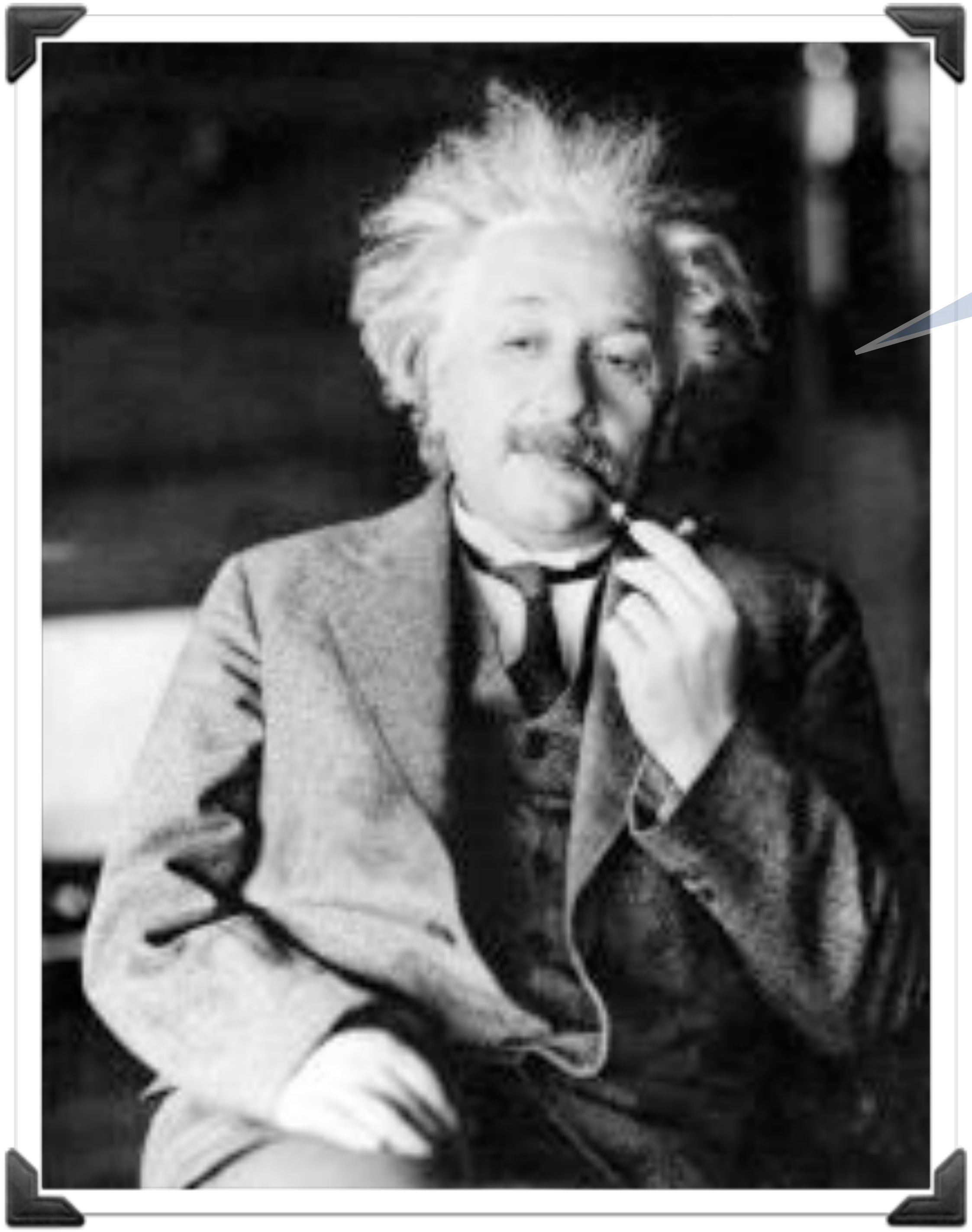


Welcome back to PHY 3305

Today's Lecture:

General Relativity

Albert Einstein
1879-1955

Professor Jodi Cooley

ANNOUNCEMENTS

- There will be no lecture video (and hence no quiz) on Tuesday, September 12th. We will be doing an in class activity that will count towards your homework grade. We will meet in FOSC 060 that day.

