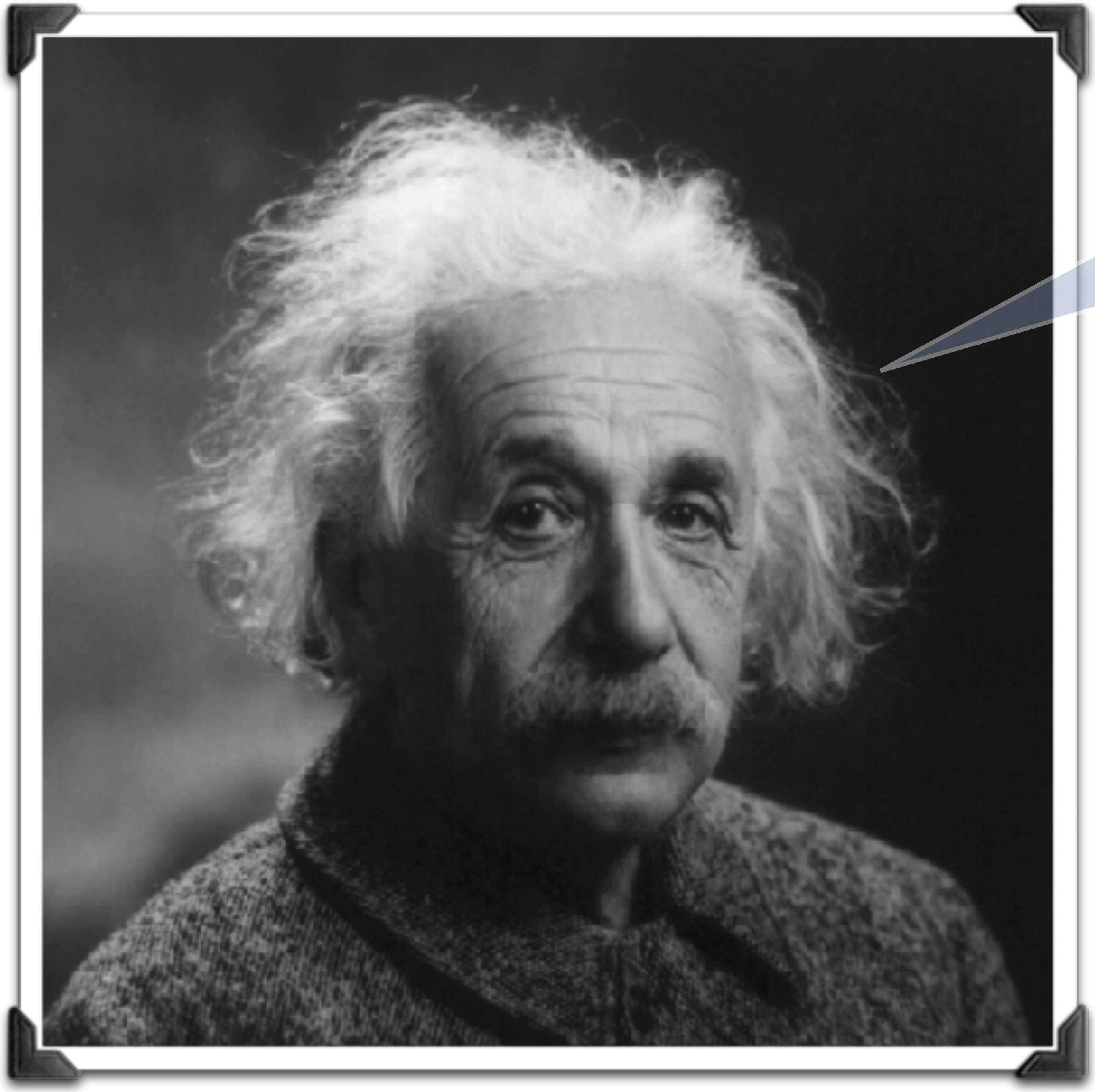
Welcome back to PHY 3305



Today's Lecture:

Applications of Energy and Momentum Conservation;

