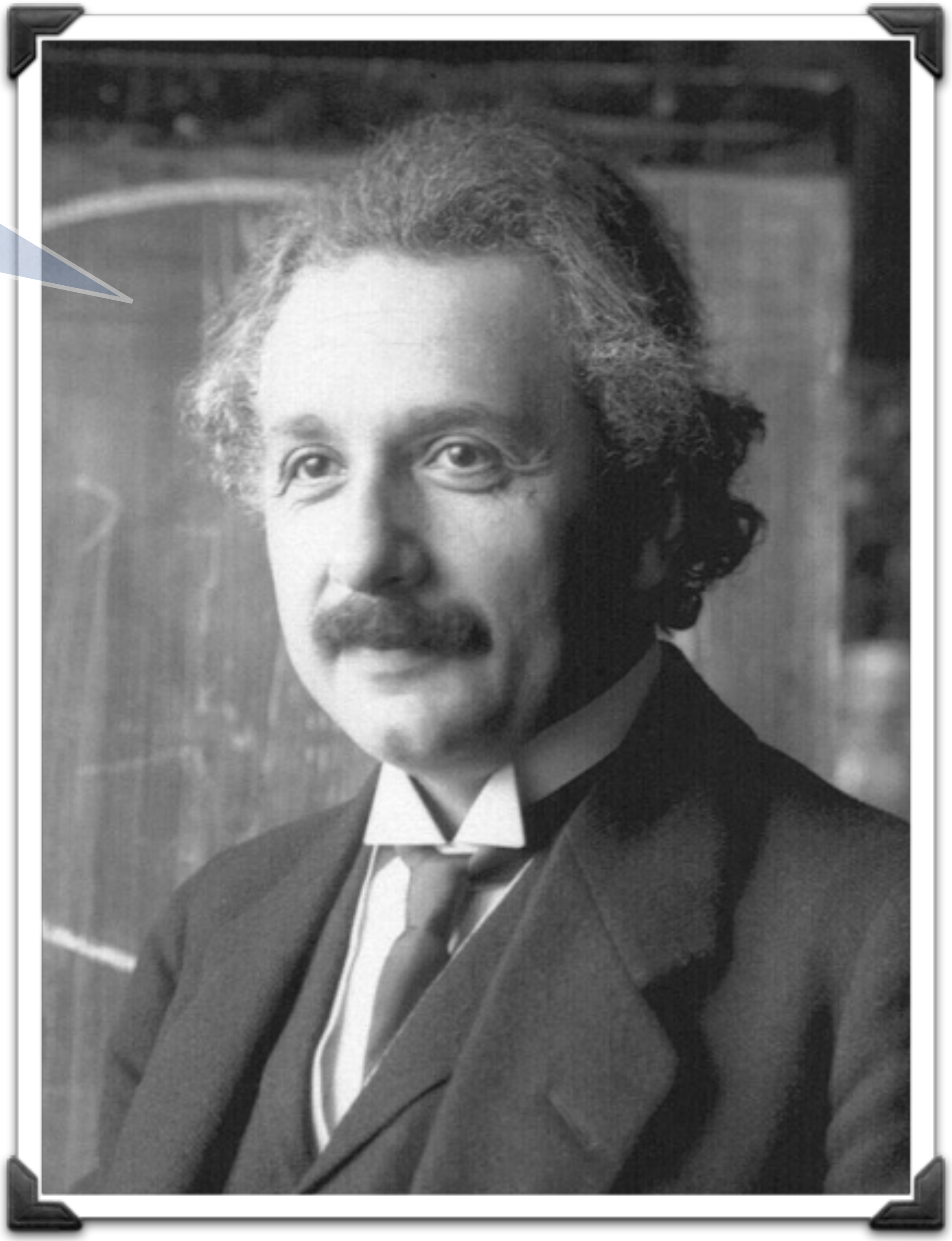
Welcome back to PHY 3305



Today's Lecture:

Consequences of Einstein's Postulates

Lorentz Transformations

Albert Einstein

1879-1955

Professor Jodi Cooley

Einstein's Postulates:

1. The laws of physics are invariant to observers in different inertial reference frames.

2. The speed of light is the same to all observers, regardless of their motion relative to the light source.

Consequence 1: Relative Simultaneity

Two events at different locations that are simultaneous in one frame of reference will not be simultaneous in a frame of reference moving relative to the first.