- Homework assignment 3 is due Tuesday, September 12th at the beginning of class.

- Regrades for assignment 2 are due Tuesday, September 12th at the beginning of class.

- Dr. Cooley will be out of town September 14th - 17th.

    - Mr. Thomas will be in class to lead the lecture and conversation on September 14th. Be sure to watch the lecture video before class (as always!)

- Extra Credit Opportunity 1 will be due Thursday, September 28th.

REVIEW QUESTION 1

An electron moves through the lab at 99% the speed of light. The lab reference frame is S and the electron's reference frame is S'. In which reference frame is the electron's rest mass larger?

A) In frame S, the lab frame.
B) In frame S', the electron's frame.
C) It is the same in both frames.

rest mass ~ stationary energy

$$m = E/c^2$$

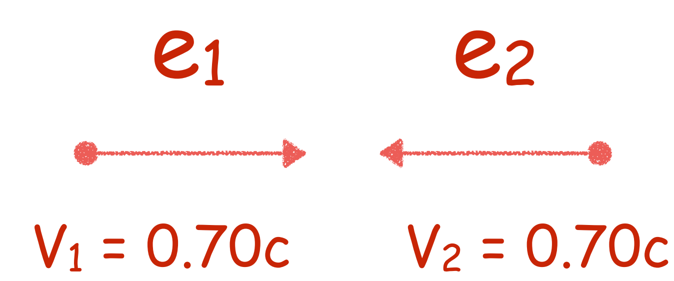
REVIEW QUESTION 2

Are the (a) kinetic energy and (b) the total energy of a 1 GeV electron more than, less than or equal to those of a 1 GeV proton?

The total energy of a 1 GeV electron equals the total energy of a 1 GeV proton. However, the rest energy of an electron is less than the rest energy of a proton. Thus, the kinetic energy of a 1 GeV electron is greater than that of the kinetic energy of a 1 GeV proton.

REVIEW QUESTION 3

Two electrons move in opposite directions at 0.70c as measured in the laboratory. Determine the speed of one electron as measured from the other.



Draw a picture and break it down:

Object = electron 2

S-frame = Laboratory

S'-frame = electron 1

This is an arbitrary choice!

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{(0.7c) - (-0.7c)}{1 - \frac{(0.7c)(-0.7c)}{c^2}} = 0.94c$$

$$\boxed{u' = 0.94c}$$

Lecture Video:

THE PRINCIPLE OF EQUIVALENCE

Einstein postulated:

A homogenous gravitational field is completely equivalent to a uniformly accelerated reference frame.

