Welcome back to PHY 3305

Today's Lecture:

Doppler Shift

Velocity Tranformations

Christian Doppler
1803-1853

ANNOUNCEMENTS

- **Office hours canceled for Monday, September 4th!**

- Reading Assignment for Tuesday, September 5: Chapter 2.7-2.9 Be sure to watch the lecture video before each class!

- Homework assignment 2 is due Tuesday, September 5th at the beginning of class.

- Regrades for assignment 1 are due Tuesday, September 5th at the beginning of class.

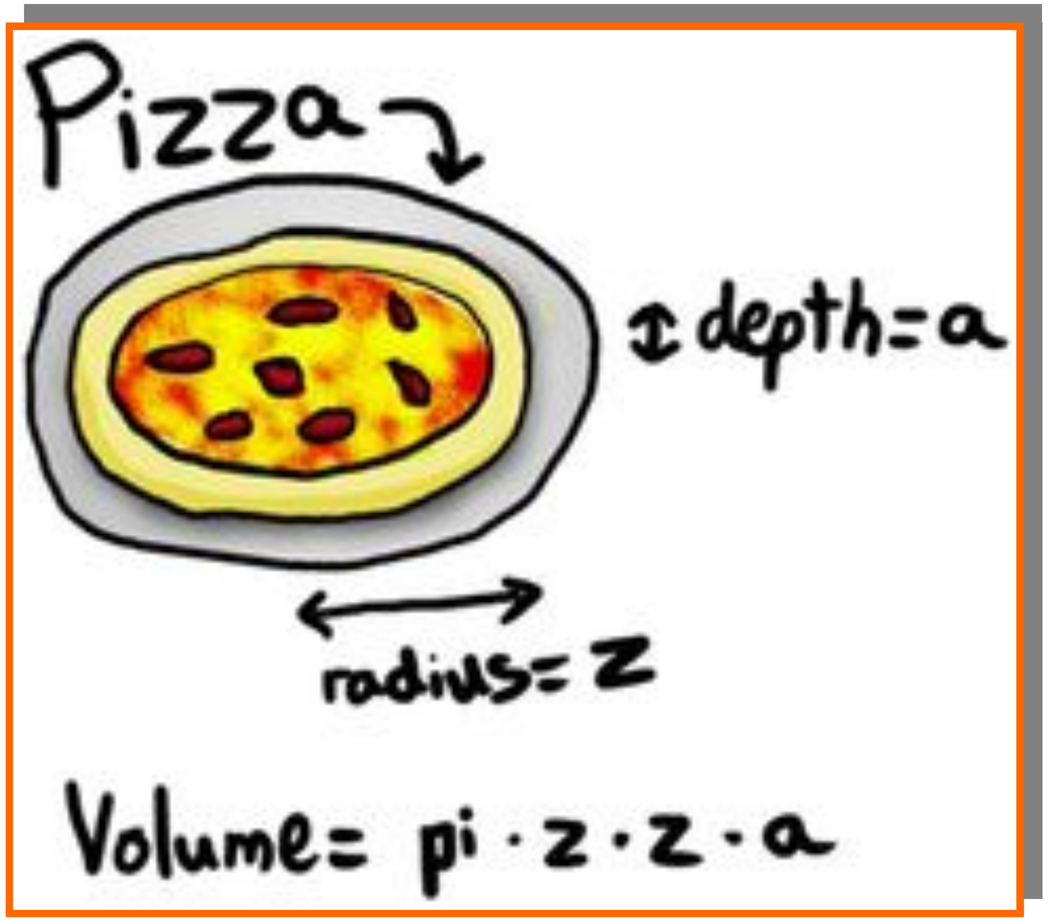
- Dr. Cooley's Office hours will be Mondays 10-11 am and Tuesdays 6 - 7 pm in FOSC 151 or by appointment.

- Mr. Thomas' Office hours will be Mondays 3-4 pm in FOSC 038A and Thursdays 2-3 pm in FOSC 060 or by appointment.

