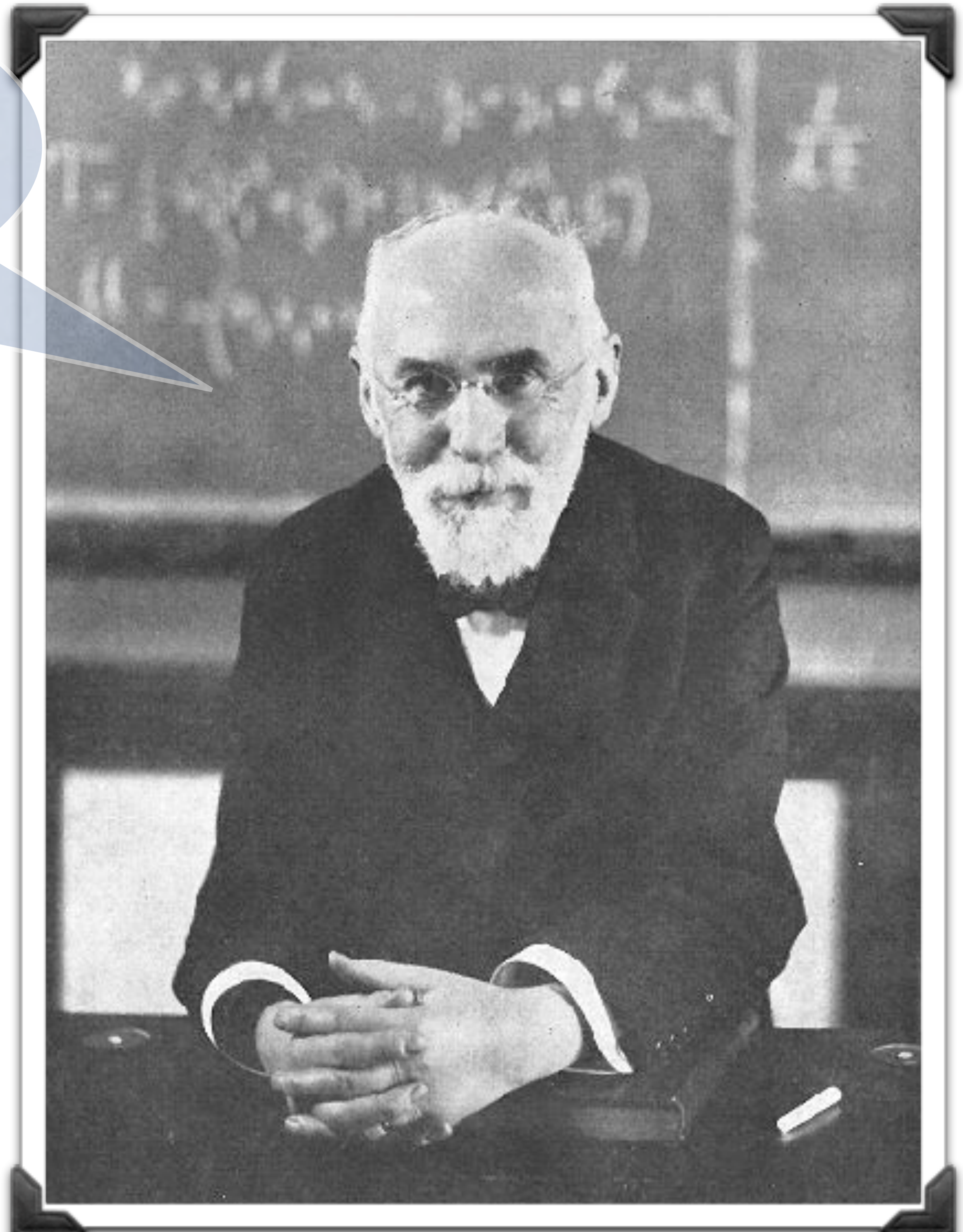
Welcome back to PHY 3305



Today's Lecture:

Applications of Lorentz Transformations

Hendrik A.Lorentz
1853-1928

ANNOUNCEMENTS

- Reading Assignment for Thursday, August 31st: Chapter 2.5-2.6 Section 2.4 (twins paradox) is optional.