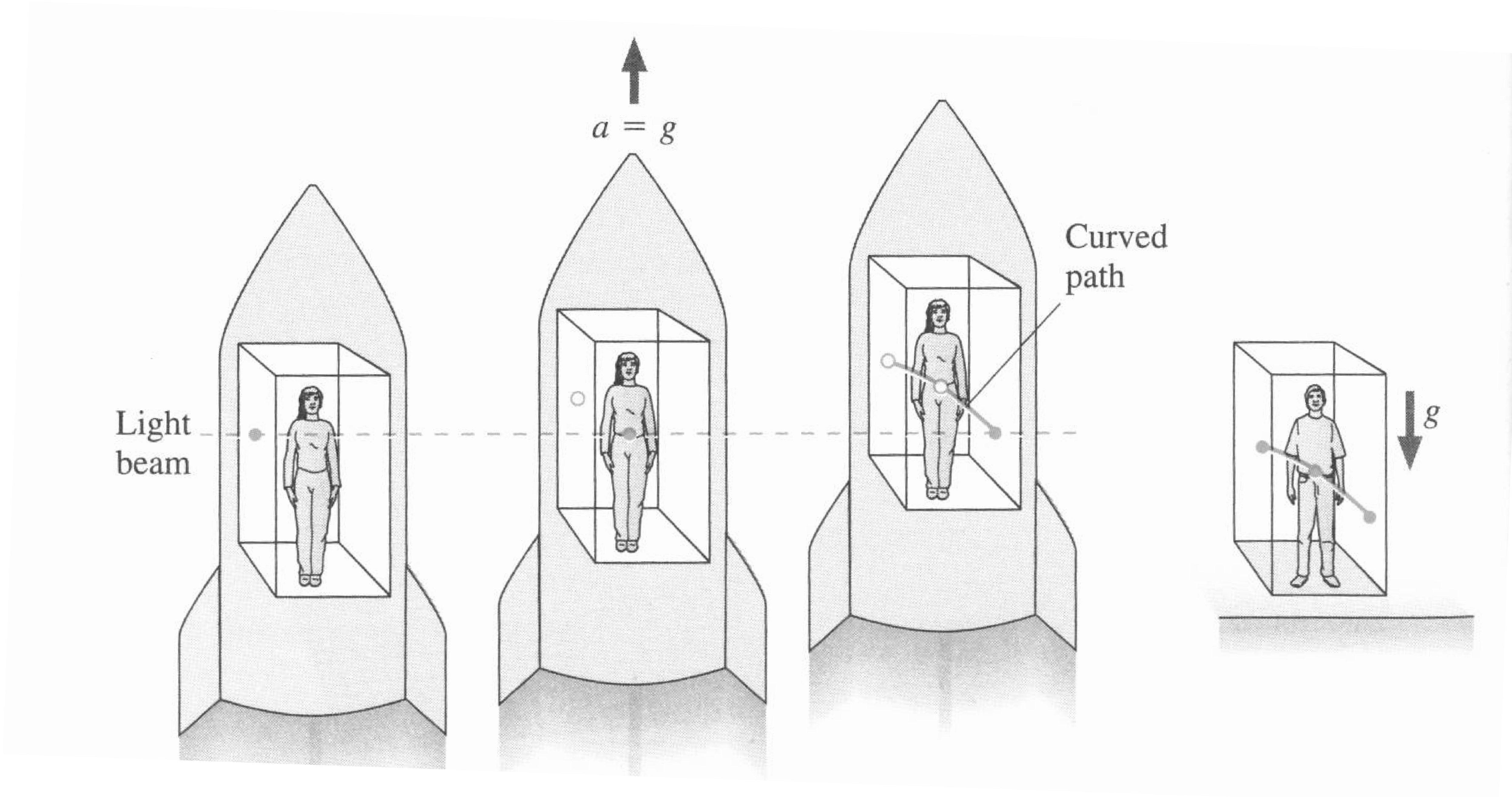
-or-

The form of each physical law is the same in all locally inertial frames.

Which of the following people will observe the same experimental results as someone who is standing on a solid lab floor on the moon?

a) An observer in a spaceship moving at constant velocity
b) An observer in an elevator on Earth moving upwards at constant velocity
c) An observer in an elevator on Earth with a broken cable (i.e. in free-fall)
d) An observer in an elevator on Earth moving downwards at constant velocity
e) An observer in a spaceship moving with constant acceleration

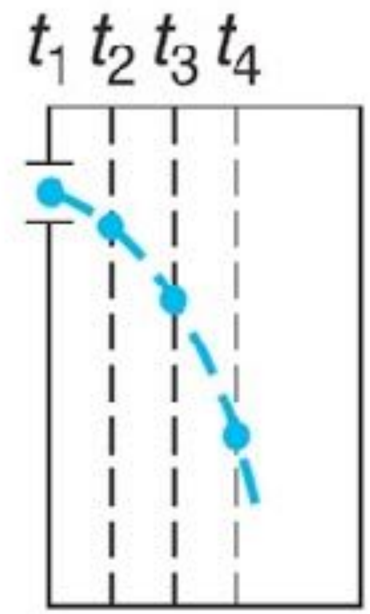
Deflection of Light in a Gravitational Field



Observing a beam of light in an accelerated reference frame.