Albert Einstein
1879-1955

ANNOUNCEMENTS

- Reading Assignment for Thursday, September 7th: chapter 2, section 2.8 - 2.9. Be sure to review the example problems from this section.
- Homework assignment 3 is due Tuesday, September 12th at the beginning of class.
- Regrades for assignment 2 are due Tuesday, September 12th at the beginning of class.
- Dr. Cooley will be out of town September 14th - 17th.
  - Mr. Thomas will be in class to lead the lecture and conversation on September 14th. Be sure to watch the lecture video before class (as always!)
- Extra Credit Opportunity 1 will be due Thursday, September 28th.
- There will be no lecture video (and hence no quiz) on Tuesday, September 12th. We will be doing an in class activity that will count towards your homework grade. We will meet in FOSC 060 that day.

Two alien spaceships are traveling at 0.95c, one directly toward the Earth and one directly away from the Earth. At one instant, both spaceships happen to be the same distance from the Earth and they fire a laser at the Earth. The light from which laser reaches the Earth first according to an observer on Earth?

a) The light from the spaceship moving toward the Earth arrives first.
b) The light from the spaceship moving away from the Earth arrives first.
c) The light from the two ships arrives at the same time.
d) The observer has no way to determine which light reaches the Earth first.

REVIEW QUESTION

A car of rest length 5 m passes through a garage of rest length 4 m. Due to the relativistic length contraction, the car is only 3 m long in the garage's rest frame. There are doors on both ends of the garage, which open automatically when the front of the car reaches them and close automatically when the rear passes them. The opening or closing of each door requires a negligible amount of time.

Which of the following statements is the best response to the question: "Was the car ever inside a closed garage?"

A) No, because the car is longer than the garage in all reference frames.

B) No, because the Lorentz contraction is not a "real" effect.

C) Yes, because the car is shorter than the garage in all reference frames.

D) Yes, because the answer to the question in the garage's rest frame must apply in all reference frames.

E) There is no unique answer to the question, as the order of door openings and closings depends on the reference frame.

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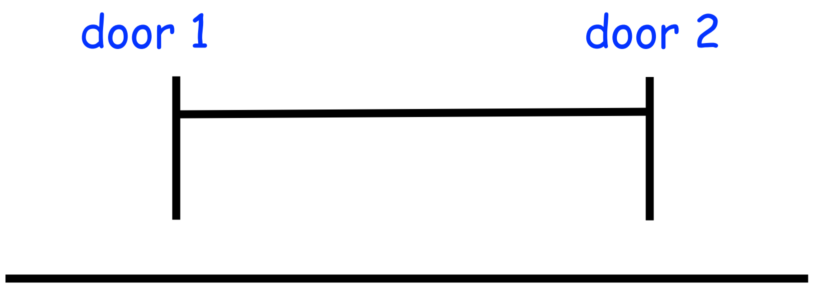
In what order does an observer in the rest frame see the garage doors open and close?



Front of car approaches door 1 and it opens,
Rear of car passes door 1 and it closes,
The car fits in the garage
Front of car approaches door 2 and it opens,
Rear of car passes door 2 and it closes.

In what order does an observer in the moving frame see the garage doors open and close?

Front of car passes door 1 and it opens,
Front of car passes door 2 and it opens,
Rear of car passes door 1 and it closes,
Rear of car passes door 2 and it closes.
The front of the car will exit the back of the garage before the rear of the car enters the front of the car.

SPEED OF QUASAR

Quasar SDSS 1030+054 produces a hydrogen emission line of wavelength λ_{rest} = 121.6nm. On Earth, this emission line is observed to have a wavelength λ_{obs} = 885.2 nm. Find the speed of the recessing quasar .