Join us for the SPS - Physics Department Pizza Social

When: 6:30 pm Thursday August 31st
Where: Heroy Hall 153

Meet current undergraduate physics majors, members of the society of physics students and physics faculty members. Learn about opportunities for undergraduate physics research.



REVIEW QUESTION 1

Molly flies her rocket past Nick at a constant velocity v. Molly and Nick both measure the time it takes the rocket from nose to tail, to pass Nick. Which of the following is true?

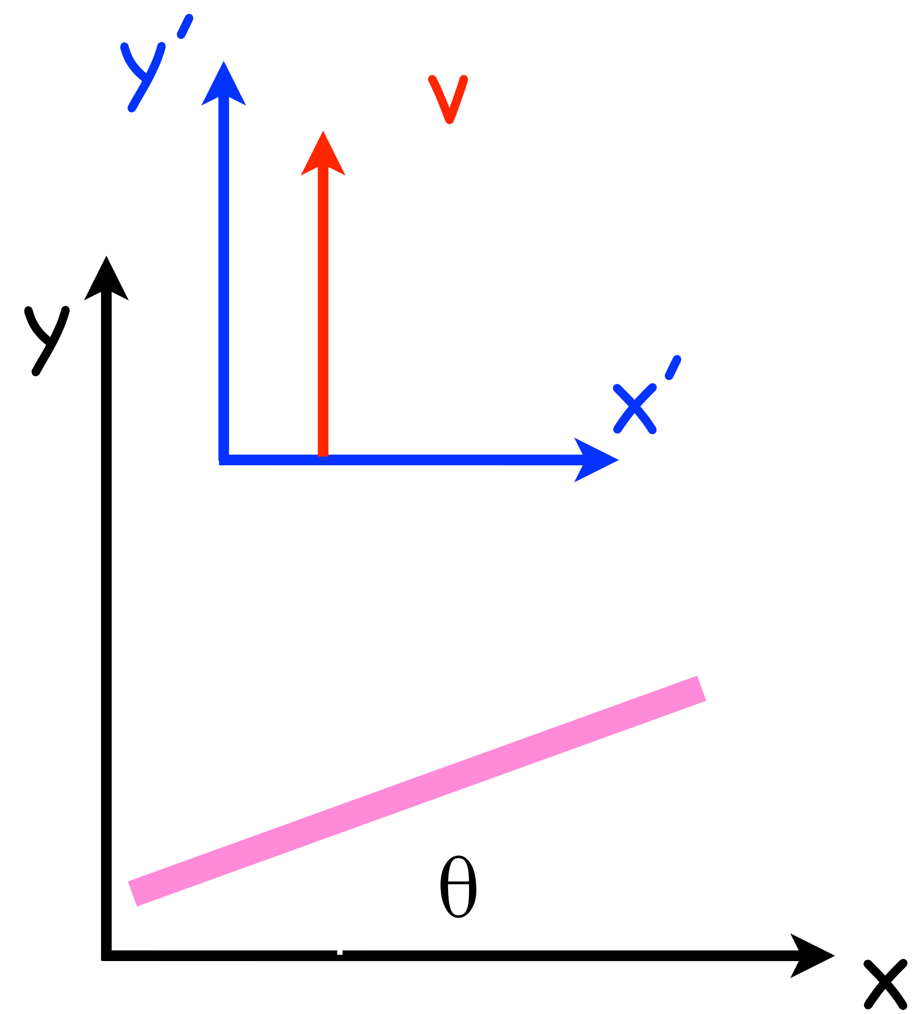
A) Both Molly and Nick measure the same amount of time.
B) Molly measures a shorter time interval than Nick.
C) Nick measures a shorter time interval than Molly.

Nick measures proper time because the "nose passes Nick" event and the "tail passes Nick" event happen at the same location ($x_1 = x_2$). Proper time is the smallest interval between the two events.

REVIEW QUESTION 2

A stick of length L lies in the x-y plane as shown in the diagram. An observer moving with velocity 0.8c in the y direction measures the length of the stick. Which of the following relations is correct regarding the components of the length as measured by the observer?