- Be sure to watch the lecture video before each class!

- Homework assignment 2 is due Tuesday, September 5th at the beginning of class.

- Regrades for assignment 1 are due Tuesday, September 5th at the beginning of class.

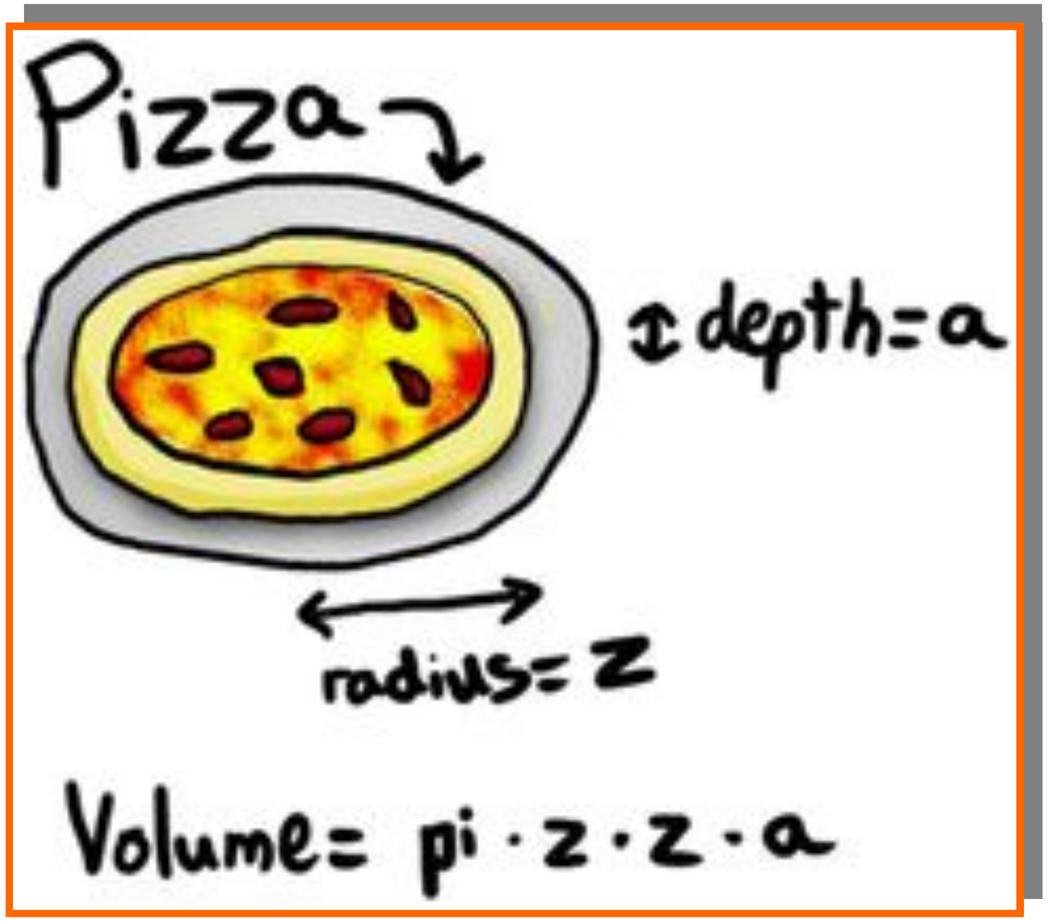
- Dr. Cooley's Office hours will be Mondays 10-11 am and Tuesdays 6 - 7 pm in FOSC 151 or by appointment.