CONSEQUENCE 2: TIME DILATION

- Einstein reported key revolution occurred while riding a streetcar in Berlin.
  - Car pulls away from clocktower
  - Light bouncing off the clock takes longer to reach his eyes.
  - Ticks of the clock appear to happen more slowly when he moves, than when he is standing still.

NOTE: This is an optical illusion!
(But it helps you appreciate the point.)

TIME DILATION

Two events occurring at the same location in one frame will be separated by a longer time interval in a frame moving relative to the first.

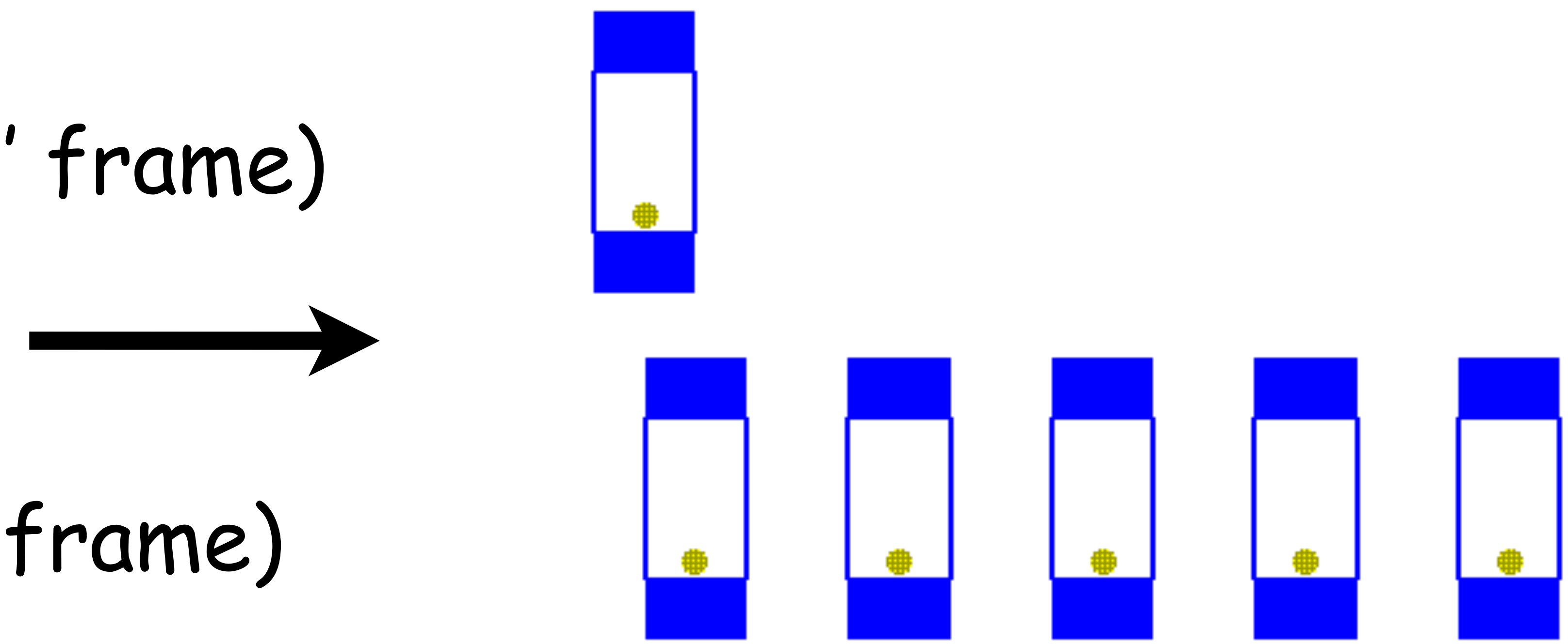
Example: Anna and Bob

Anna speeds by on a train and she waves at Bob. Bob sees Anna waving slowly. Anna contends that the wave happened very fast.