L_x	L_y
A) $< L\cos\theta$	$< L\sin\theta$
B) $< L\cos\theta$	$= L\sin\theta$
C) $> L\cos\theta$	$> L\sin\theta$
D) $> L\cos\theta$	$= L\sin\theta$
E) $= L\cos\theta$	$< L\sin\theta$



REVIEW QUESTION 3

Your friend flies from Los Angeles to New York. She carries an accurate stopwatch with her to measure the flight time. You and your assistants on the ground also measure the flight time.

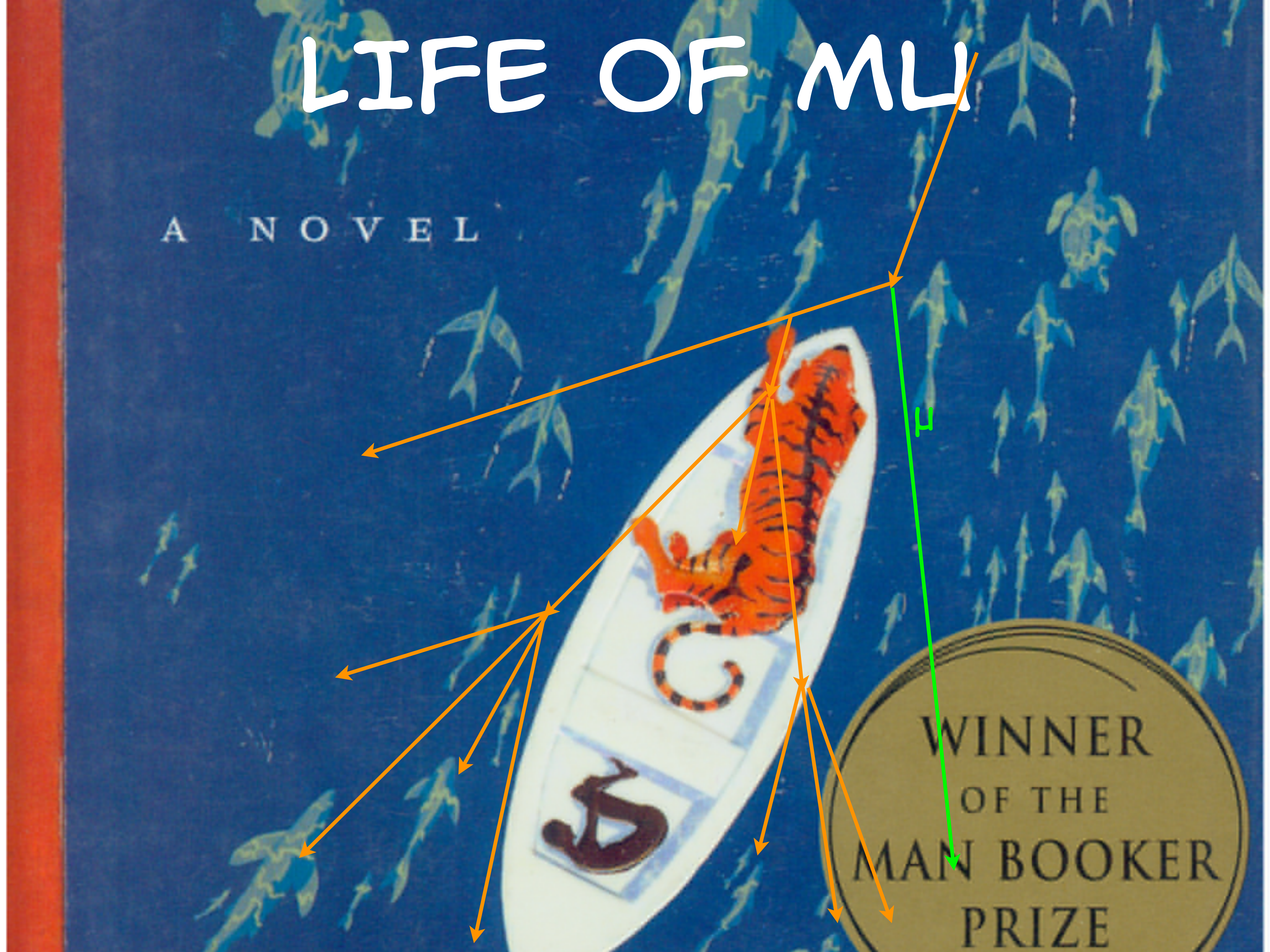
a) Identify the two events associated with the measurement.