DEFLECTION OF LIGHT IN A GRAVITATIONAL FIELD

- In an accelerating frame a light beams appears to be "falling down" in a parabolic arc

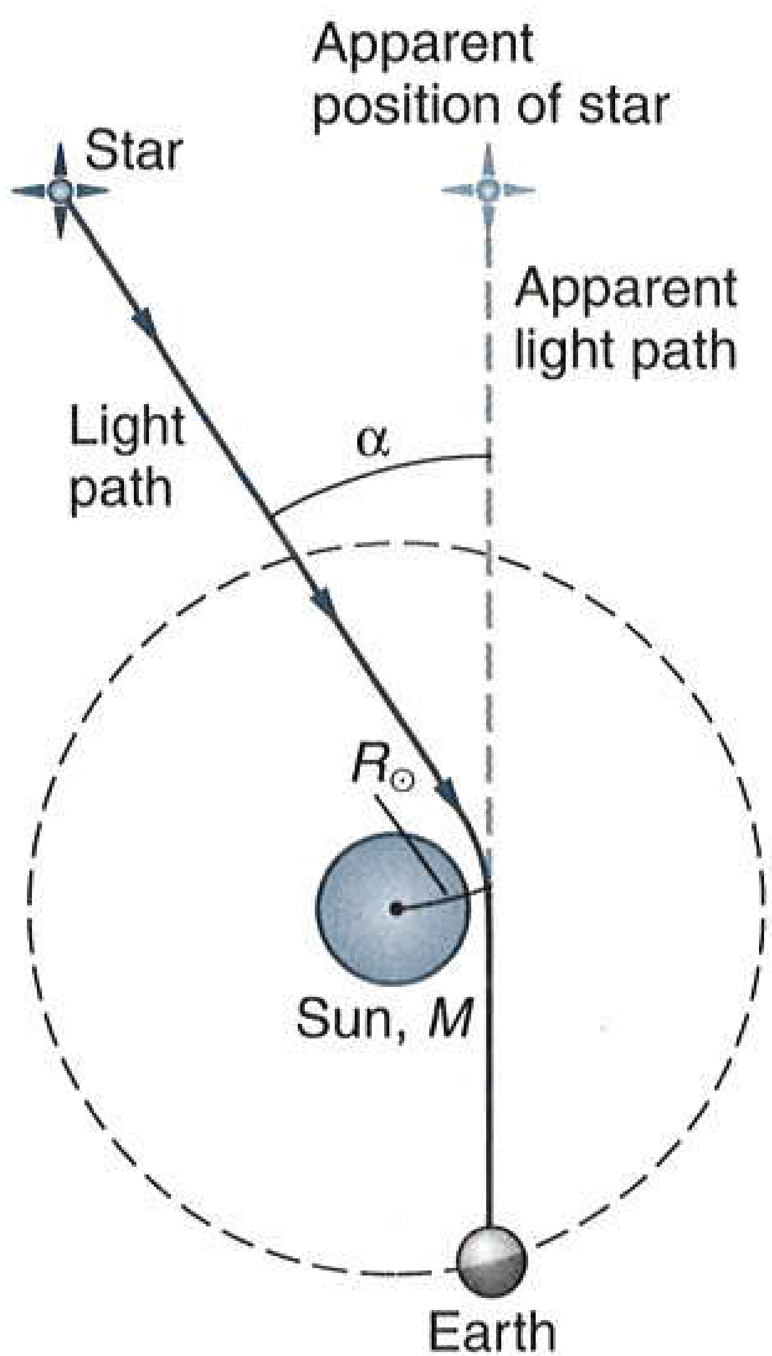


- According the the principle of equivalence: there is no physical difference between an accelerating frame and a gravitational field.

