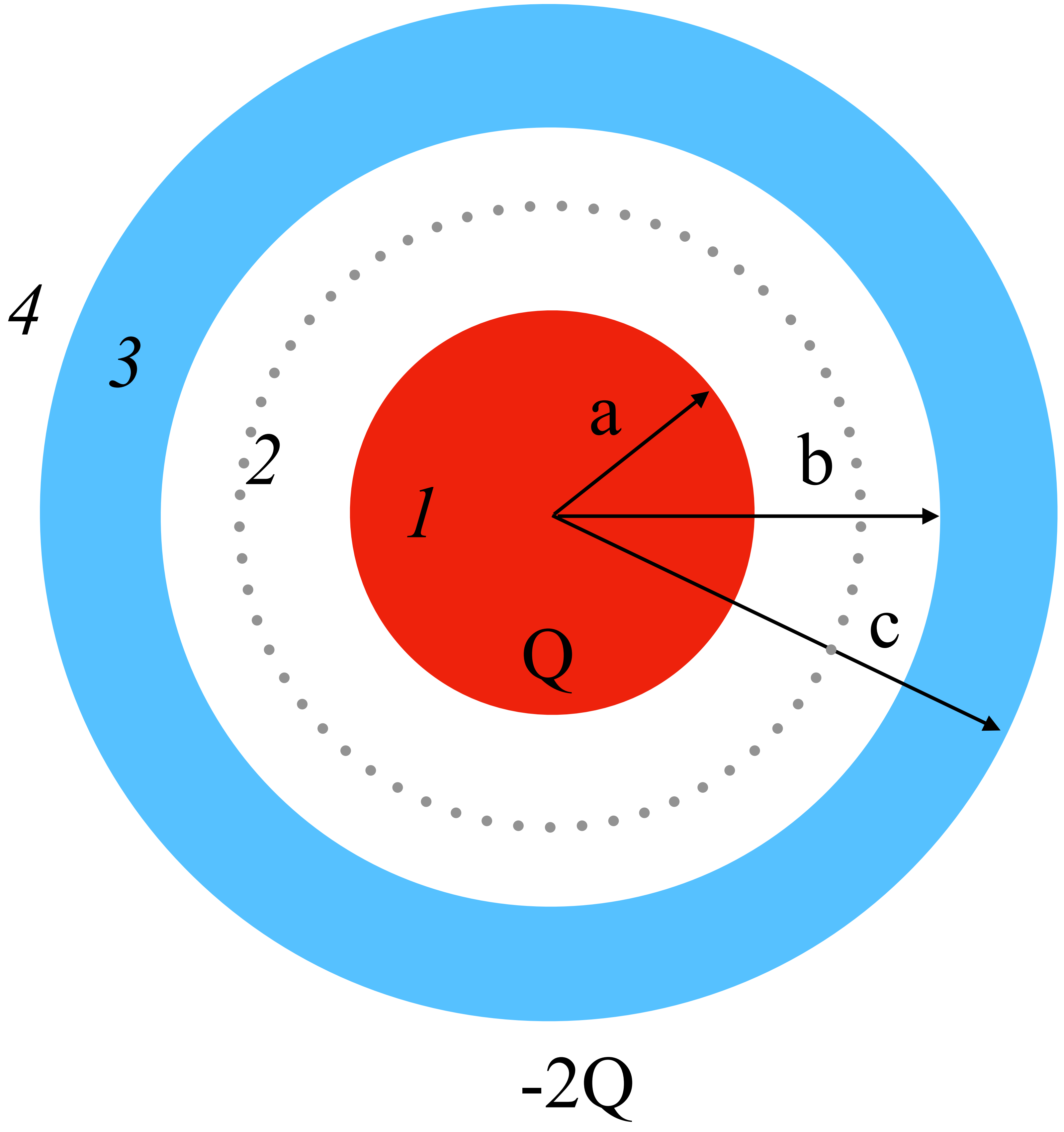
Student Problem: A Sphere Inside a Spherical Shell

A solid insulating sphere of radius a carries a net positive charge Q uniformly distributed throughout its volume. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and carries a net charge $-2Q$. Using Gauss' law, find the electric field in regions 1, 2, 3 and 4.



This is a little different than the Instructor Problem in that the charged sphere now resides in a spherical shell. How does that affect the electric field of the sphere?

The interior insulating sphere has the charge uniformly distributed throughout the sphere. The conducting shell has the charge distributed uniformly on the surfaces. Thus, the system has spherical symmetry and we can use Gauss' Law.