Friend leaves Los Angeles and friend arrives in New York.

b) Who, if anyone, measures proper time?

Proper time is the time interval between two events that occur in the same location. It does so with a single clock that is present at both events. In this case, <u>your friend</u> measures proper time.

LIFE OF MU



LIFE OF A MUON

Assume that a muon is created in the atmosphere 3 km above Earth's surface, traveling downward at 0.98c. It survives 2.2 μs in its own reference frame before decaying.

Define the S reference frame as the surface of Earth with the x-direction along the path of the muon. The S′ reference frame is that of the muon.

This means that the muon is moving at +v as seen from Earth and Earth is moving at -v as seen from the muon.

Classical Physics:

1. How far will the muon travel before it decays?

$$d = vt \; = 0.98(3.00 \times 10^8 m/s)(2.2 \times 10^{-6} s)$$

$$\boxed{d = 647m}$$

The muon would decay before reaching Earth's surface.

2. How much longer would it have to live in order to reach Earth?

$$t = \frac{d}{v} \; = \frac{3000m - 647m}{0.98(3.00 \times 10^8)m/s}$$

$$\boxed{t = 8.0 \times 10^{-6} s = 8.0 \mu s}$$

$$t = \gamma_\nu(\frac{v}{c^2}x' + t')$$

Special Relativity:

1. According to an observer on Earth, how long will the muon survive before decaying?

We can approach the problem in 2 ways - Lorentz transformation equations or time dilation formula (which is derived from Lorentz transformations).

$$\Delta t = \gamma_\nu(\underbrace{\cancel{\frac{v}{c^2}\Delta x'}}_{=0} + \Delta t')$$

In the muon's frame, it is standing still. $\Delta x' = 0$.

$$\Delta t = \gamma_\nu \Delta t'$$

Note: Proper time is assigned to the S' reference frame where events occur in the same location. This agrees with our formula here. $\Delta t = \gamma_\nu \Delta t_0$

$$\Delta t = \gamma_\nu \Delta t'$$

Now just calculate. Start with the Lorentz factor.

$$\gamma_\nu = \frac{1}{\sqrt{1 - \frac{(0.98\cancel{c})^2}{\cancel{c^2}}}} = 5.03$$

Then, apply to our case.

$$\Delta t = \gamma_\nu \Delta t' = 5.03 \times (2.2 \times 10^{-6}\ \mathrm{s}\ )$$

$$\boxed{\Delta t = 11 \mu s}$$

2. Will the muon reach the surface?

$$d = vt = 0.98(3.00 \times 10^8 \frac{m}{s}) \times (11 \times 10^{-6} s) = 3.2 km$$

Yes, the muon will reach the surface.

$$x = \gamma_\nu (x' + vt')$$

3. Now, let's ask ourselves how long it takes to reach Earth from the muon's perspective

Use the Lorentz transformation equations to find Δt'.

$$\Delta x = \gamma_\nu (\underbrace{\cancel{\Delta x'}}_{= 0} + v\Delta t')$$

In the muon's frame, it sees itself standing still. Thus, Δx' = 0.

$$\Delta t' = \frac{\Delta x}{v\gamma_\nu} = \frac{3000m}{0.98 \times (3.00 \times 10^8 \frac{m}{s}) \times 5.03}$$

$$\boxed{\Delta t' = 2.0 \times 10^{-6} s = 2.0 \mu s}$$

Thus, from the perspective of the muon, it reaches Earth, penetrates Earth, and decays 0.2 μs later.