- Therefore a light beam will "fall down" in a gravitational field

Video Lecture:

Gravitational Lensing

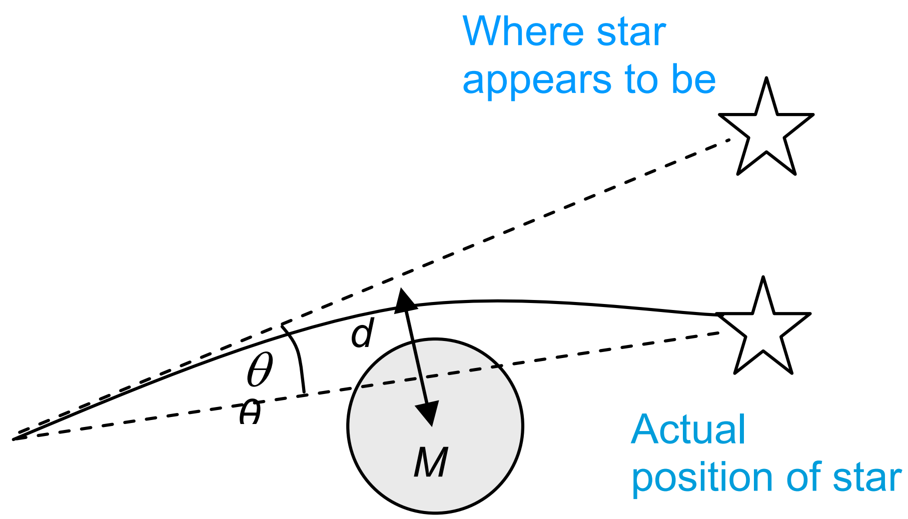


Note: This picture is an extreme exaggeration of the effect known as "Gravitational Lensing"

If you envision space-time as a fabric -- photons can not leave the fabric.

Demonstration!

Some of the best evidence that we have for the existence of dark matter comes from gravitational lensing.



$$\theta = \frac{4GM}{dc^2}$$

Calculate the deflection of light from a distant that approaches very close to the surface of the sun. The radius of the sun is 6.96 x 10^8 m.

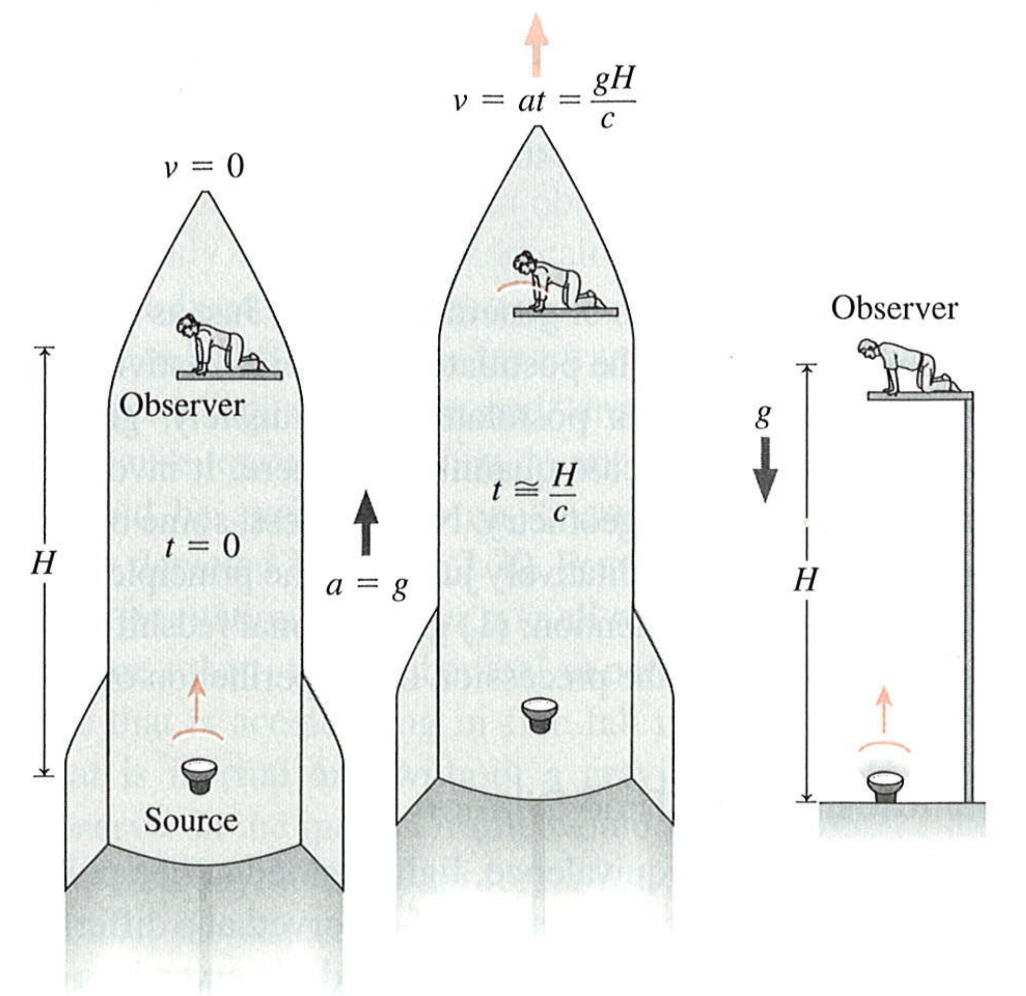
$$\theta = \frac{4(6.67\times10^{-11}\ N\cdot m^2/kg^2)(1.99\times10^{30}\ kg)}{(6.96\times10^8\ m)(3\times10^8\ m)^2} = 8.48\times10^{-6}\ rad$$