Region 2 (a < r < b):

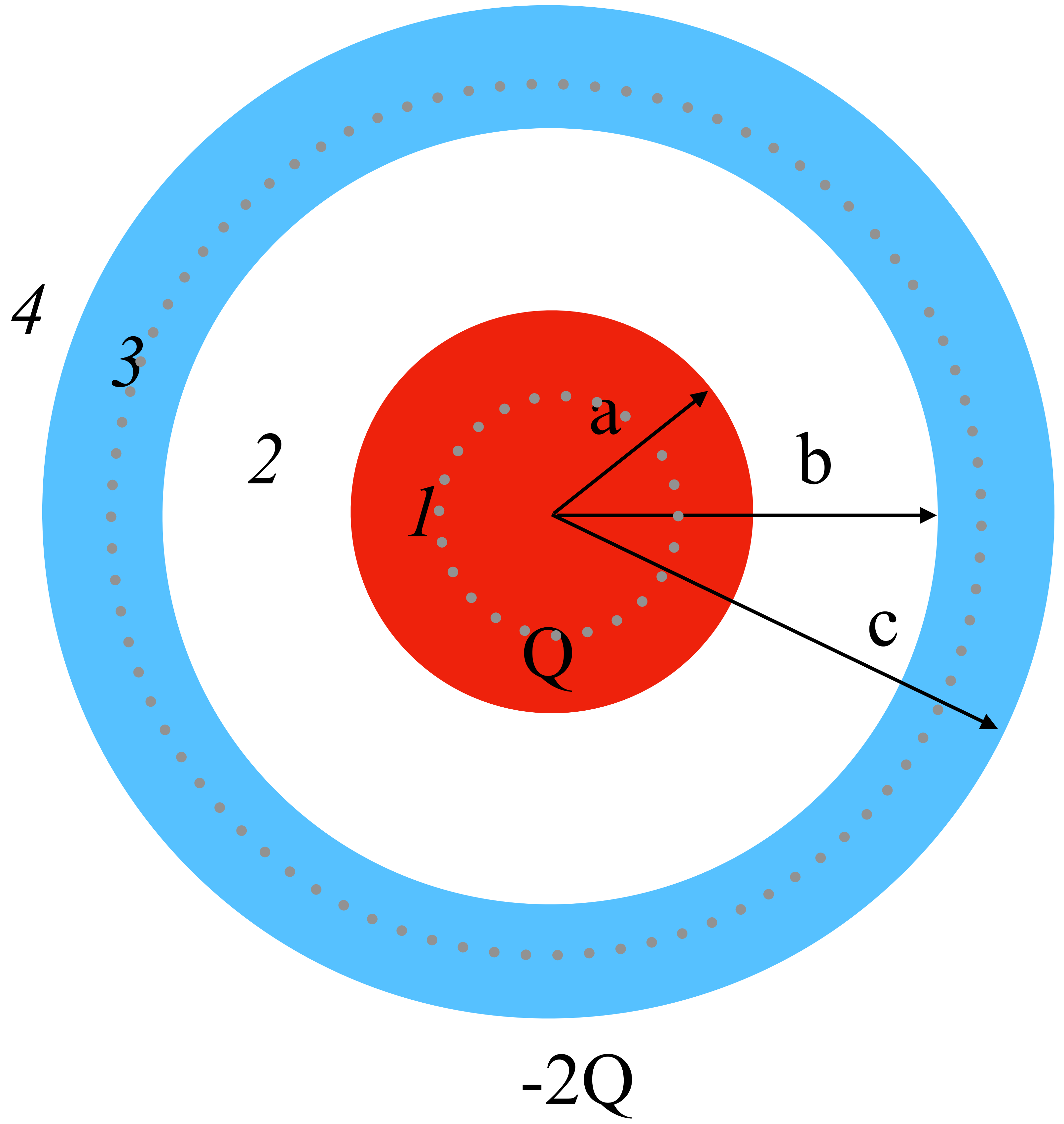
The charge on a conducting shell creates a zero electric field in the region b < 0. Thus, the shell has no effect on the sphere. Our problem simplifies to the Instructor Problem — which we just solved!

$$\longrightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0 r^2} \quad \text{for r > a}}$$

Region 1 (r < a):

Because the conducting shell has no effect on the field inside the shell, it also has no effect on the field inside the sphere. Thus, region 1 also simplifies to the Instructor Problem, which we just solved!

$$\longrightarrow \boxed{E = \frac{Qr}{4\pi\epsilon_0 a^3} \text{ for r < a}}$$



Region 3 (c < r < b):

In this region, the electric field must be zero because the electric field inside a conductor is in equilibrium.

$$\boxed{E = 0}$$

Region 4 (r > c):

First we should choose our Gaussian surface and sketch the problem.