3. Suppose that we observe 10^4 muons at an altitude of 3000 m in some time interval. How many muons would we observe at sea level?

Muons decay according to the statistical law of radioactivity.

$$N(t) = N_0 e^{(-t/\tau)}$$

N_0 = original number of muons
$N(t)$ = number of muons at time t
τ = mean lifetime of the muon (a proper time interval)

The time it takes muon at an altitude of 3000 m which are traveling 0.98c to reach sea level is

$$t = \frac{L}{v} = \frac{3000}{0.98 \times 3 \times 10^8} \sim 10\mu s$$

The mean lifetime for muons is 2.2 μs. Thus, it take approximately 4.5 lifetimes to reach sea level and the number of particles to reach sea level is

$$N = N_0 e^{-t/\tau} = 10^4 e^{-4.5} = 111$$

$$\boxed{N = 111}$$

Video Lecture Review:

f = observed
f' = source

Relativistic Doppler Effect

$$f = f' \frac{\sqrt{1 - \frac{v^2}{c^2}}}{\left(1 + \frac{v}{c}\cos\theta\right)}$$

Assuming that the source is moving away/towards (not at an angle) this can be simplified (exercise for the student).

away:

$$f = f' \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$$

towards:

$$f = f' \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$

A pilot of an airplane is flying due south at a constant speed v. She observes three sources of electromagnetic waves ahead of her. Each source emits light with the same frequency f. Source A is moving due south at a speed v, source B is moving due north at a speed v, and source C is moving due south at a speed 2v. Rank the three frequencies of the observed waves (lowest to highest).

$$f_C < f_A < f_B$$

PROBLEM: RUNNING A RED LIGHT

You are driving toward a traffic light at 0.15c. A policeman next to the light sees it as red (λ = 650 nm). What color does the light appear to you?

Red light has a wavelength of 650nm. The first step is to convert this to frequency.

$$f = \frac{c}{\lambda} \quad = \frac{3 \times 10^8 \frac{m}{s}}{650 \times 10^{-9} m}$$

$$f = 4.6 \times 10^{14} Hz$$

$$f' = 4.6 \times 10^{14} Hz$$

Now plug this into the appropriate Doppler equation.

$$f = f'\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} = (4.6 \times 10^{14} Hz) \times \sqrt{\frac{1+\frac{0.15c}{c}}{1-\frac{0.15c}{c}}}$$

$$f = 5.4 \times 10^{14} Hz$$

Convert back to wavelength.

$$\lambda = \frac{c}{f} = \frac{3 \times 10^{8} \frac{m}{s}}{5.4 \times 10^{14} Hz} = 535 nm$$

You will observe the light as green.

Notes:

A modest speed of $0.15c = 4.5 \times 10^7$ m/s (~ 100×10^6 mph) causes a dramatic shift in the character of light.

Although these high speeds are not so common in our daily life here on Earth, they are common in nature.

The Doppler shift is an essential part of modern astrophysics.

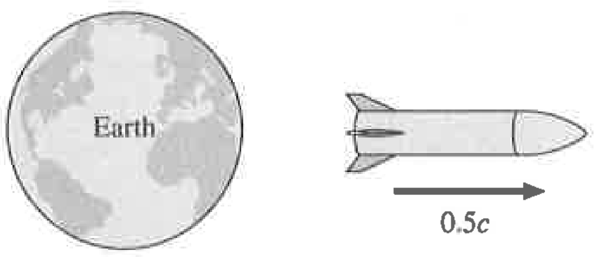
Review: Transforming Velocity

$$u = \frac{u' + v}{1 + \frac{vu'}{c^2}}$$

and

$$u' = \frac{u - v}{1 - \frac{vu}{c^2}}$$

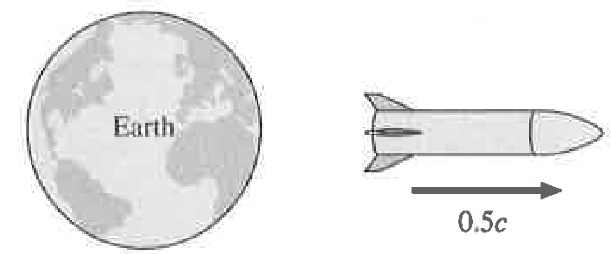
A rocket travels at speed 0.5c relative to Earth.