$$= 4.86\times10^{-4}\ degrees$$

Convert to degrees:

$$= 2.92\times10^{-2}\ arcmin$$

LAST TIME: Gravitational Redshift



(a) Accelerating frame

(b) Equivalent frame

$$\Delta t_{lower} = \Delta t_{higher}\left(1 - \frac{gH}{c^2}\right)$$

Relativity and GPS

$$\Delta t_{lower} = \Delta t_{higher}(1 - \frac{gH}{c^2})$$

General relativity tells us that a clock runs slower when it is deeper in a gravitational field (closer Earth). So, clocks on a satellite run faster than clocks at ground level. In Earth's non-uniform field ...

$$\Delta t_E = \Delta t_{sat}[1 - \frac{1}{c^2}(\frac{GM_E}{r_E} - \frac{GM_E}{r_{sat}})]$$

Apply the Physics

r = distance from center of Earth

A GPS satellite orbits at an altitude of 2.0×10^7 m and a speed of 3.0×10^3 m/s. The radius of Earth is 6.4×10^6 m.
a) By what fraction must the time be adjusted to account for both regular speed dependent and gravitational time dilation?

Speed dependent time dilation is given by

$$\Delta t_E = \Delta t_{sat}[1 - \frac{1}{c^2}(\frac{GM_E}{r_E} - \frac{GM_E}{r_{sat}})]$$

$$\Delta t = \gamma \Delta t_0 \longrightarrow \Delta t_E = \frac{\Delta t_{sat}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t_E = \frac{\Delta t_{sat}}{\sqrt{1 - \frac{(3.9\times 10^3 m/s)^2}{(3\times 10^8 m/s)^2}}} = (1 - 1.7 \times 10^{-10})^{-\frac{1}{2}} \Delta t_{sat}$$

Use Binomial Expansion: $\boxed{\Delta t_E \approx (1 + 8.5 \times 10^{-11})\Delta t_{sat}}$

Gravitational time dilation of the satellite is given by

$$\Delta t_E = \Delta t_{sat}[1 - \frac{1}{(3 \times 10^8 m/s)^2}(\frac{(6.67 \times 10^{14} \frac{m^3}{s^2})5.98 \times 10^{24} kg}{6.4 \times 10^6 m} - \frac{(6.67 \times 10^{14} \frac{m^3}{s^2})5.98 \times 10^{24} kg}{2.64 \times 10^7 m})]$$

$$\boxed{\Delta t_E = (1 - 5.26 \times 10^{-10})\Delta t_{sat}}$$

The gravitational effect makes the satellite's time larger. Its clock runs faster.
The speed-dependent effect makes the satellite's time smaller. Its clock run slower.