First we need to find the appropriate equation for a source which is moving away from the observer.

$$f = f'\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

$$f = f'\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

Rewrite the Doppler equation in terms of wavelength

$$f = \frac{c}{\lambda}$$

Thus,

$$\frac{c}{\lambda} = \frac{c}{\lambda'}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

$$\frac{\lambda'^2}{\lambda^2}(1+\frac{v}{c}) = 1-\frac{v}{c}$$

$$\frac{\lambda'^2}{\lambda^2}\frac{v}{c} + \frac{v}{c} = 1 - \frac{\lambda'^2}{\lambda^2}$$

$$\frac{v}{c} = \frac{1-\frac{\lambda'^2}{\lambda^2}}{1+\frac{\lambda'^2}{\lambda^2}}$$

$$v = c(\frac{1-\frac{\lambda'^2}{\lambda^2}}{1+\frac{\lambda'^2}{\lambda^2}})$$

$$v = c\left(\frac{1 - \frac{\lambda'^2}{\lambda^2}}{1 + \frac{\lambda'^2}{\lambda^2}}\right)$$

Plug in the numbers:

$$v = c\left(\frac{1 - \frac{121.6^2}{885.2^2}}{1 + \frac{121.6^2}{885.2^2}}\right)$$

$v = 0.963c$ **or** $v = 2.89 \times 10^8 \frac{m}{s}$

Review:

REDSHIFT PARAMETER

We know there is a relationship between the velocity at which a celestial object moves away from us and the change in the wavelength of the light it emits.

Astronomers use the **redshift parameter (z)** to describe this change in wavelength.

$$z \equiv \frac{\lambda_{obs} - \lambda'}{\lambda'}$$

If $z > 0$ the object is receding.

Review:

How to Measure Redshift

1. Supernova are dying stars which are composed of elements. Measure the spectral lines of the elements. Spectral lines are like color signatures of atoms or elements. (http://www.colorado.edu/physics/2000/quantumzone/)

2. Identify which spectral line was created by which element.

3. Measure the wavelength shift of any one of the lines w.r.t. its expected wavelength as measured in a laboratory on Earth.

4. Use a formula to relate the observed shift to the velocity of the object.

$$z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_0 - \lambda_E}{\lambda_E}$$

SPECTRAL LINES

A) Atoms give off waves of a certain frequency.

B) Each atom in the universe gives off a unique set of colors.

C) This set of colors is known as '**spectral lines**'.

D) **Spectroscopy** is the science of using spectral lines to figure out what atoms an object contains.

E) This technique is used to determine the composition of distant stars.

Hydrogen atoms in a laboratory can emit blue light that has a specific wavelength of 4.34×10^{-7} m. Hydrogen atoms in a distant galaxy far from Earth also emit this same light, but to an observer on Earth, the light appears to have a wavelength of 4.64×10^{-7} m. Find the redshift of this galaxy.

$$z \equiv \frac{\lambda_{obs} - \lambda'}{\lambda'} = \frac{4.64 \times 10^{-7} - 4.34 \times 10^{-7}}{4.34 \times 10^{-7}}$$

$$\boxed{z = 0.069}$$

Video Lecture Review:

Relativistic Momentum:

$$\vec{p} = \gamma m \vec{u}$$

The total energy of any reference frame:

$$E = \gamma mc^2 \longrightarrow E^2 = m^2c^4 + p^2c^2$$

Energy of a stationary object (internal energy):

$$E = mc^2$$

Kinetic energy = total energy - stationary object energy:

$$KE = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$$

If a particle has no mass, (i.e. photons) then

$$E = pc$$

Review:

$E = mc^2$ for an object at rest.

This is remarkable. It says that the total energy of a body at rest can be described by its mass. When energy is given off by an object, its mass decreases correspondingly by

$$\Delta m = \frac{\Delta E}{c^2}$$

Video Lecture:

UNITS

$$E = mc^2$$

Take the case of a proton:

$$m_p = 1.6726217 \times 10^{-27} \text{ kg}$$

Atomic mass unit:

$$u = 1.6605389 \times 10^{-27} \text{ kg} = 931.4941 \text{ MeV}/c^2$$

Notice that 1 proton = 1.007276 u

$$m_p = 1.67 \times 10^{-27} \text{ kg} = 1.007 \text{ u} = 938.3 \text{ MeV}/c^2$$

If I multiply by c^2, this is the proton's stationary energy.

Lecture Video:

ELECTRON VOLT

In particle and atomic physics the usual unit of energy is the **electronvolt (eV)**.

