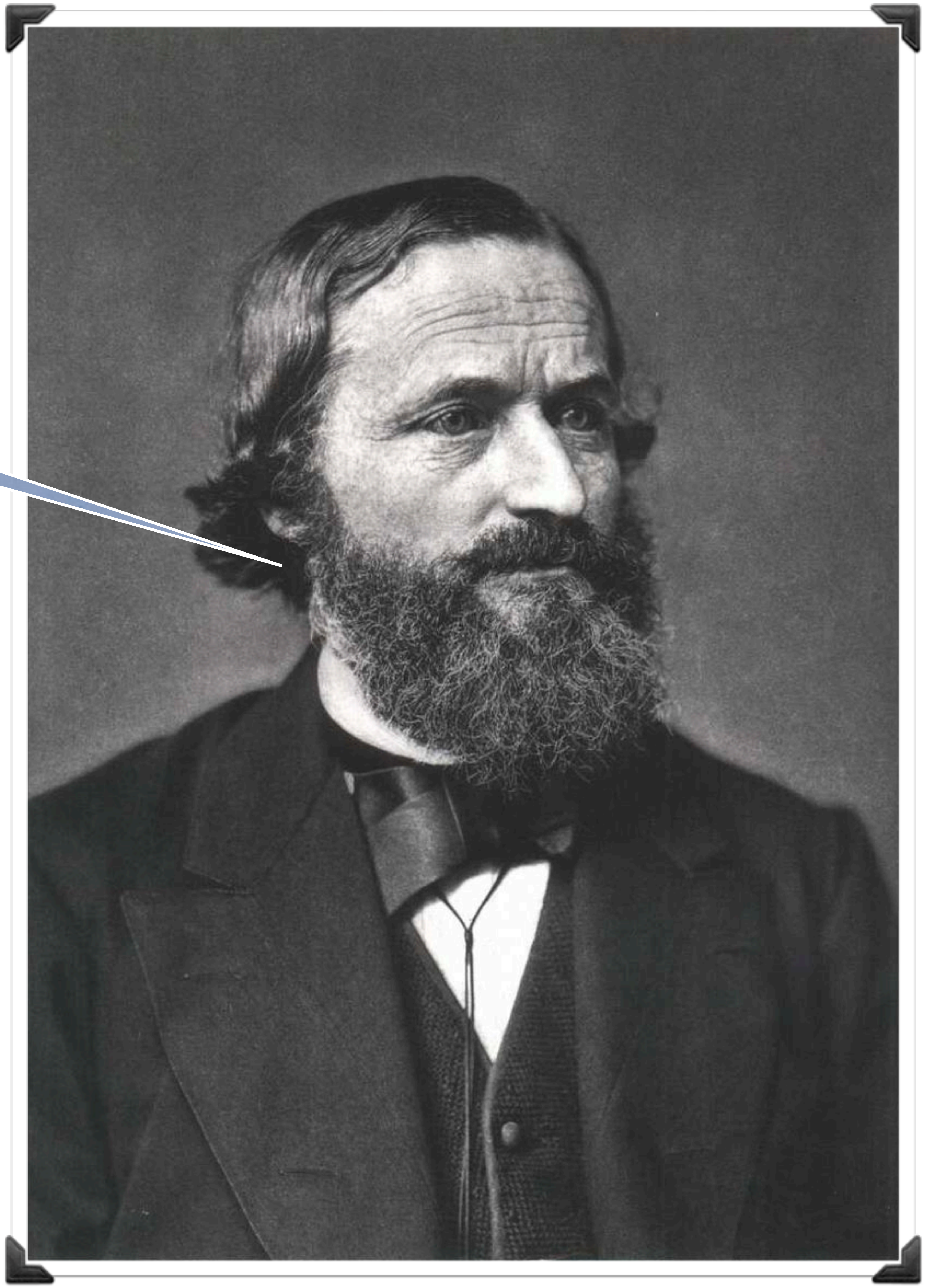
Welcome Back to
Physics 1308

Circuits



Gustav Robert Kirchhoff
12 March 1824 – 17 October 1887

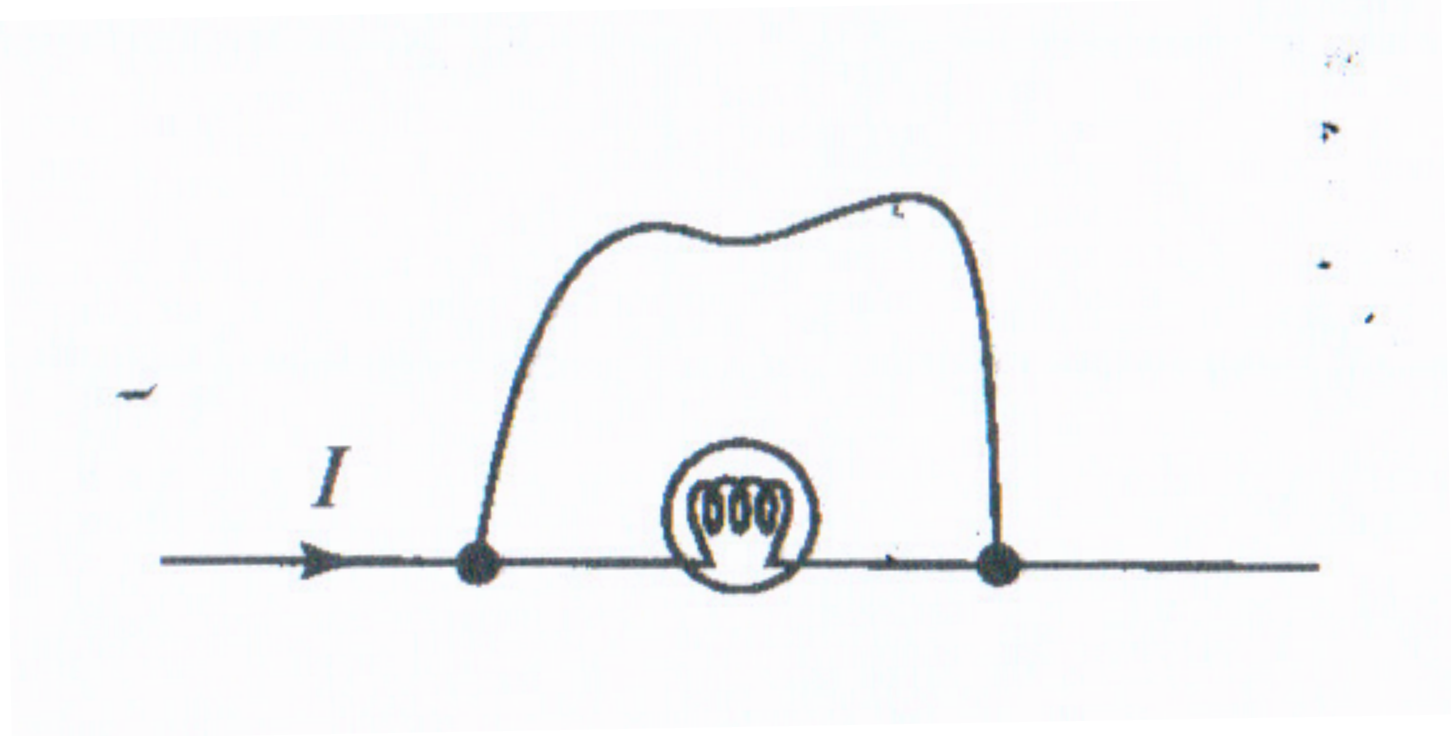
Announcements

- **Assignments for Thursday, October 18th:**
  - Reading: Chapter 28.1 - 28.2, 28.4
  - Watch Video: https://youtu.be/39VKt4Cc5nU— Lecture 14 - The Magnetic Force and Field
- **Homework 8 Assigned - due before class on Tuesday, October 23rd.**
- **Midterm Exam 2 will be in class on Tuesday, October 16th. It will explicitly cover chapters 24 - 26. However, these chapters build on the previous material and you will be expected to apply all concepts/information from the beginning of class to this point to any problem or question that you encounter on the exam. There will be a seating assignment for the exam.**
- **Dr. Cooley will be out of the office the week of October 15th. Her office hours are cancelled that week. If you need to reach her, please send an email.**


KEEP
CALM
AND
ITS
QUIZ TIME

Review Question 1

Charge flows through a lightbulb. Suppose a wire is connected across the bulb as shown. When the wire is connected



A) all of the charge continues to flow through the bulb.

B) half the charge flows though the wire, the other half continues through the bulb.

C) all of the charge flows though the wire.