The fractional change is then:

$$\frac{\Delta t_E - \Delta t_{sat}}{\Delta t_{sat}} = (8.5 \times 10^{-11}) - (5.26 \times 10^{10})$$

$$\boxed{\frac{\Delta t_E - \Delta t_{sat}}{\Delta t_{sat}} = -4.4 \times 10^{-10}}$$

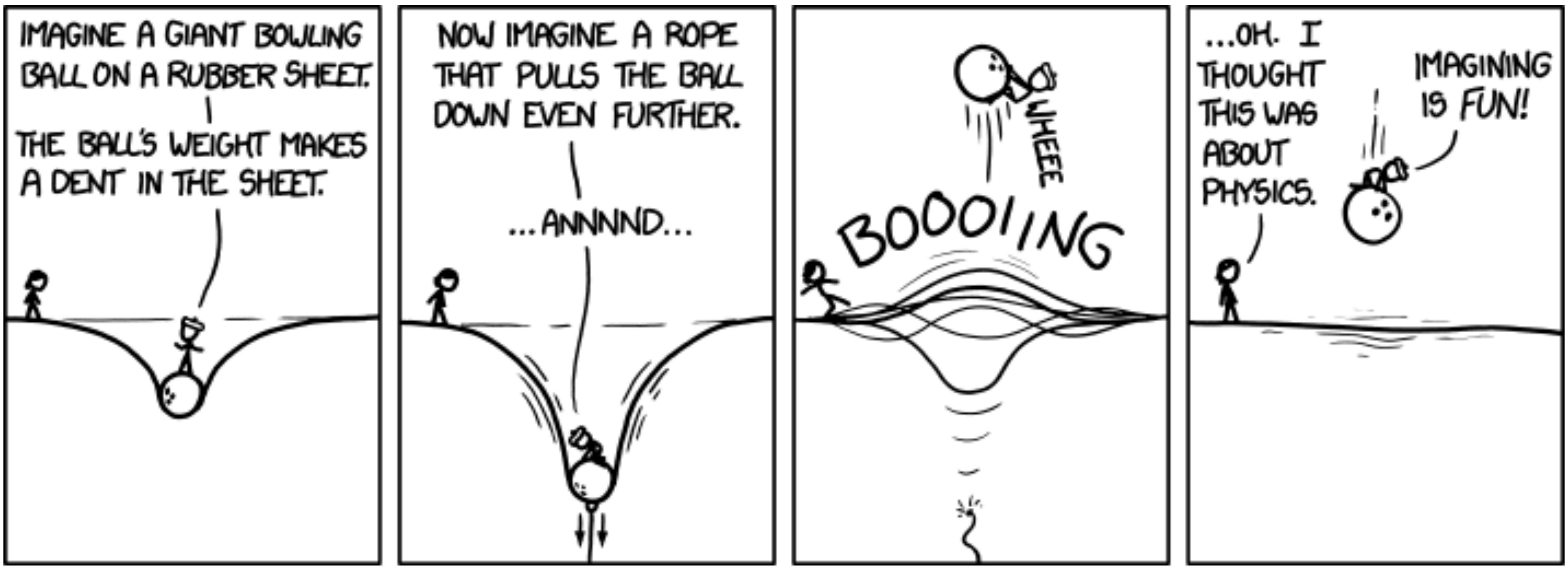
If this effect was not accounted for, how soon would the time be in error be 10 ns?

$$\frac{10 \times 10^{-9} s}{\Delta t_{sat}} = 4.4 \times 10^{-10}$$

$$\boxed{\Delta t_{sat} = 23\ s}$$

Clocks aboard GPS satellites account for the gravitational effect. Velocities and elevations of the receiver units are calculated in the receiving units.

THE END (FOR TODAY)



xkcd.com