Both agree that Anna waved. However, in Anna's frame, all time is running slowly and her perceptions are affected as well.

Time Dilation (Bob's View)

Anna's (S' frame)

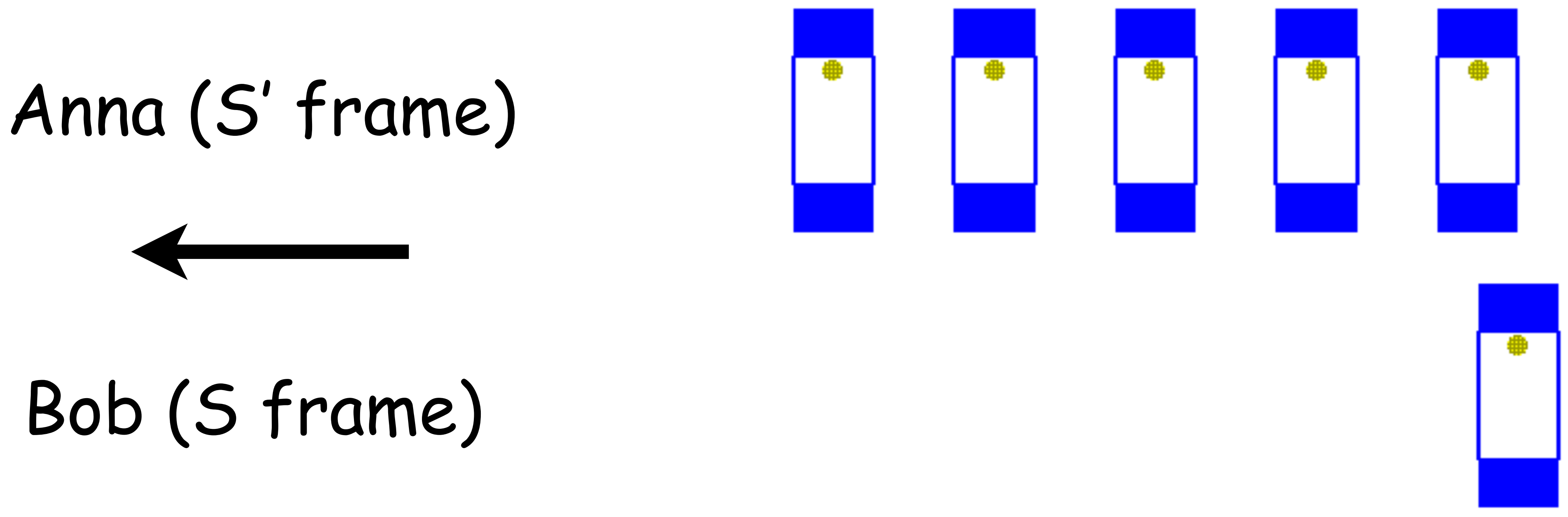


Bob's (S frame)

This is Time dilation:

A moving clock appears to tick slower to a stationary observer than a stationary clock

Time Dilation (Anna's View)



The principle of relativity requires that Anna also sees Bob's clocks going slow!

HUNDRED METER STATION

Example: Anna (S') and Bob (S)

Bob is waiting for Anna to pass on the platform at the "Hundred Meter Station" named for the length of the platform. As Anna passes, she notes how long it takes to get from one end of the platform to the other. From the conductor she gets the speed of the train. Then she calculates the length of the platform.

Will Bob and Anna agree on the length of the platform?

No!

HUNDRED METER STATION

Why do Bob and Anna disagree?

Anna's clock is running slowly. So in the time it takes for the train to pass by the station fewer seconds pass for Anna. Since, distance is

$$L' = t' \times c$$

Anna measures a shorter distance.