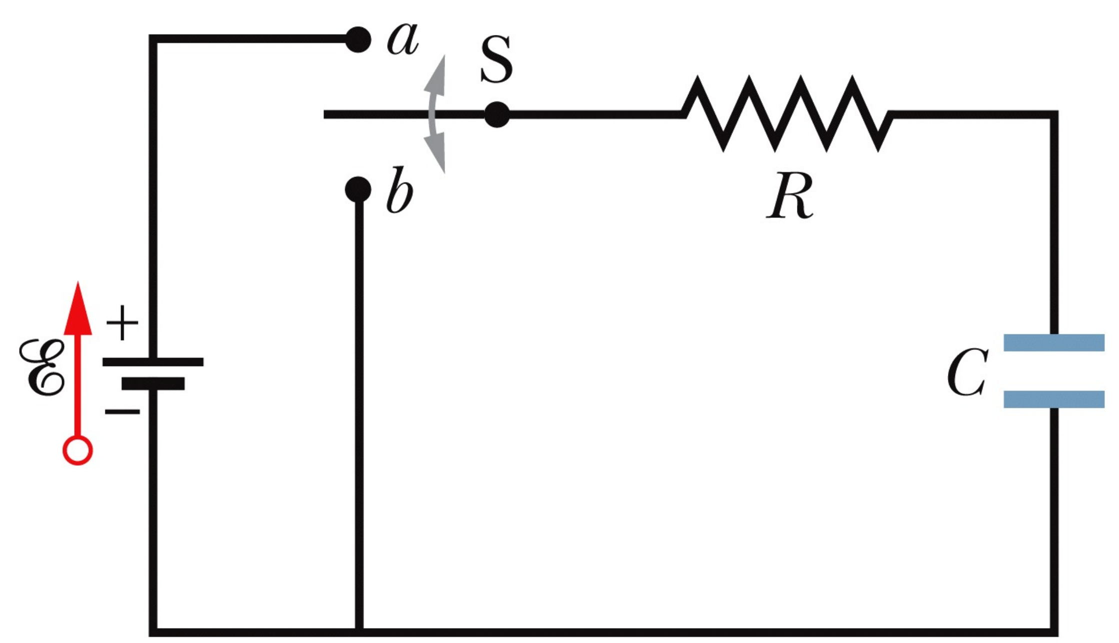
D) none of the above.

The wire had essentially zero resistance and is in parallel with the light bulb. Thus, all charge flows through the wire.

Key Concepts

Charging a Capacitor:

To charge an the capacitor in the RC circuit shown, we close switch S on point a.



The charge on the capacitor increases according to

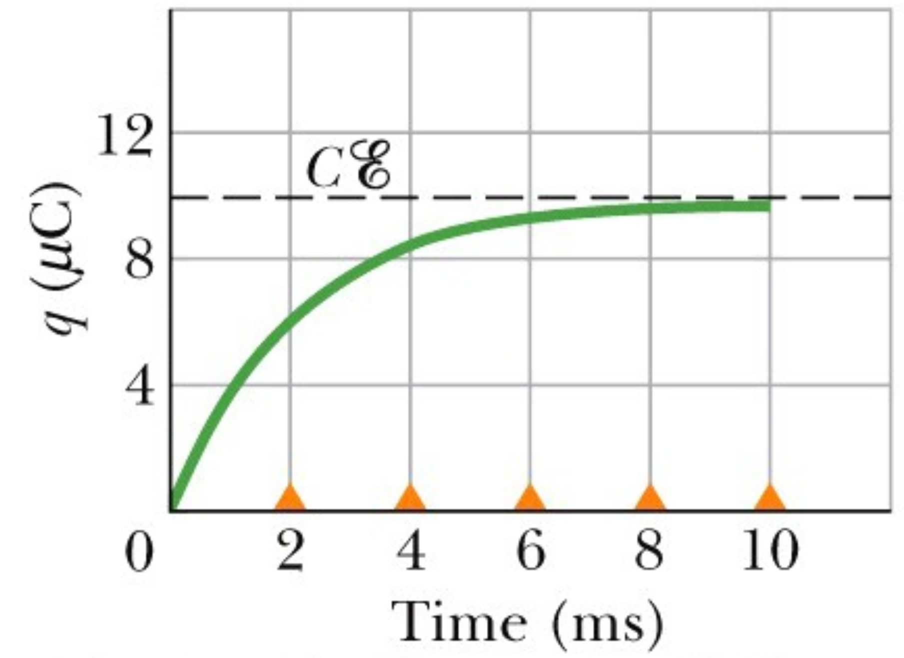
$$q = C\mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor).}$$

$C\mathscr{E} = q_0$ is the equilibrium (final) charge and $RC = \tau$ is the capacitive time constant of the circuit.

During the charging, the current and voltage are

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC} \quad \text{(charging a capacitor).}$$

$$V_C = \frac{q}{C} = \mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor).}$$