- Mr. Thomas' Office hours will be Mondays 3-4 pm in FOSC 038A and Thursdays 2-3 pm in FOSC 060 or by appointment.

Join us for the SPS - Physics Department Pizza Social

When: 6:30 pm Thursday August 31st
Where: Heroy Hall 153

Meet current undergraduate physics majors, members of the society of physics students and physics faculty members. Learn about opportunities for undergraduate physics research.

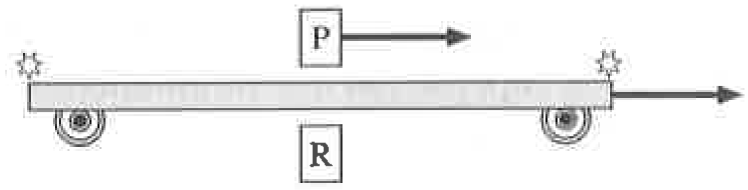


REVIEW SIMULTANEITY FROM LAST LECTURE

SIMULTANEITY

Peggy is standing at the center of her railroad car as it passes Ryan on the ground. Firecrackers attached to the ends of the car explode. A short time later, the flashes from the two explosions arrive at Peggy at the same time.

a) Were the explosions simultaneous in Peggy's reference frame? If not, which exploded first? Explain.

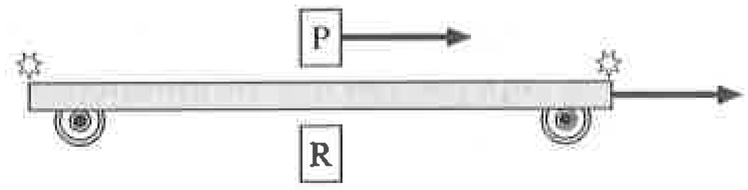


Ans: Yes. If the light arrives at Peggy at the same time, the events will be simultaneous in her frame of reference.

SIMULTANEITY

Peggy is standing at the center of her railroad car as it passes Ryan on the ground. Firecrackers attached to the ends of the car explode. A short time later, the flashes from the two explosions arrive at Peggy at the same time.

b) Were the explosions simultaneous in Ryan's reference frame? If not, which exploded first? Explain.

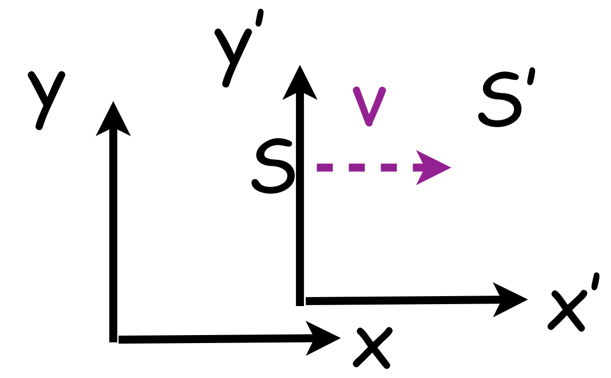


Ans: No. Ryan will see the firecracker on the left side explode before the firecracker on the right side. Ryan observes Peggy moving away from the left-side firecracker and towards the right-side firecracker. In order for the light from these firecrackers to arrive at Peggy at the same time, the left-side must have exploded first in Ryan's frame of reference.

Review from Lecture Video:

THE LORENTZ TRANSFORMATIONS

We can use γ to write our transformations.



Lorentz Factor:

$$\boxed{\gamma_\nu \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Frame S:

$$\boxed{x' = \gamma_\nu(x - vt) \quad t' = \gamma_\nu(-\frac{v}{c^2}x + t)}$$

Frame S′:

$$\boxed{x = \gamma_\nu(x' + vt') \quad t = \gamma_\nu(\frac{v}{c^2}x' + t')}$$

Review from Lecture Video:

Proper time is the time measured in the frame where all events occur in the same location **in that frame**.

$$\boxed{\Delta t = \gamma_\nu \Delta t_0}$$

t_0 is the time difference in the frame in which the events occur at the same location

Proper length is the length measured in a frame where the object being measured is at rest, so that it doesn't matter WHEN we measure the end points.

$$\boxed{L = \frac{L_0}{\gamma_v}}$$

L_0 is measured in the frame where the object is at rest.