1eV is the energy gained by an electron accelerated through a potential difference of 1 volt
(KE = W = qV).

$$1\ \text{eV} = (1.602 \times 10^{-19}\ C)(1.000\ V) = 1.602 \times 10^{-19}\ J$$

and the usual prefixes apply

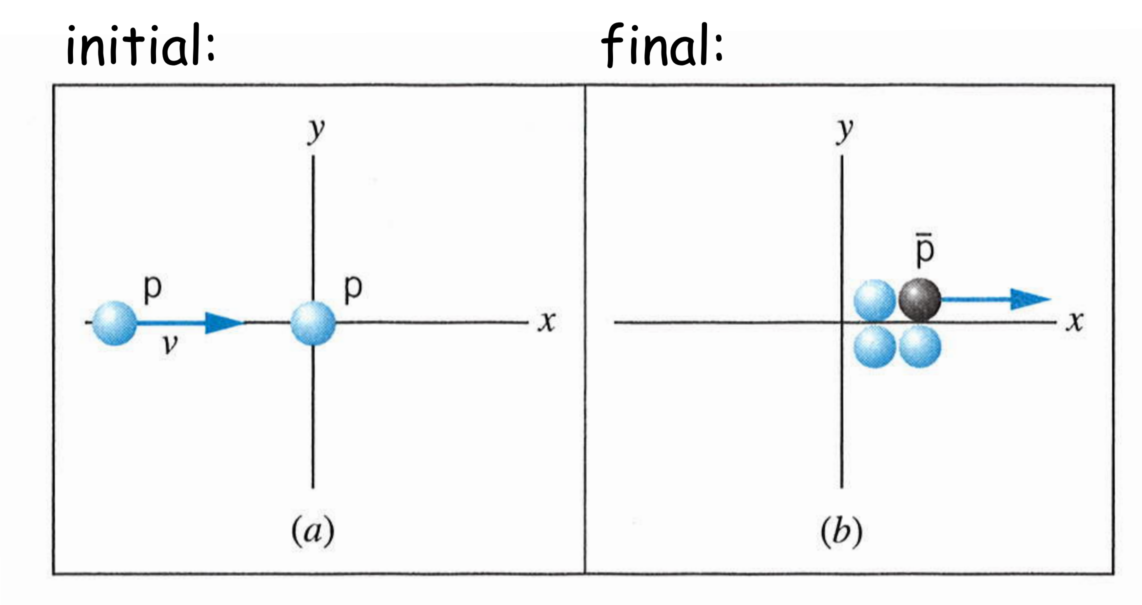
$1\ \text{MeV} = 10^6\ \text{eV}$ $\qquad$ $1\ \text{GeV} = 10^9\ \text{eV}$

DISCOVERY OF THE ANTIPROTON

The discovery of the antiproton $\bar{p}$ (a particle with the same rest energy as a proton, 938 MeV, but with opposite electric charge) took place in 1956 through the following reaction

$$p + p \longrightarrow p + p + p + \bar{p}$$

in which accelerated protons were incident on a target of protons at rest in the laboratory. The minimum incident energy needed to produce the reaction is called the threshold kinetic energy, for which all the particles move together as if they were a single unit. Find the threshold kinetic energy to produce antiprotons in this reaction.



This problem is solved by straightforward application of energy and momentum conservation.

- E_p = total energy of each particle before the collision
- p_p = total momentum of each particle before the collision
- E_p' = total energy of each particle after the collision
- p_p' = total momentum of each particle after the collision

Conservation of energy than gives

$$E_p + m_p c^2 = 4E_p'$$

proton 1 (E_p)

proton 2 at rest ($m_p c^2$)

3 protons + 1 antiproton ($4E_p'$)

$$E_p + m_p c^2 = 4E_p'$$

Conservation of momentum gives

$$p_p = 4p_p'$$

$$p_p c = 4p_p' c \quad \text{...(1)}$$

Rewrite Einstein's equation for total energy in terms of momentum.

$$E^2 = m^2c^4 + p^2c^2$$

$$pc = \sqrt{E^2 - (mc^2)^2} \quad \text{...(2)}$$

Substitute (2) into (1)

$$\sqrt{E_p^2 - (m_p c^2)^2} = 4\sqrt{E_p'^2 - (m_p c^2)^2} \quad \text{...(3)}$$

Two equations with 2 unknowns.