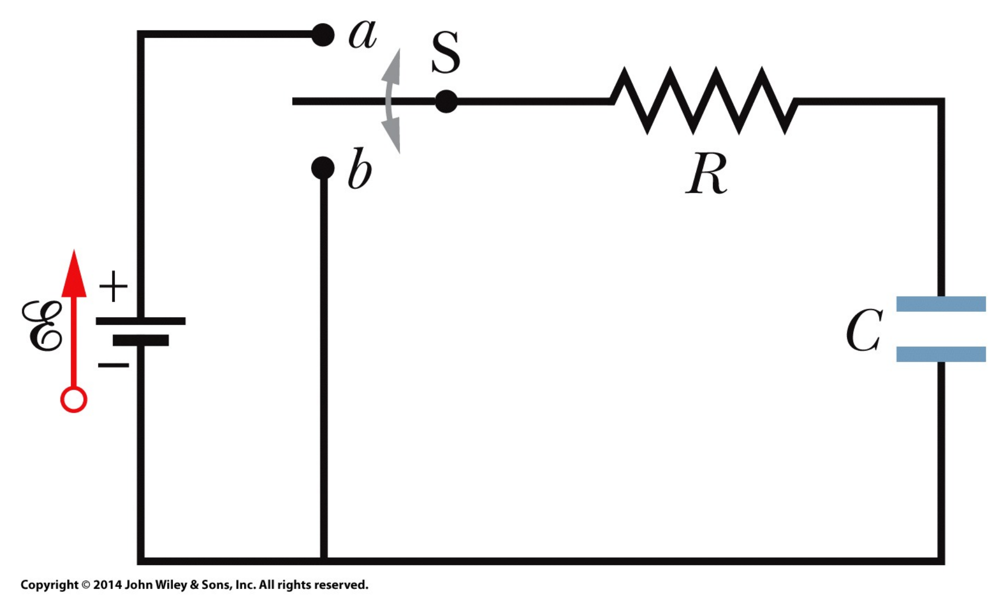
capacitive time constant:

$$\tau = RC \quad \text{(time constant).}$$

Key Concepts

Discharging a Capacitor:

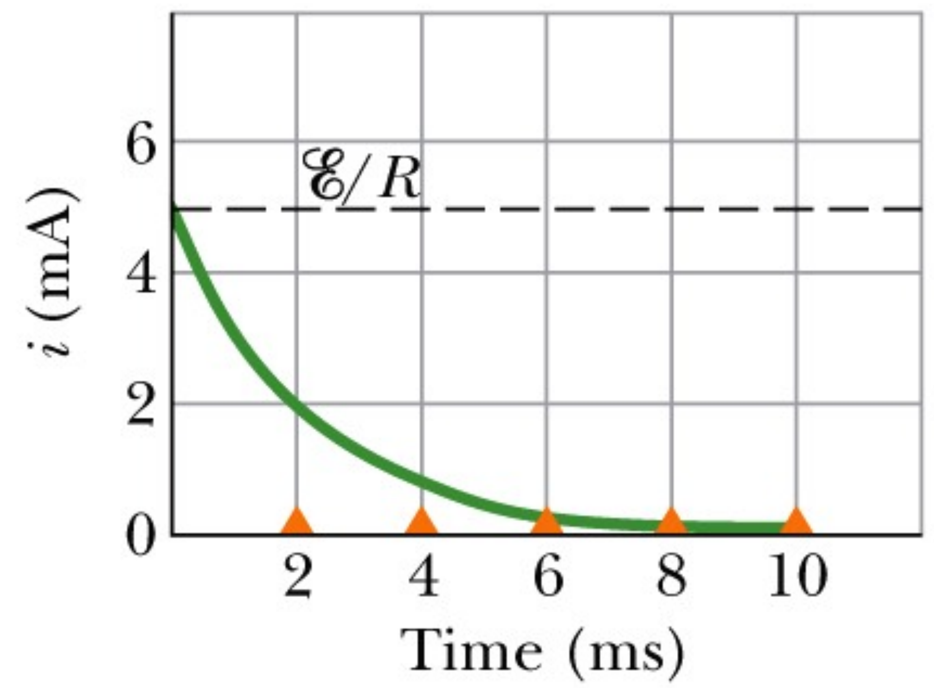
To dicharge an the capacitor in the RC circuit shown, we open switch S on point a.



When a capacitor discharges through a resistance R, the charge on the capacitor decays according to

$$q = q_0 e^{-t/RC} \quad \text{(discharging a capacitor),}$$

where q_0 ($=CV_0$) is the initial charge on the capacitor.



During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right) e^{-t/RC} \quad \text{(discharging a capacitor).}$$