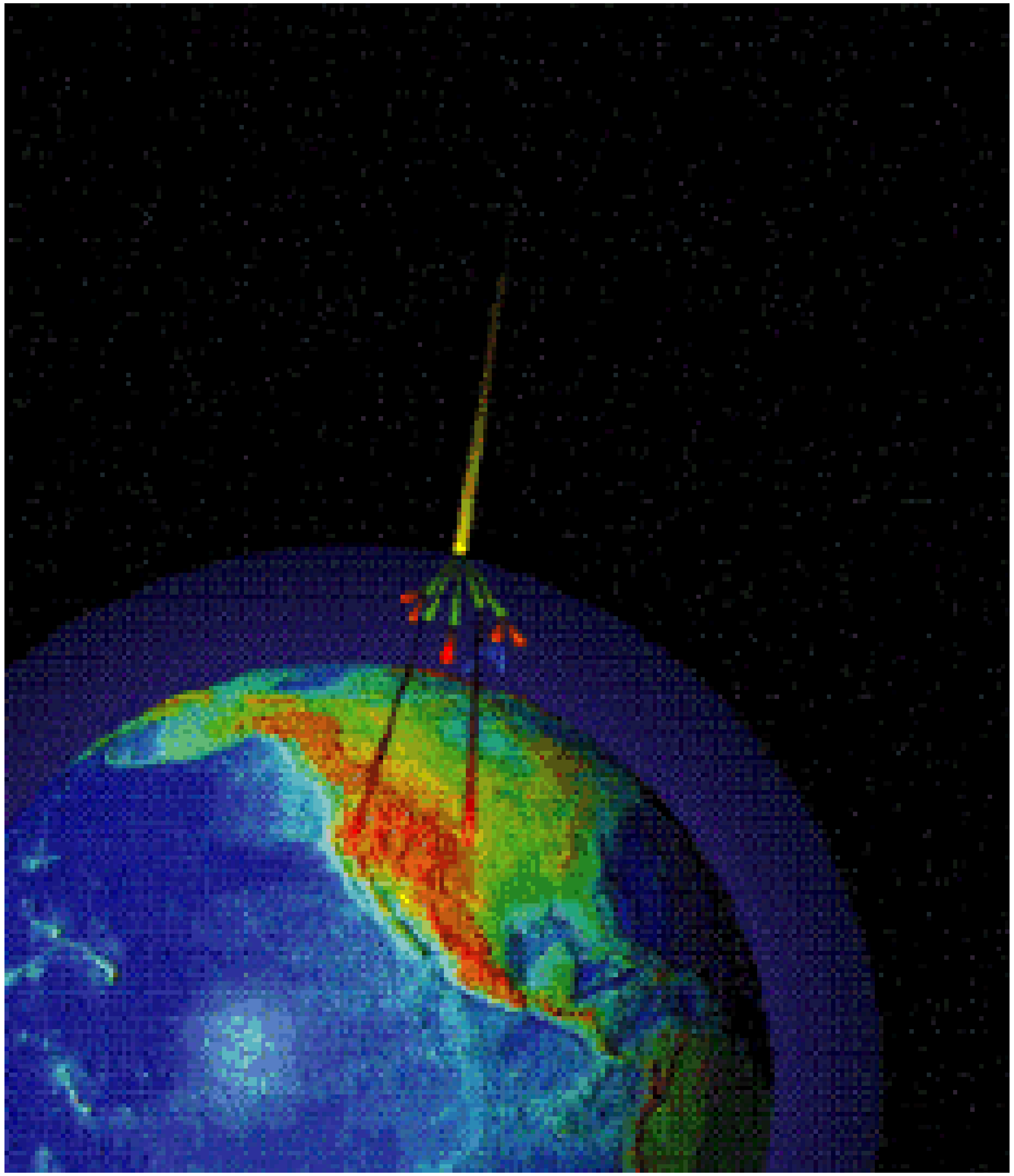
Consequence 3: Length Contraction

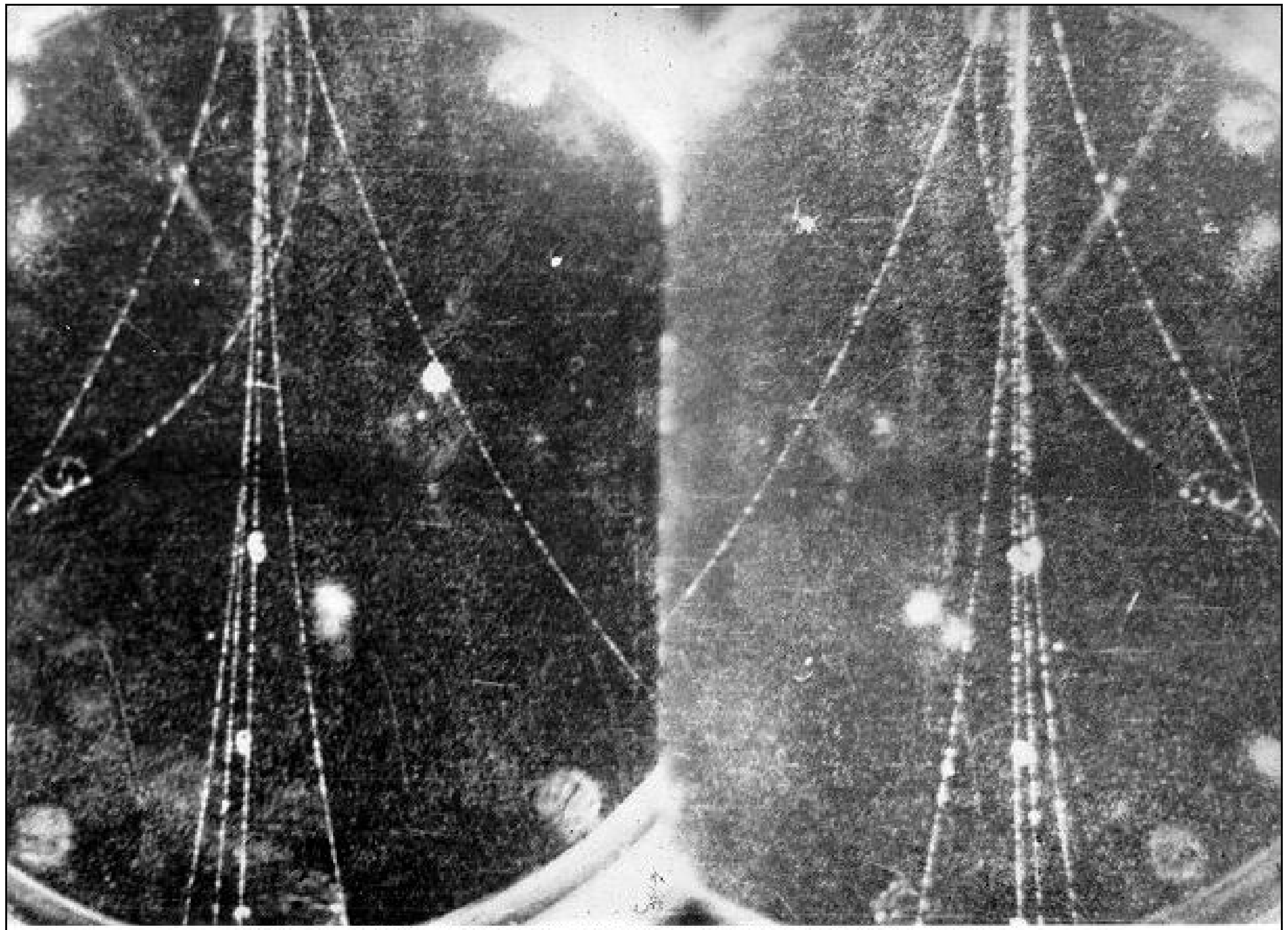
The length of an object in a frame through which the object moves is smaller than its length in the frame which it is at rest.

- or -

Length of a moving object contracts in the direction of motion.

EVIDENCE FOR RELATIVISTIC EFFECTS





Muons: subatomic particles produced by cosmic rays in the upper atmosphere.

COSMIC RAY MUONS

- Created in the upper atmosphere (10 km above ground) when protons impact the atmosphere.
- A muon's lifetime has been measured to be 2.2 μs. They travel at nearly the speed of light. How far do we expect that can they travel during their life-time?

$$d = vt$$

$$= (3 \times 10^{8}\ m/s) \times (2.2 \times 10^{-6}\ s)$$

$$\boxed{d = 726\ m}$$

This calculation tells us that muons should not make it to Earth. But we know that they do. (more later)

Special Relativity

The theory that governs physical phenomena when one object or reference frame moves relative to another at speeds comparable to that of light: $c = 3 \times 10^8$ m/s.