HOW FAST ARE YOU MOVING?

How fast are you moving right now?

Depends on your relative frame of reference!!!

Relative to your seat, $v = 0$ m/s

Relative to the sun, $v = 3 \times 10^4$ m/s

Relative to the center of the galaxy, $v = 2 \times 10^5$ m/s

Relative to a cosmic-ray muon, $v = 2.94 \times 10^8$ m/s

HOW LONG IS THIS ROD?

THE MEANING OF MEASUREMENT

Consider a metal rod of unknown length, L, and a meter stick of length 200cm. What does it mean to "make a measurement" of the unknown length of the rod when...

- the rod is in your inertial reference frame (that is to say, "at rest with respect to you")?

  Relax! Take all the time you want. Align the meter stick with the rod in such a way that you can locate both ends of the rod in any time you please.

- the rod is in another inertial reference frame (that is to say, "in motion with respect to you at speed v")?

  Can't relax! Have to be clever! Have to move fast! Have to either (a) locate the two ends of the rod simultaneously or (b) wait for one end, then another, to pass the same point in space and measure the time of passage!

Questions

Consider two inertial reference frames, S and S′. Let us assign S to be "at rest" and S′ to be in motion with respect to S at a speed v along the positive x-direction. An observer in frame S′ is holding a stick, of length L, such that it lies entirely along the x′ axis.

Questions:

- In what frame would simultaneity be needed in order to measure the length of the stick?

  Frame S: since, relative to that frame, the stick is in motion, to measure its length it might be necessary to locate both of its ends simultaneously (at the same moment in time in frame S)

- In what frame do the passage of the front and back ends of the stick happen in the same spatial location?

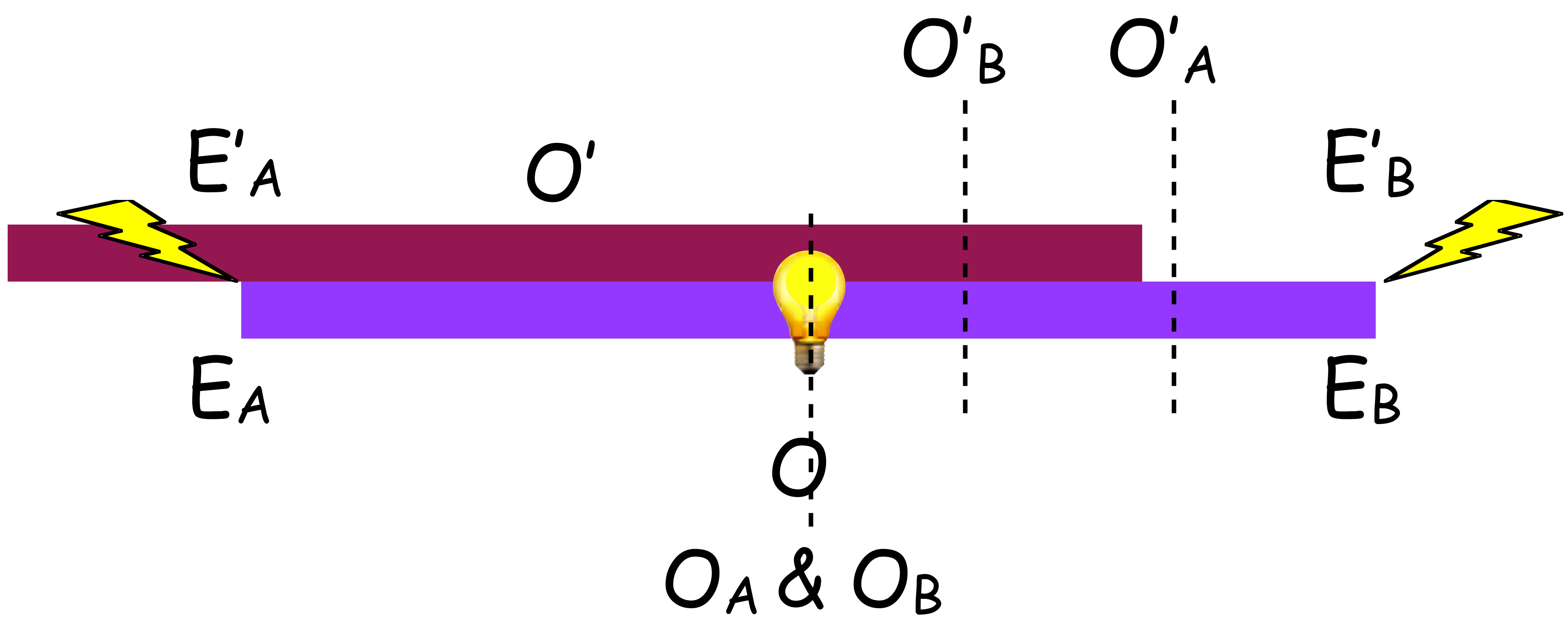
  Frame S: the ends of the stick can never be in the same spatial location in S′, but since it is moving relative to S it's possible for the front, and then later the back, of the stick to pass a common point, x, in frame S.