A) The rocket shoots a bullet in the forward direction at a speed 0.5c relative to the rocket. Is the bullet's speed relative to Earth less than, greater than or equal to c?

Less than c

A rocket travels at speed 0.5c relative to Earth.



B) The rocket shoots a second bullet in the backward direction at a speed 0.5c relative to the rocket. In Earth's frame, is the bullet moving right, moving left, or at rest? At Rest.

Object = bullet

velocity of S'-frame:

v = 0.5c

S-frame = Earth

u =?

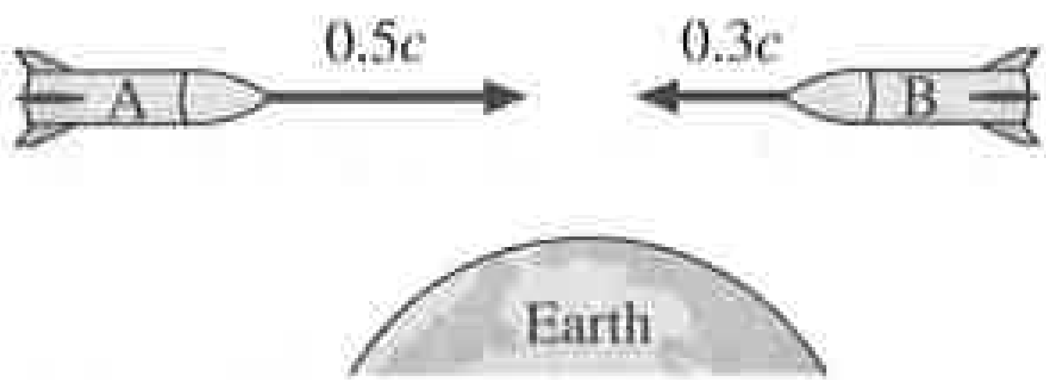
S'-frame = rocket

u' = -0.5c

Want speed of bullet in Earth frame:

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

A rocket travels at speed 0.5c relative to Earth.



C) The rocket speeds are shown relative to Earth. Is the speed of rocket A relative to rocket B greater than, less than or equal to 0.8c?

Greater Than 0.8c

Object = rocket A

velocity of S′-frame:

v = -0.3c

S-frame = Earth

u = 0.5c

S′-frame = rocket B

u′ = ?

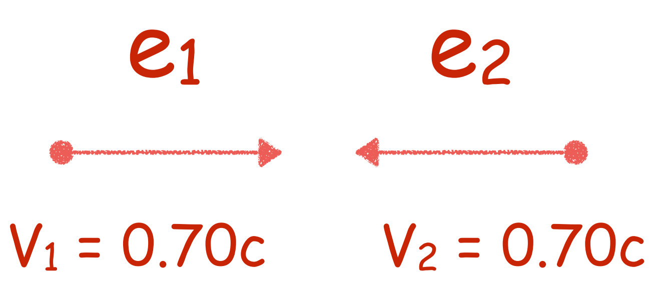
Want speed of rocket A in rocket B frame:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{0.5c - (-0.3c)}{1 - \frac{(0.5)(-0.3)}{c^2}}$$

$$u' = 0.94c$$

Two electrons move in opposite directions at 0.70c as measured in the laboratory. Determine the speed of one electron as measured from the other.