Next — use Gauss' Law to find the E-field.

$$\Phi = \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

The E-field is normal to the surface everywhere. Thus ,

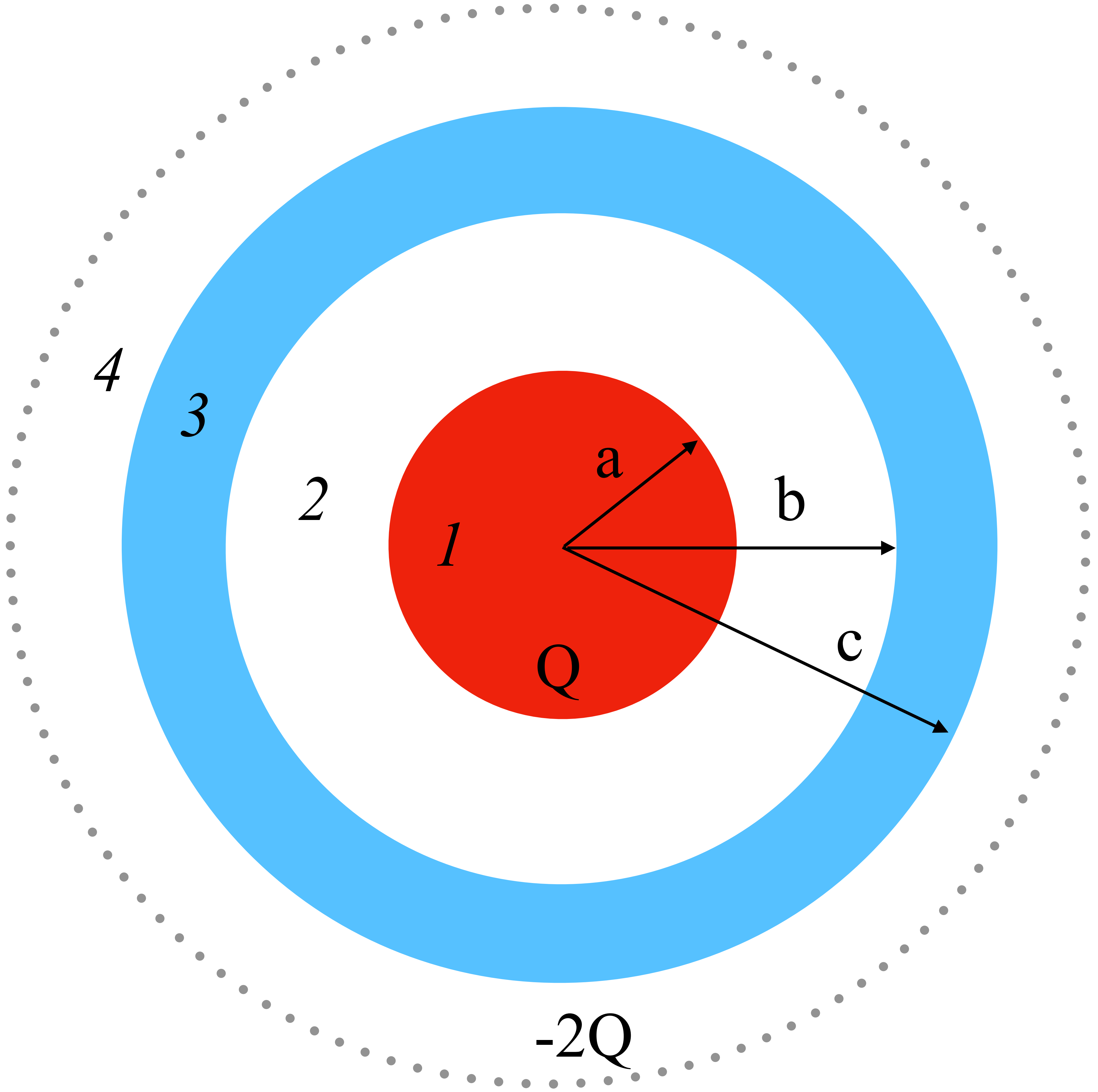
$$\epsilon_0 \oint E dA = q_{enc}$$

By symmetry E has the same value everywhere on the surface. Thus, we can take it outside the integral.

$$\epsilon_0 E \oint dA = q_{enc}$$

Next we need to calculate the total area of the sphere and the total charge enclosed.

$$\epsilon_0 E(4\pi r^2) = -2Q \quad \longrightarrow \boxed{E = -\frac{Q}{2\pi\epsilon_0 r^2}}$$



The End for Today!