$$E_p + m_p c^2 = 4E_p'$$

$$\sqrt{E_p^2 - (m_p c^2)^2} = 4\sqrt{E_p'^2 - (m_p c^2)^2} \longrightarrow E_p^2 - (m_p c^2)^2 = 16(E_p'^2 - (m_p c^2)^2)$$

Solve top equation for E'.

$$E_p' = \frac{1}{4}(E_p + m_p c^2)$$

Substitute into bottom equation.

$$E_p^2 - (m_p c^2)^2 = 16(\frac{1}{16}(E_p + m_p c^2)^2 - (m_p c^2)^2)$$

$$\cancel{E_p^2} - (m_p c^2)\cancel{^2} = (\cancel{E_p^2} + (m_p c^2)\cancel{^2} + 2E\cancel{m_p c^2} - 16(m_p c^2)\cancel{^2})$$

$$2E = 14 m_p c^2$$

$$E = 7 m_p c^2$$

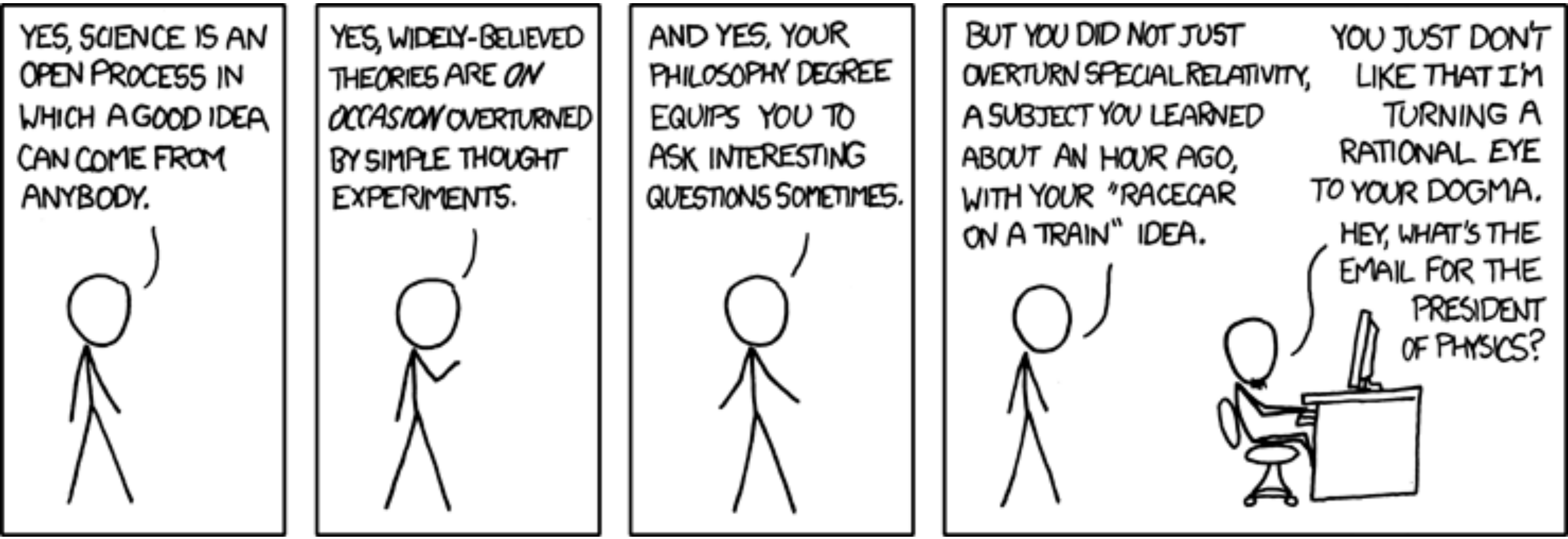
$$E = 7m_pc^2$$

Finally, from this we can calculate the kinetic energy of the incident proton.

$$\begin{aligned} KE &= E_p - m_pc^2 \\ &= 7m_pc^2 - m_pc^2 \\ &= 6m_pc^2 \\ &= 6(938\frac{MeV}{c^2})c^2 = 5628 MeV \end{aligned}$$

KE = 5.63 GeV

RELATIVITY IS REAL!



THE END
(FOR TODAY)