Draw a picture and break it down:



Object = electron 2

S-frame = Laboratory

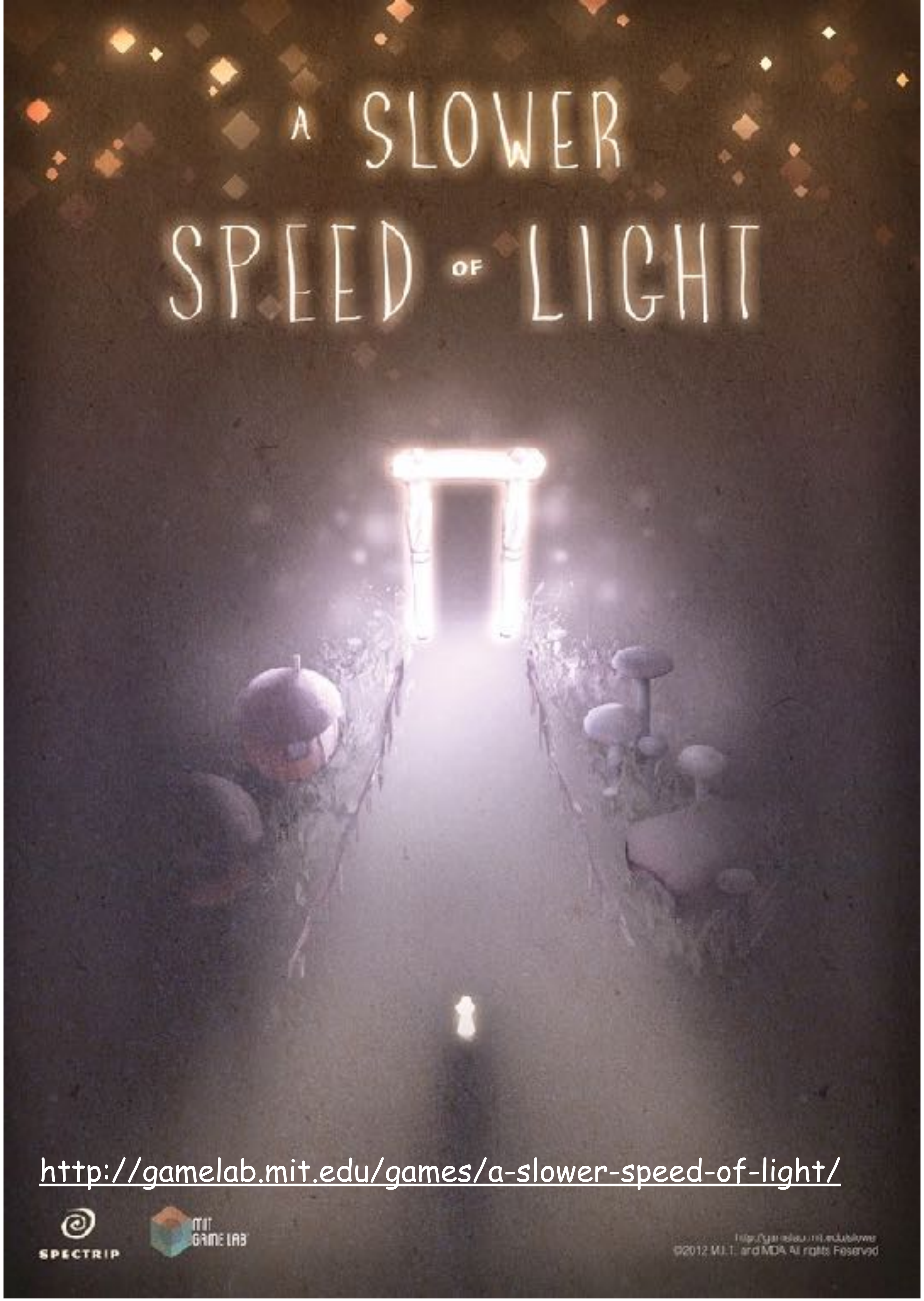
S′-frame = electron 1

This is an arbitrary choice!

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{(0.7c) - (-0.7c)}{1 - \frac{(0.7c)(-0.7c)}{c^2}} = 0.94c$$

$$\boxed{u' = 0.94c}$$

The End (for today!)



http://gamelab.mit.edu/games/a-slower-speed-of-light/