A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

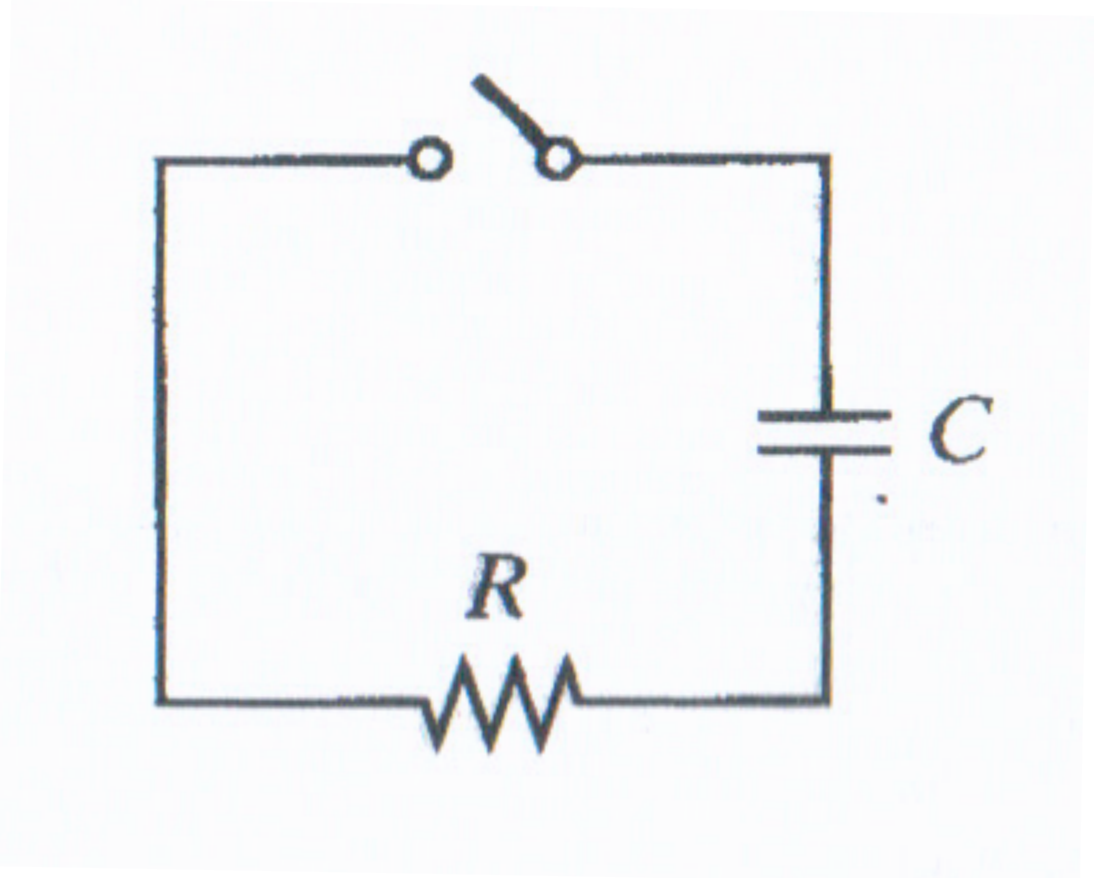
Question 1

A simple circuit consists of a resistor R, a capacitor C charged to a potential V_0, and a switch that is initially open but then thrown closed. Immediately after the switch is thrown closed, the current in the circuit is

A) V_0/R

B) zero.

C) need more information.



Immediately after the switch is thrown closed, the current is V_0/R. It decreases from this value to zero exponentially, with a time constant equal to RC.

Question 2

What effect, if any, does increasing the battery emf in an RC circuit have on the time to charge the capacitor?

A) The charging time will decrease because the rate of charge flowing to the plates will increase.

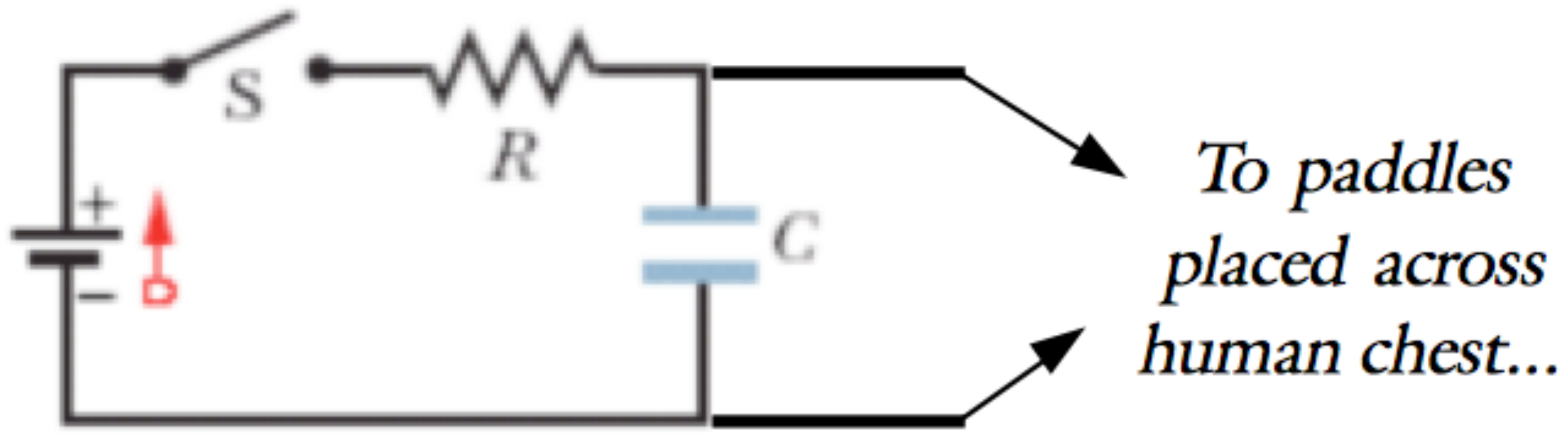
B) The charging time will decrease because the rate of charge flowing to the plates will decrease.