- What does an observer in frame S observe about the length of the stick?

  They observe the stick to have a length smaller than L, due to length contraction.

REVISIT CONSEQUENCES

At the instant Anna and Bob pass each other, lightening strikes both ends of the train. Do Anna and Bob agree on the ordering of the lightening strikes?



(Anna) O' sees E'B
(Bob) O sees EB and EA
(Anna) O' sees E'A

Anna = S'
Bob = S

LIGHTENING STRIKE

Let's say the lightening strikes are 1000 km (east, event B) and -1000 km (west, event A) from Bob and the train is moving at 0.87c. What time difference does Anna see in the strikes?

From our Lorentz transformations we know

$$t'_B - t'_A = \gamma_\nu[-\frac{v}{c^2}(x_B - x_A) + \cancelto{= 0}{(t_B - t_A)}]$$

$$t'_B - t'_A = \gamma_\nu[-\frac{v}{c^2}(x_B - x_A)]$$

$$t'_B - t'_A = \gamma_\nu[-\frac{v}{c^2}(x_B - x_A)]$$

Calculate γ_v

$$\gamma_\nu = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{0.87^2 c^2}{c^2}}} = 2$$

Substitute in v = 0.87c, x_A = -1000m and x_B = 1000 m

$$t'_B - t'_A = 2[-\frac{0.87c}{c^2}(1000m - (-1000m))]$$

$$\boxed{t'_B - t'_A = -1.2 \times 10^{-5} s}$$

Anna determines event B (east) happens before event A (west)

SLOW WAVE

Anna speeds by on a train traveling at 0.87c and she waves at Bob. To Anna, the wave happens in the same place and takes 1 second. How much time does Bob think the wave takes?

We can use our definition of proper time to solve.

$$\Delta t = \gamma_\nu \Delta t_0$$

Who's frame of reference is assigned proper time?

Anna's - The event happens in the same location in her (S') reference frame.

$$\Delta t = \gamma_\nu \Delta t_0$$

$$t_2 - t_1 = \gamma_\nu[(t'_2 - t'_1)] \quad \text{...(1)}$$

Calculate γ_ν

$$\gamma_\nu = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{0.87^2 \cancel{c^2}}{\cancel{c^2}}}} = 2$$

Substitute into (1)

$$t_2 - t_1 = 2(1s) = 2s$$

$$\boxed{t_2 - t_1 = 2s}$$

Bob concludes the wave took 2 seconds to finish.

