Modern Physics
Problem Set 14
Due: Thursday, Nov. 29 at 12:30 pm
No Regrades!

JC-61) **Pion Decay**
And pion spontaneously decays into a muon and a muon antineutrino according to the following -

$$\pi^- \rightarrow \mu^- + \bar{\nu_\mu}$$

Recent experimental evidence indicates that the mass *m* of the antineutrino is no larger than 190 keV/c^2 and may be as small as zero. In this problem, assume that the pion decays at rest in the laboratory frame and compute the energies and momentum of the muon and muon antineutrino

a) (20 points) if the mass of the antineutrino is zero.
b) (20 points) if the mass of the antineutrino is were 190 keV/c^2.

The mass of the pion is 139.56755 MeV/c^2 and the mass of the muon is 105.65839 MeV/c^2.

JC-62) **Velocity Transformation Revisited**
(20 points) In class we outlined the steps for deriving the velocity transformation in the x-direction. This transformation was given as

$$u' = \frac{u - v}{1 - \frac{vu}{c^2}}$$

Now use the *Lorentz transformations* to derive expressions for the velocity transformation in the y- and z-directions. Assume the O' frame is moving in the positive x-direction.

JC-63) **Doppler Broadening**
Before a positron and an electron annihilate, they form a sort of "atom" in which each orbits about their common center of mass with identical speeds. As a result of this motion, the photons emitted in the annihilation show a small Doppler shift. In one experiment, the Doppler shift in energy of the photons was observed to be 2.41 keV.

a) (10 points) What would be the speed of the electron or positron before the annihilation to produce this Doppler shift?
b) (10 points) The positrons form these atom-like structures with the nearly "free" electrons in a solid. Assuming the positron and the electron must have about the same speed to form this structure, find the kinetic energy of the electron.

This technique, called *Doppler broadening,* is an important method for learning about the energy of electrons in materials.

Modern Physics
Problem Set 14
Due: Thursday, Nov. 29 at 12:30 pm
No Regrades!

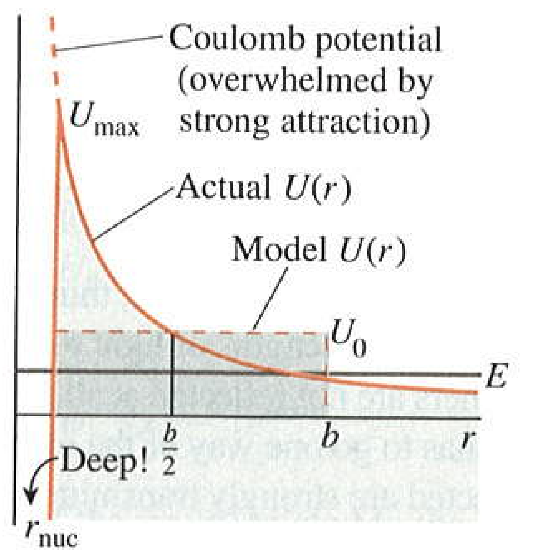
JC-64) **Review: Expectation Value and Probability**

a) (10 points) Calculate the most probable radius to find an electron in a hydrogen atom in its ground state.
b) (10 points) Calculate the radial expectation value $\langle r \rangle$ for the electron in the ground state of hydrogen.
c) (10 points) Comment on your answer.

JC-68) **Fusion in the Sun**

Without tunneling, our Sun would fail us. The source of its energy is nuclear fusion, and a crucial step is the fusion of a light-hydrogen nucleus, which is just a proton, and a heavy-hydrogen nucleus, which is one of the same charge but twice the mass. When these nuclei get close enough, their short-range attraction via the strong force overcomes their Coulomb repulsion. This allows them to stick together, resulting in a reduced total mass/internal energy and a consequent release of kinetic energy. However, the Sun's temperature is too low to ensure that nuclei move fast enough to overcome their repulsion.

a) (10 points) Show that a temperature of 10^9 K would be needed to overcome the Coulomb repulsion by equating the average thermal kinetic energy that the nuclei would have when distant, $[(3/2)k_bT]$, and the Coulomb potential energy they would have when 2 fm apart (roughly the separation at which they stick).



b) (10 points) The Sun's core temperature is only about 10^7 K. If nuclei can't make it "over the top" of the potential barrier, they must tunnel through. Consider the following model, illustrated in the figure above. One nucleus is fixed at the origin, while the other approaches from far away

with energy E. As r decreases, the Coulomb potential energy increases until the separation r is roughly the nuclear radius, r_{nuc}, whereupon the potential energy is U_{max} and then quickly drops into a very deep "hole" as the strong-force attraction takes over. Given that $U << U_{max}$, the point b, where tunneling must begin, will be very large compared with r_{nuc}, so we approximate the barrier's width L as b. Its height U_0, we approximate by the Coulomb potential evaluated at $b/2$. Finally, for the energy E, which fixes b, let us use $4 \times (3/2)k_bT$, which is a reasonable limit given the natural range of speeds in a thermodynamic system. Combining these approximations, show that the exponential factor in the wide-barrier tunneling probability is

$$\exp\left[\frac{-e^2}{(4\pi\varepsilon_0)\hbar}\sqrt{\frac{4m}{3k_B T}}\right]$$

c) (10 points) Using the proton mass for m, evaluate this factor for a temperature of 10^7 K. Then evaluate it at 3000 K (the temperature of an incadescent filament or hot flame). Discuss the consequences of your findings.