C) The charging time will not change because the charging time does not depend on the battery emf.

D) The charging time will increase because the emf is increased.

E) The charging time will decrease because potential difference across the plates will be larger.

Instructor Problem: Charging a Defibrillator



Simple model of a "defibrillator" energy storage circuit.

The energy storage part of the defibrillator can be modeled as an RC Circuit (see lower image). It is powered by a 2.5 kV potential supplied by an energy storage device (e.g. a battery). The capacitor needs to be able to store a maximum charge of 150 μC. Before the switch is closed, the capacitor stores no charge.

a) What capacitance is needed to accomplish this goal?
b) What resistance is required in order to achieve 99% of maximum charge in 3.0 seconds?
c) After the switch is closed, at what time, t', is the voltage across the resistor equal to that across the capacitor?

Given: $\mathcal{E} = 2.5\ kV = 2.5 \times 10^3\ V$

$q_{max} = 150\ \mu C = 1.5 \times 10^{-4}\ C$

Part A:

The capacitance needed will be the one such that

$$q_{max} = C\mathcal{E}$$

Since at t = ∞ s, the voltage across the capacitor will be that supplied by the emf.

$$C = \frac{q_{max}}{\mathcal{E}}$$

$$= \frac{1.5 \times 10^{-4}\ C}{2.5 \times 10^3\ V}$$

$$C = 6.0 \times 10^{-8}\ F = 60\ nF$$

Part B:

Given: $t' = 3.0\ s$ Find: R

$V_c = 0.99\mathcal{E}$

The charging equation is

$$V_c = \mathcal{E}(1 - e^{-\tau/RC})$$

Apply to our problem. At t = 3.0 s:

$$0.99\mathcal{E} = \mathcal{E}(1 - e^{-t'/RC})$$

$$0.99 - 1 = e^{-t'/RC}$$

$$-0.01 = e^{-t'/RC}$$

$$\ln(0.01) = \frac{-t'}{RC}$$

$$R = \frac{-t'}{\ln(0.01)C} = \frac{-3.0\ s}{\ln(0.01)(6.0 \times 6.0 \times 10^{-8}\ F)}$$

$$R = 1.08 \times 10^{7}\ \Omega = 11M\Omega$$

Part C:

Find: Time t' at which $V_c = V_R$.

We know:

$$V_C = \mathcal{E}(1 - e^{-t/RC}) \quad \text{(rises over time)}$$

$$V_R = \mathcal{E}e^{-t/RC} \quad \text{(declines over time)}$$

We need to solve when these are equal.