100 METER STATION

Bob knows the length of the station to be 100 m. Anna passes by on a train traveling at 0.87c. What length does Anna find the station to be?

We can use our definition of proper length to solve.

$$L = \frac{L_0}{\gamma_\nu}$$

Who's frame of reference is assigned proper length?

Bob's - The station is a rest. Anna, in contrast, must observe both ends of the station at the same time in order to measure the length.

$$L' = \frac{L}{\gamma_\nu} \quad = \frac{100m}{2}$$

$$L = \frac{L_0}{\gamma_\nu}$$

$$\boxed{L' = 50m}$$

Alternative method - Use the Lorentz transformation. Anna will make the measurement by timing how long it takes to pass.

$$\begin{aligned} L' &= x_2' - x_1' \ = v(t_2' - t_1') \\ &= v\gamma_\nu[-\frac{v}{c^2}(x_2 - x_1) + (t_2 - t_1)] \\ &= \gamma_\nu[-\frac{v^2}{c^2}(x_2 - x_1) + v(t_2 - t_1)] \\ &= \gamma_\nu[-\frac{v^2}{c^2}(x_2 - x_1) + (x_2 - x_1)] \end{aligned}$$

$$L' = \gamma_\nu[-\frac{v^2}{c^2}(x_2 - x_1) + (x_2 - x_1)]$$

Simplify.

$$L' = \gamma_\nu[\underbrace{(x_2 - x_1)}_{= L}\underbrace{(-\frac{v^2}{c^2} + 1)}_{= \frac{1}{\gamma^2}}]$$

$$L' = \frac{L}{\gamma_\nu} = \frac{100m}{2}$$

$$\boxed{L' = 50m}$$

WILL COMMUTING MAKE YOU YOUNGER?

When you commute, time passes more slowly for you than those at rest with the surface of Earth. Assume that you commute for 1 hour each day at 75.00 mph. Skipping weekends, how much younger are you than you would otherwise be?

First calculate the gamma factor for commuting. Since the commuting speed is much smaller than light, need a trick. (Numbers are too small for your calculator to handle.)

Binomial Expansion

$$\sqrt{(1-x^2)} = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - ...$$

Now apply time dilation.

$$\Delta t = \gamma_\nu \Delta t_0$$

Which frame is assigned proper time?

Commuter frame S' - time is shorter for the commuter.

$$\Delta t' = \frac{\Delta t}{\gamma_\nu} \quad = \Delta t \times \sqrt{1 - \frac{v^2}{c^2}}$$

$$= \Delta t \times (1 - \frac{1}{2}\frac{v^2}{c^2})$$

Binomial Expansion

$$= \Delta t - \frac{1}{2}\frac{v^2}{c^2}\Delta t$$

Put it all together.

$$\Delta t' = \Delta t - \frac{1}{2}\frac{v^2}{c^2}\Delta t$$

We commute 1 hour = 3600 s per day. The second term in the equation for Δt' is the amount per day that the commuters clocks fall behind.

$$\begin{aligned}\frac{1}{2}\frac{(33.53\mathrm{m/s})^2}{(2.998\times10^8\mathrm{m/s})^2}\times 3600\mathrm{s} &= 6.25\times10^{-15}\times3600\mathrm{s}\\ &= 2.25\times10^{-11}\mathrm{s}\end{aligned}$$

Assuming that the commuter commutes 5 days a week over 45 years.

$$2.25\times10^{-11}\frac{\mathrm{s}}{\mathrm{day}}\times5\frac{\mathrm{day}}{\mathrm{week}}\times52\frac{\mathrm{week}}{\mathrm{year}}\times45\mathrm{year} = 2.63\times10^{-7}\mathrm{s}$$

The commuter is younger by ~0.3μs!

THE END
(FOR TODAY)