Einstein's Postulates:

1. The laws of physics are invariant to observers in different inertial reference frames.

2. The speed of light is the same to all observers, regardless of their motion relative to the light source.

LORENTZ TRANSFORMATION

a) Anna and Bob agree on events but disagree on the space and time patterns of those events.

b) We need a way to transform what they see that fits within the postulates of special relativity.

c) In special relativity, time is not absolute. It dooms the use of a simple Galilean transformation.

LORENTZ TRANSFORMATION

Consider the transformation between frames S and S′ of an object with no net forces moving at a constant velocity.

General relationship relating both frames.

$$x' = Ax + Bt$$
$$t' = Cx + Dt$$

Note: We require only that the transformation be linear. It is not required that time be absolute.

LORENTZ TRANSFORMATION

In each frame (S and S') the space, speed and time for the object are related.

$$x = ut \qquad \text{and} \qquad x' = u't'$$

From previous slide

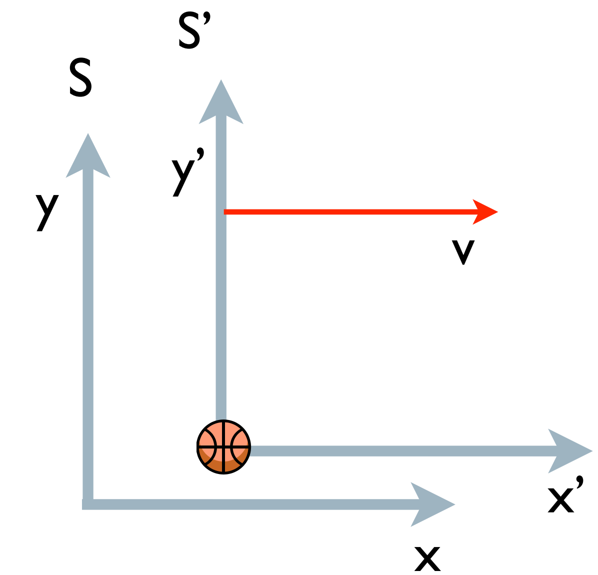
$$x' = Ax + Bt$$
$$t' = Cx + Dt$$

Need to determine A, B, C and D.

Determine the coefficients by considering 3 special cases.

CASE 1:

The object is pinned to the origin of S'.



What is the speed and position as seen by an observer in the S' frame?

$$u' = 0 \qquad x' = 0$$

What is the speed and position as seen by an observer in the S frame?

$$u = v \qquad x = vt$$

Going back to our linear relationships, we have

$$x' = Ax + Bt$$

$$0 = Av\cancel{t} + B\cancel{t}$$

$$\boxed{B = -Av} \qquad \text{...(1)}$$

Case 2: Object is pinned to the origin of S.

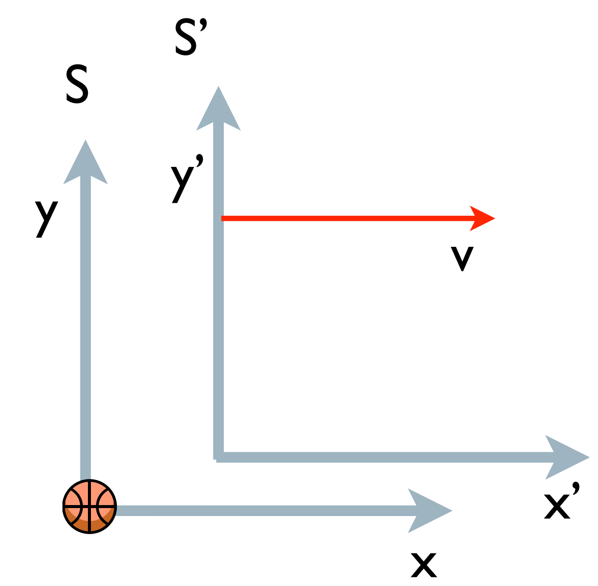
$$B = -Av$$

What is the speed and position as seen by an observer in the S frame?