$$V_C(t') = V_R(t')$$

$$\mathcal{E}(1 - e^{-t'/RC}) = \mathcal{E}e^{-t/RC}$$

$$1 = 2e^{-t/RC}$$

$$\frac{1}{2} = e^{-t/RC}$$

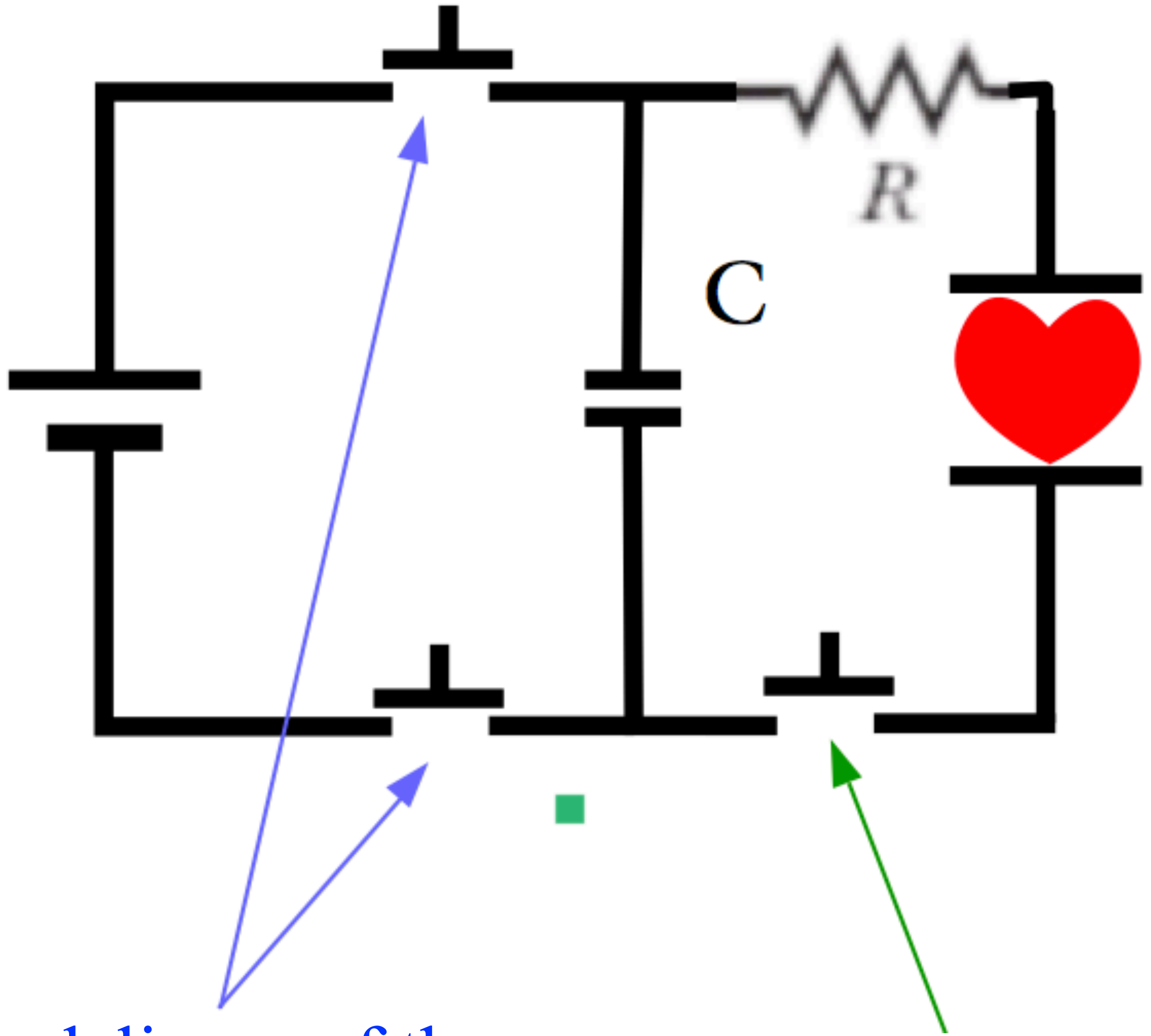
$$\ln\left(\frac{1}{2}\right) = -t/RC$$

$$t = -RC\ln\left(\frac{1}{2}\right)$$

$$= -(1.08 \times 10^7\ \Omega)(6.0 \times 10^{-8}\ F)\ln\left(\frac{1}{2}\right)$$

$$t' = 0.45\ s$$

Student Problem: Discharging a Defibrillator



To discharge the stored energy across the heart, switches to the original voltage are opened (disconnecting the capacitor from its energy source) and instead the capacitor is connected to itself, across the heart, by closing another switch. At time t=0, the voltage across the capacitor is 2.5 kV.

a) If the capacitance, C, is 60.0 nF, what resistance, R, is needed inside the circuit to deliver 99% of the stored charge in the capacitor across the heart in 0.20 s? Treat the human chest and heart as just a big resistor, and for its resistance use $R_{chest} = 9.0 \times 10^2\ \Omega$.

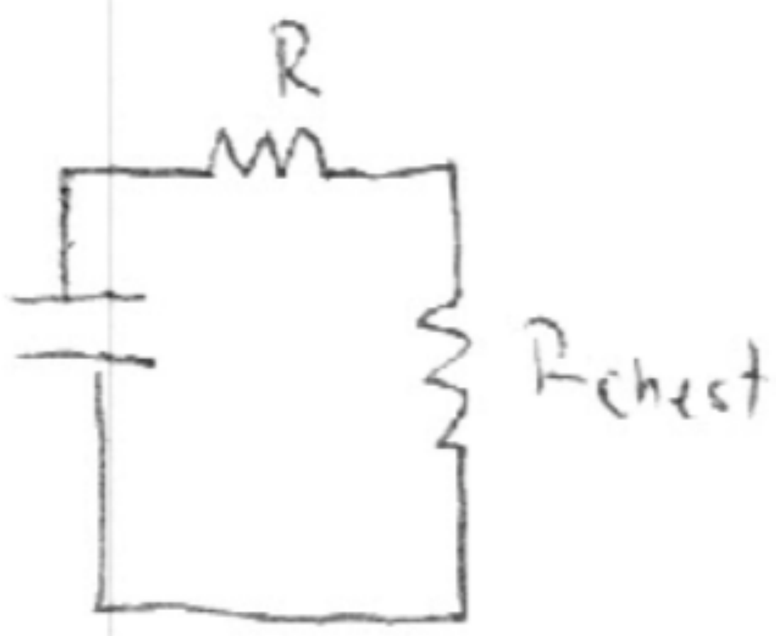
b) Just as the switch is closed and the current begins to flow through R and R_{chest}, what is the voltage across R_{chest}? Is it dangerous? As a guide, to cause internal burns the flesh has to be exposed to at least 500 V.

Part A:

Given: $R_{chest} = 900\ \Omega$

$C = 60.0\ nF = 60.0 \times 10^{-9}\ F$

$V = 2.5\ kV = 2.5 \times 10^{3}\ V$



Find:

R such that 99% of the charge is delivered across the heart in 0.2 s.

We know a couple of other things as well. At

$t = 0.02s \longrightarrow 0.01q_0$ where $q_0 = CV_0$

$t = 0\ s \longrightarrow V_0 = 2.5kV$

The capacitor is discharging so, the charge goes as

$$q = q_0 e^{-t/R_T C}$$

Since our resistors are in series: $R_T = R + R_{chest}$

$$q = q_0 e^{-t/R_T C}$$

If $q = 0.01q_0$ when $t = 0.2$ s, then:

$$0.01q_0 = q_0 e^{-t/R_T C}$$

$$ln(0.01) = -\frac{t}{R_T C}$$

$$R_T = -\frac{t}{\ln(0.01)C}$$

$$R_{chest} + R = -\frac{t}{\ln(0.01)C}$$

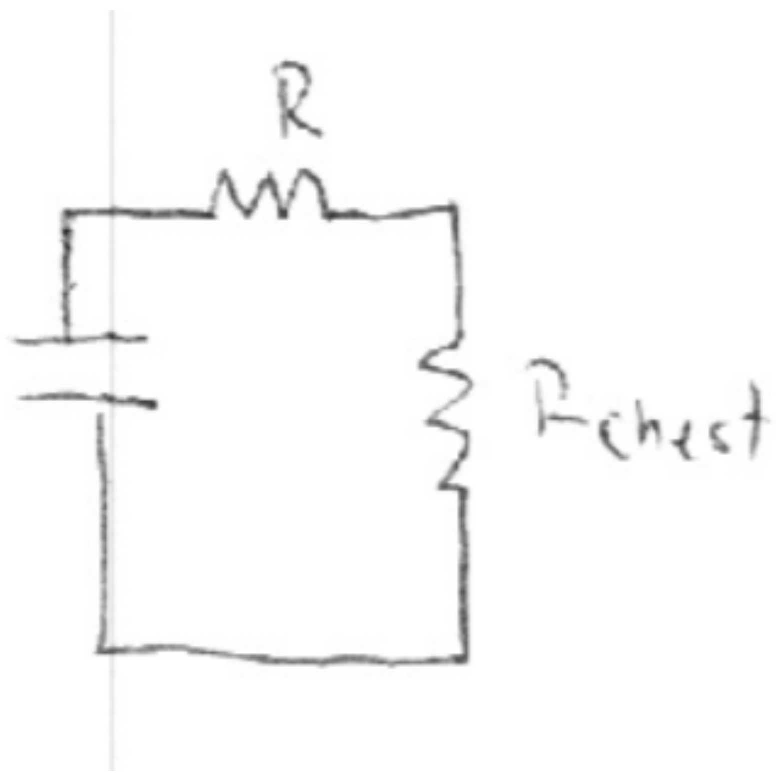
$$R = -\frac{t}{\ln(0.01)C} - R_{chest}$$

$$R = -\frac{0.2\ s}{\ln(0.01)(60 \times 10^{-9}\ F)} - 900\ \Omega \longrightarrow R = 7.2 \times 10^5\ \Omega$$

Part B:

After the switch is closed, V_c is still 2.5 kV. Since R and R_{chest} are in parallel, they share the voltage. Thus,

$$V_C = V_R + V_{chest}$$



In order to find V_{chest} we need to eliminate V_R.

$$V_R = i_R R$$

$$V_{chest} = i_{chest} R_{chest}$$

For circuits in series — we know that $i_r = i_{chest}$, thus

$$\frac{V_R}{R} = i_R = i_{chest} = \frac{V_{chest}}{R_{chest}} \longrightarrow V_R = \frac{V_{chest}}{R_{chest}} R$$

Now plug V_R into our expression for V_c.

Now plug V_R into our expression for V_c.

$$V_C = V_R + V_{chest}$$

$$= \frac{V_{chest}}{R_{chest}} R + V_{chest}$$

$$= V_{chest}\left(\frac{R}{R_{chest}} + 1\right)$$

$$V_c = V_{chest} \frac{1}{R_{chest}} (R + R_{chest})$$

$$V_{chest} = \frac{R_{chest}}{R + R_{chest}} V_c \quad = 2.5\ kV \frac{900\ \Omega}{7.2 \times 10^5\ \Omega + 900\ \Omega}$$

$$V_{chest} = 3.1\ V$$

The End for Today!