$$u = 0 \qquad x = 0$$

What is the speed and position as seen by an observer in the S' frame?

$$u' = -v \qquad x' = -vt'$$



From our linear relationships

$$t' = \cancel{Cx} + Dt$$

$$t' = Dt$$

$$x' = Ax + Bt$$

$$-vt' = Ax + Bt$$

Sub t' = Dt and x = 0

$$-vDt = Bt$$

$$D = \frac{-B}{v}$$

Since,

$$B = -Av$$

We have

$$D = \frac{-B}{\frac{-B}{A}}$$

$$\boxed{D = A}$$

$$D = A$$
$$B = -Av$$

Case 3:

Assume the object is moving at the speed of light.

$$x' = ct' \qquad x = ct$$

From our linear relationships

$$t' = Cx + Dt$$
$$t' = Cct + Dt$$
$$ct' = c(Cct + Dt)$$

Recall A = D

$$ct' = Cc^2t + Act$$

$$x' = Ax + Bt$$
$$ct' = Act + Bt$$

Thus,

$$Cc^2\cancel{t} + \cancel{Act} = \cancel{Act} + B\cancel{t}$$

$$C = \frac{B}{c^2}$$

Since, B = -Av

$$\boxed{C = \frac{-Av}{c^2}}$$

$$C = \frac{-Av}{c^2} \; \& \; B = -Av$$

Step 4:

Rewrite our linear transformation.

$$x' = Ax + Bt$$

$$= Ax - Avt$$

$$\boxed{x' = A(x - vt)}$$

$$t' = Cx + Dt$$

$$= \frac{-Av}{c^2}x + At$$

$$\boxed{t' = A(\frac{-vx}{c^2} + t)}$$

In order to find A, we solve these equations for x and t.

Solve for x first.

$$x' = A(x - vt)$$

$$x - vt = \frac{x'}{A} \qquad \text{...(1)}$$

$$t' = A(\frac{-vx}{c^2} + t)$$

$$t = \frac{t'}{A} + \frac{v}{c^2}x \qquad \text{...(2)}$$

Substitute (2) into (1)

$$x - v(\frac{t'}{A} + \frac{v}{c^2}x) = \frac{x'}{A}$$

$$x - \frac{vt'}{A} - \frac{v^2}{c^2}x = \frac{x'}{A}$$

$$x(1 - \frac{v^2}{c^2}) = \frac{x'}{A} + \frac{vt'}{A}$$

$$\boxed{x = \frac{1}{A(1 - \frac{v^2}{c^2})}(x' + vt')}$$

Solve for t.

$$x = \frac{1}{A(1-\frac{v^2}{c^2})}(x' + vt')$$

$$t' = A(\frac{-vx}{c^2} + t) \;\; = A[-\frac{v(x'+vt')}{c^2 A(1-\frac{v^2}{c^2})} + t]$$

$$t = \frac{v(x'+vt')}{c^2 A(1-\frac{v^2}{c^2})} + \frac{t'}{A}$$

$$= \frac{vx'}{Ac^2(1-\frac{v^2}{c^2})} + \frac{v^2 t'}{Ac^2(1-\frac{v^2}{c^2})} + \frac{c^2(1-\frac{v^2}{c^2})t'}{Ac^2(1-\frac{v^2}{c^2})}$$

$$= \frac{1}{A(1-\frac{v^2}{c^2})}[\frac{vx'}{c^2} + \cancel{\frac{v^2 t'}{c^2}} + \frac{\cancel{c^2} t'}{\cancel{c^2}} - \cancel{\frac{v^2 t'}{c^2}}]$$

$$\boxed{t = \frac{1}{A(1-\frac{v^2}{c^2})}(\frac{vx'}{c^2} + t')}$$

Examine our equations:

$$x = \frac{1}{A(1-\frac{v^2}{c^2})}(x' + vt') \quad \& \quad t = \frac{1}{A(1-\frac{v^2}{c^2})}(\frac{vx'}{c^2} + t')$$

$$x' = A(x - vt) \qquad t' = A(\frac{-vx}{c^2} + t)$$

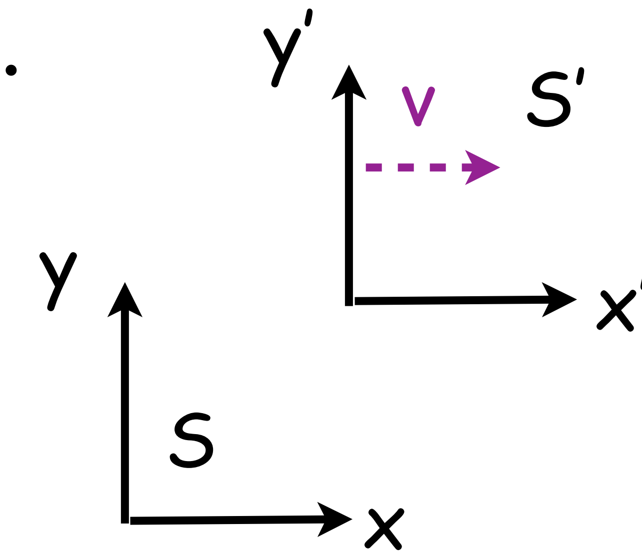
Since the two frames are in constant relative motion w.r.t each other, the **ONLY** difference is the direction each sees the other moving.

Lorentz Factor:

$$A = \frac{1}{A(1-\frac{v^2}{c^2})} \longrightarrow A = \boxed{\frac{1}{\sqrt{(1-\frac{v^2}{c^2})}} = \gamma}$$

THE LORENTZ TRANSFORMATIONS

We can use γ to write our transformations.



$$\gamma_\nu \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Frame S:

$$x' = \gamma_\nu(x - vt) \quad t' = \gamma_\nu(-\frac{v}{c^2}x + t)$$

Frame S':

$$x = \gamma_\nu(x' + vt') \quad t = \gamma_\nu(\frac{v}{c^2}x' + t')$$

What if v << c?

$$\gamma_\nu \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\gamma_\nu = 1$$

$$t' = \underbrace{\gamma_\nu}_{=1}(-\underbrace{\frac{v}{c^2}}_{=0}x + t)$$

$$t = \gamma_\nu(\frac{v}{c^2}x' + t')$$

$$t' = t$$

$$x' = \underbrace{\gamma_\nu}_{=1}(x - vt)$$

$$x = \underbrace{\gamma_\nu}_{=1}(x' + vt')$$

$$x' = x - vt$$

$$x = x' + vt'$$

We get the classical result!

$$\gamma_\nu \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Final Notes on the Lorentz Factor:

1. $v = 0$, $\gamma_v = 1$
2. $v > 0$, $\gamma_v > 1$
3. $v = c$, the gamma function is undefined (infinity). The Lorentz Transformation can not be applied.

PROPER TIME

Proper time is the time measured in the frame where all events occur in the same location **in that frame**.

$x'_1 = x'_2$

The time difference from our Lorentz Transformation would then be

$$t_2 - t_1 = \gamma_\nu(\cancel{\frac{v}{c^2}x'_2} + t'_2) - \gamma_\nu(\cancel{\frac{v}{c^2}x'_1} + t'_1)$$

$$\Delta t = \gamma_\nu \Delta t'$$

In more general terms: $\boxed{\Delta t = \gamma_\nu \Delta t_0}$

t_0 is the time difference in the frame in which the events occur at the same location

PROPER LENGTH

Proper length is the length measured in a frame where the object being measured is at rest, so that it doesn't matter WHEN we measure the end points. But in any MOVING frame, S', the ends must be established SIMULTANEOUSLY to obtain a meaningful length:

$$t'_1 = t'_2$$

The length difference from our Lorentz Transformation would then be

$$x_2 - x_1 = \gamma_v(x'_2 - x'_1 + v(\cancel{t'_2} - \cancel{t'_1}))$$

$$\Delta x = \gamma_v \Delta x' \quad \text{or} \quad \Delta x' = \frac{\Delta x}{\gamma_v}$$

In more general terms:

$$\boxed{L = \frac{L_0}{\gamma_v}}$$

L_0 is measured in the frame where the object is at rest.

THE END
(FOR TODAY)